COURSE MATERIAL
ON
PHYSICS
For
First Year B.Tech Students
(According to BPUT Syllabus)

Prepared by

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PREFACE

Engineering and technology would not exist without physics. Any technology involving energy (heat, light, sound, electromagnetic, mechanical),involves physics! All manufactured items originate in physics-based technology.

In Engineering Physics you will learn about the physics concepts that form the foundation of any mechanical design. To be an engineer, you not only have to be able to come up with the ideas, but you must also understand the physics behind those ideas so that you can design and manufacture a successful product.

This study material is designed to provide students in Pure & Applied Science who wish to study engineering or physical sciences at university with an enhanced background in order to improve their chances of success in their chosen program. The material will be presented using the normal mix of lectures and problem-solving sessions. Some useful data and information are given in the appendices.

I hope that the study material will prove helpful and will meet the needs of the students at undergraduate level.

Suggestions and criticisms for further improvement of the study material are most welcome.

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CONTENTS

PHYSICS

For 1st Semester Code (RPH1A001) For 2nd Semester Code (RPH2A001)

Module I

Oscillation & Waves (8 Hours)

Simple Harmonic Oscillation: velocity of motion, acceleration, time period, frequency, phase; damped harmonic oscillation: Wave equation of damped vibration, logarithmic decrement, quality factor, relaxation time; Forced oscillation, resonance, velocity resonance and amplitude resonance, coupled oscillation, Normal coordinates and normal frequencies, In- phase and out-of-Phase Oscillation, Concept of wave and wave equation,, reflection and transmission of longitudinal waves at boundaries.

Module II

OPTICS (10 Hours)

Concept of interference, two sources interference pattern, Bi-prism, Fringe width, uses of biprism, Newton's ring & measurement of wavelength and refractive index. Diffraction: Huygen's principle, Fresnel's Diffraction and Fraunhoffer's diffraction, Half period zone, Zone plate, construction, principle, multiple foci, comparison ofzone plate with convex lens, Fraunhoper's diffraction of Single slit, intensity distribution

Module III

LASER and Fibre Optics :(6 Hours)

Atomic excitation and energy states, Interaction of external energy with atomic energy states, Absorption, spontaneous emission and stimulated emission, Population inversion, Pumping mechanism, optical pumping, Electrical Pumping, Components of laser system, active medium, population inversion, Ruby laser, Helium-Neon laser, Semiconductor laser (basic concepts, and Engineering application only),Structure of optical fibre, Principle of propagation and numerical aperture, Acceptance angle, classification of optical fibre (Single mode and Multimode, SINand GRIN), FOCL (Fiber Optic Communication Link)

Solid State Physics (4 Hours)

Crystalline and Amorphous solid, unit cell, lattice parameter, Miller Indices, Reciprocal Lattice(Only Concept), Bragg's law, Concept of fermions and Bosons and their distribution Functions, Band theory of Solids(Qualitative),Classification of materials: metals, semiconductor and insulator in terms of band theory.

Module IV Electromagnetism (8 Hours)

(Student will be familiarized with some basic used in vector calculus prior to Development of Maxwell's electromagnetic wave equations. No proof of theorems and laws included in this unit expected- statement and interpretation should sufficient.)

Introduction; Scalar & vector fields, Gradient Of Scalar Field, divergence and curl of Vector Field, Gauss divergence theorem, Stokes theorem (Only Statements, noproof), Gauss's law of electrostatics in free space and in a medium (Onlystatements), Faraday's law of electromagnetic induction (Only statements)Displacement current, Ampere's circuital law, Maxwell's equation in Differential andIntegral form, Electromagnetic wave equation in E and, Electromagnetic Energy,Poynting theorem and Poynting vector(no derivation)

Module V

Quantum Physics: (10 Hours)

Elementary concepts of quantum physics formulation to deal with physical systems. Need for Quantum physics- historical overviews (For concept), Einstein equation, de Broglie Hypothesis of matter waves, Compton Scattering, Pair production (no derivation), Uncertainty Principle, Application of Uncertainty Principle, Non-existence of electrons in the Nucleus, Ground state energy of a harmonic oscillator.Basic Features of Quantum Mechanics: Transition from deterministic to Probabilistic,Wave function, probability density, Normalization of wave function (Simple problem),observables and operators, expectation values (Simple problem), Schrodingerequation-Time dependent and time independent equation Application: Free Particle and Particle in a box

A BRIEF INTRODUCTION OF ENGINEERING PHYSICS

Physics is the study of the mechanical universe. It is the basic science that underlies all the natural sciences. It is a search for the basic rules of the behaviour of matter and energy on every scale: from the interaction of subatomic particles, to the motion of everyday objects, to the evolution of galaxies. Physics consists of many sub-fields, including particle and nuclear physics, atomic and molecular spectroscopy, optics, solid state physics, biological and medical physics, computational physics, acoustics, astrophysics and cosmology.

Engineering physics is the study of the combined disciplines of physics, engineering and mathematics in order to develop an understanding of the interrelationships of these three disciplines. Fundamental physics is combined with problem solving and engineering skills, which then has broad applications. Career paths for Engineering physics is usually (broadly) "engineering, applied science or applied physics through research, teaching or entrepreneurial engineering". This interdisciplinary knowledge is designed for the continuous innovation occurring with technology.

Unlike traditional engineering disciplines, engineering science/physics is not necessarily confined to a particular branch of science or physics. Instead, engineering science/physics is meant to provide a more thorough grounding in applied physics for a selected specialty such as optics, quantum physics, materials science, applied mechanics, nanotechnology, micro fabrication, mechanical engineering, electrical engineering, biophysics, control theory, aerodynamics, energy, solid-state physics, etc. It is the discipline devoted to creating and optimizing engineering solutions through enhanced understanding and integrated application of mathematical, scientific, statistical, and engineering principles.

It is a bridge between pure and applied science, utilizing fundamental concepts in today's rapidly changing and highly technical engineering environment. An engineering physicist is motivated by the application of science for advancing technology and sustainability. The program emphasizes the solid foundations of modern scientific principles, mathematical rigour, technical know-how in designing, building and doing experiments, the knowledge essential for a successful professional career in science and technology. The program is recommended for students interested in newly developing areas of physics, modern technology, instrumentation, and experimentation. It also enriches a student with analytical skills of mathematics and scientific reasoning; technical skills of design, construction and operation of systems including nanotechnology, space instrumentation, particle accelerators and more; leadership skills as engineering physicists are called to manage projects involving electrical, mechanical or chemical components and tasks. They tend to be versatile and adaptable to the projects as they evolve.

Undergraduate program in engineering science focuses on the creation and use of more advanced experimental or computational techniques where standard approaches are inadequate (i.e., development of engineering solutions to contemporary problems in the physical and life sciences by applying fundamental principles).The study of Engineering Physics emphasizes the application of basic scientific principles to the design of equipment, which includes electronic and electro-mechanical systems, for use in measurements, communications, and data acquisition.

The program is recommended for students interested in newly developing areas of physics, high technology, instrumentation and communications. Our program is fully accredited by the Canadian Engineering Accreditation Board so graduates will be eligible to be certified as a professional engineer. Graduates are also qualified for entry into graduate schools in Physics or other disciplines.

Engineering Physics Educational Outcomes:

- an ability to apply knowledge of mathematics, science, and engineering.
- an ability to design and conduct experiments, as well as to analyze and interpret data.
- an ability to function on multi-disciplinary teams.
- an ability to identify, formulate, and solve engineering problems.
- an understanding of professional and ethical responsibility.
- an ability to communicate effectively.
- the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and social context.
- a recognition of the need for, and an ability to engage in life-long learning.
- a knowledge of contemporary issues.
- an ability to use techniques, skills, and modern engineering tools necessary for engineering practice.
- knowledge of fundamental physical principles and their applications.
- an ability to use the computer to solve engineering physics problems.
- knowledge and application of advanced mathematics.

Engineering Physics offers a wide range of exciting opportunities for students who are curious about the way things work and who want to use their talents to make the world a better place. Engineers are inventors and problem-solvers. They use science and technology to find faster, better, and cheaper ways of doing things. They take ideas and raw materials and design machinery and systems that increase efficiency and productivity. They develop new products to simplify household tasks. They find new energy sources and ways to protect the environment. Almost everything we use today has been designed and produced by engineers.

Discoveries by physicists, like quantum phenomena and the theory of the Big Bang, have literally transformed have literally transformed our view of the natural world. Inventions like the transistor and the laser have fuelled the modern technological revolution. We can look forward to even more exhilarating breakthroughs in the future - a future that holds exciting opportunities for the physics students of today.

MODULE I

OSCILLATIONS

Introduction

A typical example of an oscillation is provided by the simple pendulum, i.e. a mass attached to a vertical string. When the mass is displaced slightly sideways and then released, the mass will begin to oscillate. In an oscillation the motion is *repetitive*, i.e. *periodic*, and the body moves back and forth around an equilibrium position. A characteristic of oscillatory motion is the time taken to complete one full oscillation. This is the time taken to move from one extreme position of the motion and back to the same position. This is called the **period**. We will mostly be interested in those oscillations where the period stays constant, i.e. when successive oscillations take the same time to complete. Many oscillations do not share this property. For example, the leaf of a tree blowing in the wind oscillates, but its oscillations do not have a fixed period, and the amount by which the leaf moves away from its equilibrium position is not a regular function of time.

Examples of oscillations include:

- the motion of a mass at the end of a horizontal or vertical spring after the mass is displaced away from its equilibrium position;
- the motion of a ball inside a bowl after it has been displaced away from its equilibrium position at the bottom of the bowl;
- the motion of a body floating in a liquid after it has been pushed downwards and then released;
- a tight guitar string that is set in motion by plucking the string;
- the motion of a diving board as a diver prepares to dive;
- the motion of an aeroplane wing;
- the motion of a tree branch or a skyscraper under the action of the wind.

The examples mentioned above are all mechanical, but there are of course other kinds of oscillation, for example electrical. A very special periodic oscillation is called **simple harmonic motion** (**SHM**) and is the main topic of this chapter. We shall consider three examples of SHM in the main text, and some others in the example questions.

Simple harmonic motion

Oscillations are a very common phenomenon in all areas of physics. They are interesting in their own right, but they are also needed to understand many diverse phenomena, from sound to light. This chapter introduces a very special and important type of oscillatory motion, called simple harmonic motion (SHM). We discuss the case of free oscillations in detail, and qualitatively discuss the effect of damping and of an external periodic force on the oscillations.

In mechanics and physics, simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement. It can serve as a mathematical model of a variety of motions, such as the oscillation of a spring. In addition, other phenomena can be approximated by simple harmonic motion, including the motion of a simple pendulum as well as molecular vibration. Simple harmonic motion is typified by the motion of a mass on a spring when it is subject to the linear elastic restoring force given by Hooke's Law.

The motion is sinusoidal in time and demonstrates a single resonant frequency. In order for simple harmonic motion to take place, the net force of the object at the end of the pendulum must be proportional to the displacement.

Simple harmonic motion provides a basis for the characterization of more complicated motions through the techniques of Fourier analysis.

In the diagram a simple harmonic oscillator, comprising a mass attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, a restoring elastic force which obeys Hooke's law is exerted by the spring.

Mathematically, the restoring force F is given by

$$\mathbf{F} = -k\mathbf{x},$$

where F is the restoring elastic force exerted by the spring (in SI units: N), k is the spring constant ($N \bullet m^{-1}$), and x is the displacement from the equilibrium position (in m).

For any simple harmonic oscillator:

When the system is displaced from its equilibrium position, a restoring force which resembles Hooke's law tends to restore the system to equilibrium.

Once the mass is displaced from its equilibrium position, it experiences a net restoring force. As a result, it accelerates and starts going back to the equilibrium position. When the mass moves closer to the equilibrium position, the restoring force decreases. At the equilibrium position, the net restoring force vanishes. However, at x= 0, the mass has momentum because of the impulse that the restoring force has imparted. Therefore, the mass continues past the equilibrium position, compressing the spring. A net restoring force then tends to slow it down, until its velocity reaches zero, whereby it will attempt to reach equilibrium position again.

As long as the system has no energy loss, the mass will continue to oscillate. Thus, simple harmonic motion is a type of periodic motion.

Dynamics of simple harmonic motion

For one-dimensional simple harmonic motion, the equation of motion, which is a second-order linear ordinary differential equation with constant coefficients, could be obtained by means of Newton's second law and Hooke's law.

$$F_{net} = m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -kx,$$

where m is the inertial mass of the oscillating body, x is its displacement from the equilibrium (or mean) position, and k is the spring constant. Therefore,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\left(\frac{k}{m}\right)x,$$

Solving the differential equation above, a solution which is a sinusoidal function is obtained.

$$x(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t) = A \cos(\omega t - \varphi),$$

where

$$\omega = \sqrt{\frac{k}{m}},$$
$$A = \sqrt{{c_1}^2 + {c_2}^2},$$
$$\tan\varphi = \left(\frac{c_2}{c_1}\right),$$

In the solution, c_1 and c_2 are two constants determined by the initial conditions, and the origin is set to be the equilibrium position. Each of these constants carries a physical meaning of the motion: A is the amplitude (maximum displacement from the equilibrium position), $\omega = 2\pi f$ is the angular frequency, and φ is the phase.

Using the techniques of differential calculus, the velocity and acceleration as a function of time can be found:

$$v(t) = \frac{\mathrm{d}x}{\mathrm{d}t} = -A\omega \sin(\omega t - \varphi),$$
$$a(t) = \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -A\omega^2 \cos(\omega t - \varphi).$$

Maximum acceleration = $A\omega^2$ (at extreme points)

Acceleration can also be expressed as a function of displacement: $a(x) = -\omega^2 x.$

Then since $\omega = 2\pi f$,

Frequency $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}},$ is

and, since $T = 1/f$ where T is the time period, $T = 2\pi\sqrt{\frac{m}{k}}.$

These equations demonstrate that the simple harmonic motion is isochronous (the period and frequency are independent of the amplitude and the initial phase of the motion).

Energy of simple harmonic motion

The kinetic energy K of the system at time t is

$$K(t) = \frac{1}{2}mv^2(t) = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t - \varphi) = \frac{1}{2}kA^2 \sin^2(\omega t - \varphi),$$

and the potential energy is

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kA^2 \cos^2(\omega t - \varphi).$$

The total mechanical energy of the system therefore has the constant value

$$E = K + U = \frac{1}{2}kA^2.$$

Examples

An undamped spring–mass system undergoes simple harmonic motion.

The following physical systems are some examples of simple harmonic oscillator.

Mass on a spring

A mass *m* attached to a spring of spring constant *k* exhibits simple harmonic motion in closed space. The equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

shows that the period of oscillation is independent of both the amplitude and gravitational acceleration

Uniform circular motion

Simple harmonic motion can in some cases be considered to be the one-dimensional projection of uniform circular motion. If an object moves with angular speed ω around a circle of radius r centered at the origin of the x-y plane, then its motion along each coordinate is simple harmonic motion with amplitude r and angular frequency ω.

Mass on a simple pendulum

The motion of an undamped pendulum approximates to simple harmonic motion if the amplitude is very small relative to the length of the rod.

In the small-angle approximation, the motion of a simple pendulum is approximated by simple harmonic motion. The period of a mass attached to a pendulum of length ℓ with gravitational acceleration g is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

This shows that the period of oscillation is independent of the amplitude and mass of the pendulum but not the acceleration due to gravity (g), therefore a pendulum of the same length on the Moon would swing more slowly due to the Moon's lower gravitational field strength.

This approximation is accurate only in small angles because of the expression for angular acceleration α being proportional to the sine of position:

$$mg\ell\sin(\theta) = I\alpha,$$

where I is the moment of inertia. When θ is small, sin θ ≈ θ and therefore the expression becomes

$$-mg\ell\theta = I\alpha$$

which makes angular acceleration directly proportional to θ, satisfying the definition of simple harmonic motion.

Damped Harmonic Oscillation

Damping

The SHM described above is unrealistic in that we have completely ignored frictional and other resistance forces. The effect of these forces on an oscillating system is that the oscillations will eventually stop and the energy of the system will be dissipated mainly as thermal energy to the environment and the system itself. Oscillations taking place in the presence of resistance forces are called damped oscillations. The behaviour of the system depends on the degree of damping. We may distinguish three distinct cases: under-damping, critical damping and over-damping.

Under-damping

Whenever the resistance forces are small, the system will continue to oscillate but with a frequency that is somewhat smaller than that in the absence of damping. The amplitude gradually decreases until it approaches zero and the oscillations stop. The amplitude decreases exponentially. Typical examples of under-damped SHM are shown in Figure. The case represented by (b) corresponds to heavier damping than (a) and the oscillations die out faster. Note that the period of oscillation in the case of the heavier damping (b) is larger than that in the case of lighter damping (a).

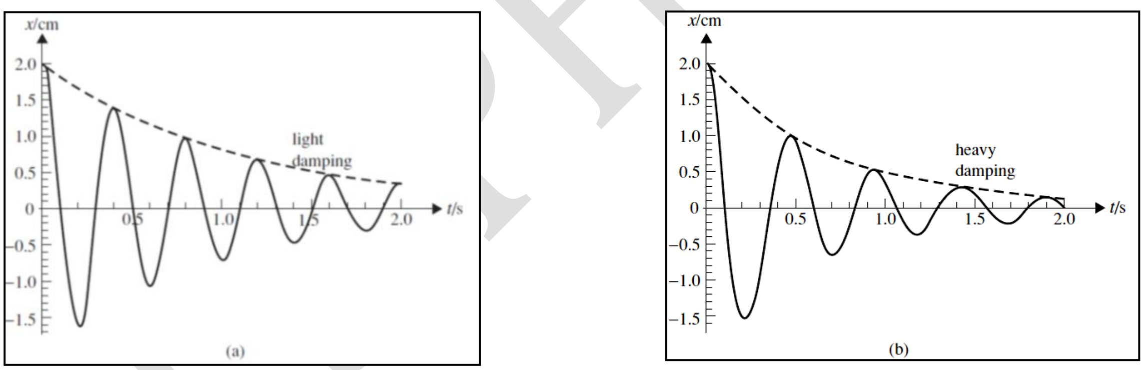
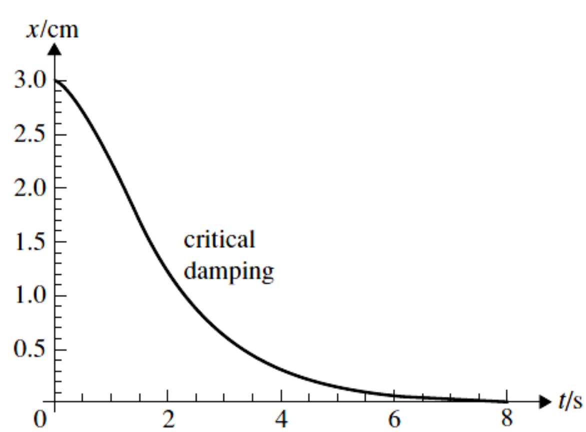


Figure Graphs showing the variation with time of the displacement of a particle in damped SHM. The curve in (b) corresponds to heavier damping than in (a), and has a slightly longer period.

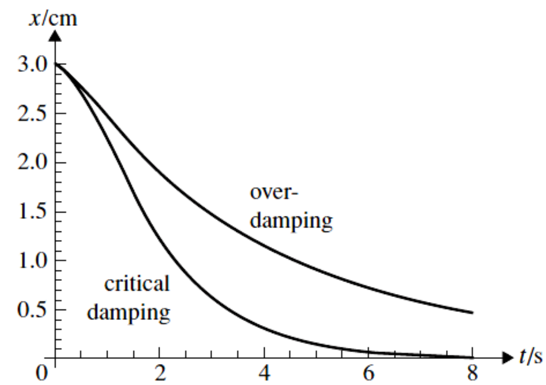
Critical damping

In this case the amount of damping is large enough that the system returns to its equilibrium state as fast as possible *without* performing oscillations. A typical case of critical damping is shown in Figure.



Over-damping

In this case the degree of damping is so great that the system returns to equilibrium without oscillations (as in the case of critical damping) but *much slower* than in the case of critical damping. The system shown in Figure 1.18 if over-damped would behave as the upper curve in Figure.



Forced oscillations and resonance

We will now examine qualitatively the effect of an externally applied force F on a system that is free to oscillate with frequency f_0. The force F will be assumed to vary periodically with time with a frequency (the driving frequency) f_D, for example as $F = F_0 \cos(2\pi f_D t)$. The question is how the oscillating system will respond to the presence of the external driving force. The oscillations that take place in this case are called forced oscillations. In general, sometime after the external force is applied, the system will switch to oscillations with a frequency equal to the driving frequency f_D. However, the amplitude of the oscillations will depend on the relation between f_D and f_0, and the amount of damping. We might expect that, because the system wants to oscillate at its own natural frequency, when the external force has the same frequency as the natural frequency, large oscillations will take place. On the other hand, at very low frequencies, $f_D \approx 0$, and so $F= F_0 \cos(2\pi f_D t) \approx F_0$, i.e. it is constant. A constant force applied to a spring, for example, will extend the spring by a constant amount. A detailed analysis produces the graph in Figure showing how the amplitude of oscillation of a system with natural frequency f_0 varies as it is subjected to a periodic force of frequency f_D. The degree of damping increases as we move from the top curve down.

Resonance

The state in which the frequency of the externally applied periodic force equals the natural frequency of the system is called *resonance*. This results in oscillations with large amplitude.

Resonance can be disastrous: we do not want an aeroplane wing to resonate; nor is it good for a building to be set into resonance by an earthquake. Resonance can be irritating: if the car in which you drive is set into resonance by bumps on the road or a poorly tuned engine. But resonance can also be a good thing: resonance is used by a microwave oven to warm food; and your radio uses resonance to tune into one specific station and not another. Another useful example of electrical resonance is the quartz oscillator, a crystal made out of quartz that can be made to vibrate at a specific frequency. The resonant frequency of the quartz oscillator depends on how it is cut from the original crystal.

These crystals are used as the timing device in electronic watches and many other devices in electronics. They are cheap and keep their characteristics with time. The operation of the quartz oscillator uses a phenomenon called *piezoelectricity* in which an electrical signal applied to the crystal forces the crystal to vibrate. In turn, the mechanical vibration is fed back as another electrical signal at the crystal's resonant frequency.

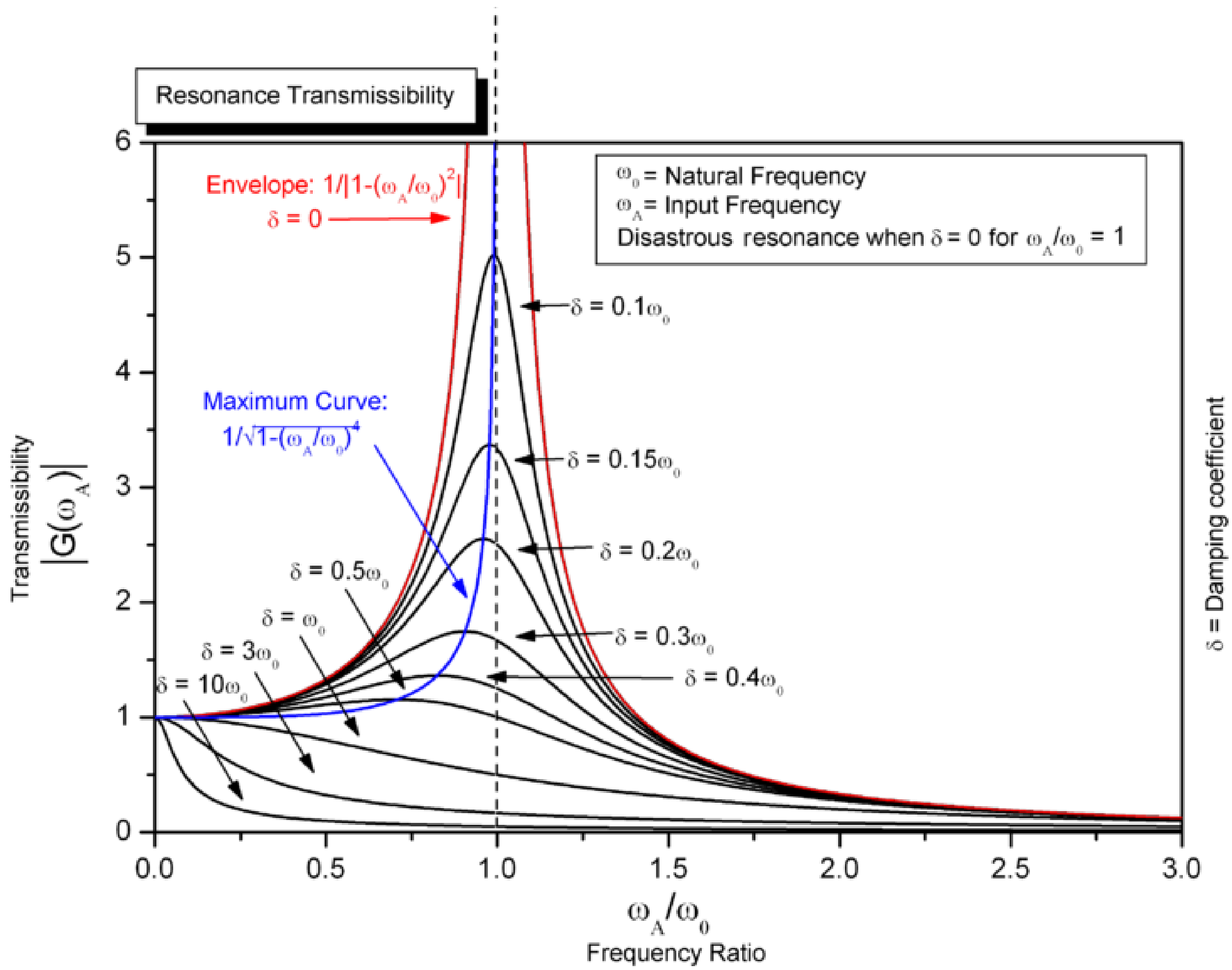


Figure: Graph showing the variation with driving frequency of the amplitude of forced SHM when the system is driven by an external periodic force.

The general features of the graph in Figure are as follows:

- For a small degree of damping, the peak of the curve occurs at the natural frequency of the system, f_0.
- The lower the degree of damping, the higher and narrower the curve.
- As the amount of damping increases, the peak shifts to lower frequencies.
- At very low frequencies, the amplitude is essentially constant. If f_D is very different from f_0, the amplitude of oscillation will be small.

On the other hand, if f_D is approximately the same as f_0, and the degree of damping is small, the resulting driven oscillations will have large amplitude. The largest amplitude is obtained when f_D is equal to f_0, in which case we say that the system is in resonance.

WAVES

Introduction

Waves are everywhere. Whether we recognize it or not, we encounter waves on a daily basis. Sound waves, visible light waves, radio waves, microwaves, water waves, sine waves, cosine waves, stadium waves, earthquake waves, waves on a string, and slinky waves and are just a few of the examples of our daily encounters with waves.

For many people, the first thought concerning waves conjures up a picture of a wave moving across the surface of an ocean, lake, pond or other body of water. The waves are created by some form of a disturbance, such as a rock thrown into the water, a duck shaking its tail in the water or a boat moving through the water. The water wave has a crest and a trough and travels from one location to another. One crest is often followed by a second crest that is often followed by a third crest. Every crest is separated by a trough to create an alternating pattern of crests and troughs. A duck or gull at rest on the surface of the water is observed to bob up-and-down at rather regular time intervals as the wave passes by. The waves may appear to be plane waves that travel together as a front in a straight-line direction, perhaps towards a sandy shore. Or the waves may be circular waves that originate from the point where the disturbances occur; such circular waves travel across the surface of the water in all directions. These mental pictures of water waves are useful for understanding the nature of a wave and will be revisited later when we begin our formal discussion of the topic.

A rock tossed into the water will create a circular disturbance which travels outwards in all directions.

We also encountered waves in Math class in the form of the sine and cosine function. We often plotted y = B•sin (A•x) on our calculator or by hand and observed that its graphical shape resembled the characteristic shape of a wave. There was a crest and a trough and a repeating pattern. If we changed the constant A in the equation, we noticed that we could change the length of the repeating pattern. And if we changed B in the equation, we noticed that we changed the height of the pattern. In math class, we encountered the underlying mathematical functions that describe the physical nature of waves.

What is a Wave?

A wave can be described as a disturbance that travels through a medium from one location to another location. Consider a slinky wave as an example of a wave. When the slinky is stretched from end to end and is held at rest, it assumes a natural position known as the equilibrium or rest position. The coils of the slinky naturally assume this position, spaced equally far apart.

When a slinky is stretched, the individual coils assume an equilibrium or rest position.

When the first coil of the slinky is repeatedly vibrated back and forth, a disturbance is created which travels through the slinky from one end to the other.

To introduce a wave into the slinky, the first particle is displaced or moved from its equilibrium or rest position. The particle might be moved upwards or downwards, forwards or backwards; but once moved, it is returned to its original equilibrium or rest position. The act of moving the first coil of the slinky in a given direction and then returning it to its equilibrium position creates a disturbance in the slinky. We can then observe this disturbance moving through the slinky from one end to the other. If the first coil of the slinky is given a single back-and-forth vibration, then we call the observed motion of the disturbance through the slinky a slinky pulse. A pulse is a single disturbance moving through a medium from one location to another location. However, if the first coil of the slinky is continuously and periodically vibrated in a back-and-forth manner, we would observe a repeating disturbance moving within the slinky that endures over some prolonged period of time. The repeating and periodic disturbance that moves through a medium from one location to another is referred to as a wave.

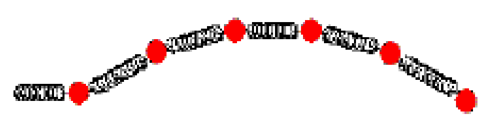
What is a Medium?

But what is meant by the word medium? A medium is a substance or material that carries the wave. You have perhaps heard of the phrase news media. The news media refers to the various institutions (newspaper offices, television stations, radio stations, etc.) within our society that carry the news from one location to another. The news moves through the media. The media doesn't make the news and the media isn't the same as the news. The news media is merely thething that carries the news from its source to various locations. In a similar manner, a wave medium is the substance that carries a wave (or disturbance) from one location to another. The wave medium is not the wave and it doesn't make the wave; it merely carries or transports the wave from its source to other locations. In the case of our slinky wave, the medium through that the wave travels is the slinky coils. In the case of a water wave in the ocean, the medium through which the wave travels is the ocean water. In the case of a sound wave moving from the church choir to the pews, the medium through which the sound wave travels is the air in the room. And in the case of the stadium wave, the medium through which the stadium wave travels is the fans that are in the stadium.

Particle-to-Particle Interaction

To fully understand the nature of a wave, it is important to consider the medium as a collection of interacting particles. In other words, the medium is composed of parts that are capable of interacting with each other. The interactions of one particle of the medium with the next adjacent particle allow the disturbance to travel through the medium. In the case of the slinky wave, the particles or interacting parts of the medium are the individual coils of the slinky. In the case of a sound wave in air, the particles or interacting parts of the medium are the individual molecules of air. And in the case of a stadium wave, the particles or interacting parts of the medium are the fans in the stadium.

A medium can be modeled by a series of particles connected by springs. As one particle is displaced, ...



... the spring attaching it to the next is stretched and begins to exert a force on its neighbor, thus displacing the neighbor from its rest position.

Consider the presence of a wave in a slinky. The first coil becomes disturbed and begins to push or pull on the second coil; this push or pull on the second coil will displace the second coil from its equilibrium position. As the second coil becomes displaced, it begins to push or pull on the third coil; the push or pull on the third coil displaces it from its equilibrium position.

As the third coil becomes displaced, it begins to push or pull on the fourth coil. This process continues in consecutive fashion, with each individual particle acting to displace the adjacent particle. Subsequently, the disturbance travels through the medium. The medium can be pictured as a series of particles connected by springs. As one particle moves, the spring connecting it to the next particle begins to stretch and apply a force to its adjacent neighbour. As this neighbour begins to move, the spring attaching this neighbour to its neighbour begins to stretch and apply a force on its adjacent neighbour.

A Wave Transports Energy and Not Matter:

When a wave is present in a medium (that is, when there is a disturbance moving through a medium), the individual particles of the medium are only temporarily displaced from their rest position. There is always a force acting upon the particles that restores them to their original position. In a slinky wave, each coil of the slinky ultimately returns to its original position. In a water wave, each molecule of the water ultimately returns to its original position. And in a stadium wave, each fan in the bleacher ultimately returns to its original position. It is for this reason, that a wave is said to involve the movement of a disturbance without the movement of matter. The particles of the medium (water molecules, slinky coils, stadium fans) simply vibrate about a fixed position as the pattern of the disturbance moves from one location to another location.

Waves are said to be an energy transport phenomenon. As a disturbance moves through a medium from one particle to its adjacent particle, energy is being transported from one end of the medium to the other. In a slinky wave, a person imparts energy to the first coil by doing work upon it. The first coil receives a large amount of energy that it subsequently transfers to the second coil. When the first coil returns to its original position, it possesses the same amount of energy as it had before it was displaced. The first coil transferred its energy to the second coil. The second coil then has a large amount of energy that it subsequently transfers to the third coil. When the second coil returns to its original position, it possesses the same amount of energy as it had before it was displaced. The third coil has received the energy of the second coil. This process of energy transfer continues as each coil interacts with its neighbour. In this manner, energy is transported from one end of the slinky to the other, from its source to another location.

This characteristic of a wave as an energy transport phenomenon distinguishes waves from other types of phenomenon. Consider a common phenomenon observed at a softball game - the collision of a bat with a ball. A batter is able to transport energy from her to the softball by means of a bat. The batter applies a force to the bat, thus imparting energy to the bat in the form of kinetic energy. The bat then carries this energy to the softball and transports the energy to the softball upon collision. In this example, a bat is used to transport energy from the player to the softball. However, unlike wave phenomena, this phenomenon involves the transport of matter. The bat must move from its starting location to the contact location in order to transport energy. In a wave phenomenon, energy can move from one location to another, yet the particles of matter in the medium return to their fixed position. A wave transports its energy without transporting matter.

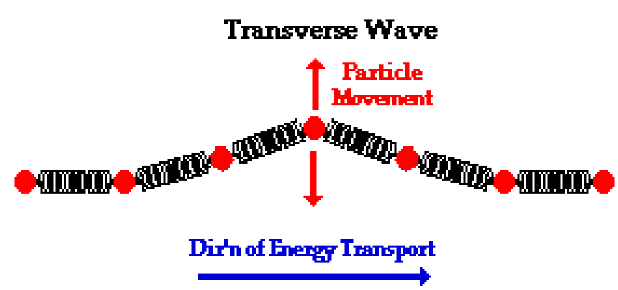
Waves are seen to move through an ocean or lake; yet the water always returns to its rest position. Energy is transported through the medium, yet the water molecules are not transported. Proof of this is the fact that there is still water in the middle of the ocean. The water has not moved from the middle of the ocean to the shore. If we were to observe a gull or duck at rest on the water, it would merely bob up-and-down in a somewhat circular fashion as the disturbance moves through the water. The gull or duck always returns to its original position. The gull or duck is not transported to the shore because the water on which it rests is not transported to the shore. In a water wave, energy is transported without the transport of water.
The same thing can be said about a stadium wave. In a stadium wave, the fans do not get out of their seats and walk around the stadium. We all recognize that it would be silly (and embarrassing) for any fan to even contemplate such a thought. In a stadium wave, each fan rises up and returns to the original seat. The disturbance moves through the stadium, yet the fans are not transported. Waves involve the transport of energy without the transport of matter.

In conclusion, a wave can be described as a disturbance that travels through a medium, transporting energy from one location (its source) to another location without transporting matter. Each individual particle of the medium is temporarily displaced and then returns to its original equilibrium positioned.
Categories of Waves
Waves come in many shapes and forms. While all waves share some basic characteristic properties and behaviors, some waves can be distinguished from others based on some observable (and some non-observable) characteristics. It is common to categorize waves based on these distinguishing characteristics.

Longitudinal versus Transverse Waves versus Surface Waves

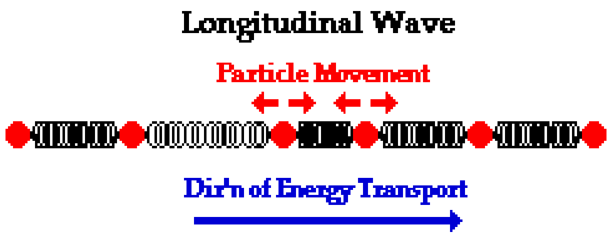
One way to categorize waves is on the basis of the direction of movement of the individual particles of the medium relative to the direction that the waves travel. Categorizing waves on this basis leads to three notable categories: transverse waves, longitudinal waves, and surface waves.

Transverse Waves:

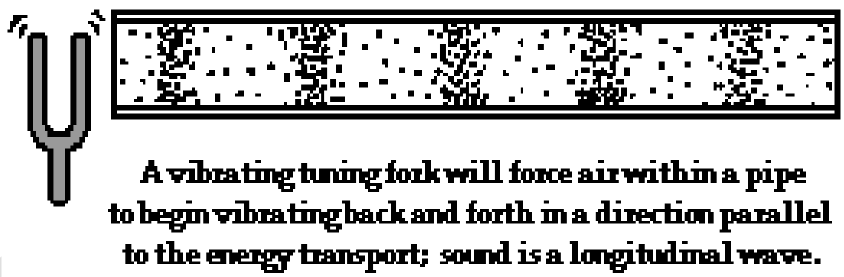


A transverse wave is a wave in which particles of the medium move in a direction perpendicular to the direction that the wave moves. Suppose that a slinky is stretched out in a horizontal direction across the classroom and that a pulse is introduced into the slinky on the left end by vibrating the first coil up and down. Energy will begin to be transported through the slinky from left to right. As the energy is transported from left to right, the individual coils of the medium will be displaced upwards and downwards. In this case, the particles of the medium move perpendicular to the direction that the pulse moves. This type of wave is a transverse wave. Transverse waves are always characterized by particle motion being perpendicular to wave motion.

Longitudinal Waves



A longitudinal wave is a wave in which particles of the medium move in a direction parallel to the direction that the wave moves. Suppose that a slinky is stretched out in a horizontal direction across the classroom and that a pulse is introduced into the slinky on the left end by vibrating the first coil left and right. Energy will begin to be transported through the slinky from left to right. As the energy is transported from left to right, the individual coils of the medium will be displaced leftwards and rightwards. In this case, the particles of the medium move parallel to the direction that the pulse moves. This type of wave is a longitudinal wave. Longitudinal waves are always characterized by particle motion being parallel to wave motion.



A sound wave traveling through air is a classic example of a longitudinal wave. As a sound wave moves from the lips of a speaker to the ear of a listener, particles of air vibrate back and forth in the same direction and the opposite direction of energy transport. Each individual particle pushes on its neighboring particle so as to push it forward. The collision of particle #1 with its neighbor serves to restore particle #1 to its original position and displace particle #2 in a forward direction. This back and forth motion of particles in the direction of energy transport creates regions within the medium where the particles are pressed together and other regions where the particles are spread apart. Longitudinal waves can always be quickly identified by the presence of such regions. This process continues along the chain of particles until the sound wave reaches the ear of the listener. A detailed discussion of sound is presented in another unit of The Physics Classroom Tutorial.

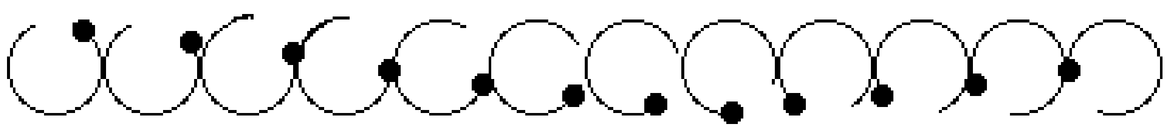
Waves traveling through a solid medium can be either transverse waves or longitudinal waves. Yet waves traveling through the bulk of a fluid (such as a liquid or a gas) are always longitudinal waves. Transverse waves require a relatively rigid medium in order to transmit their energy. As one particle begins to move it must be able to exert a pull on its nearest neighbour. If the medium is not rigid as is the case with fluids, the particles will slide past each other. This sliding action that is characteristic of liquids and gases prevents one particle from displacing its neighbour in a direction perpendicular to the energy transport. It is for this reason that only longitudinal waves are observed moving through the bulk of liquids such as our oceans.

Earthquakes are capable of producing both transverse and longitudinal waves that travel through the solid structures of the Earth. When seismologists began to study earthquake waves they noticed that only longitudinal waves were capable of traveling through the core of the Earth. For this reason, geologists believe that the Earth's core consists of a liquid - most likely molten iron.

While waves that travel within the depths of the ocean are longitudinal waves, the waves that travel along the surface of the oceans are referred to as surface waves. A surface wave is a wave in which particles of the medium undergo a circular motion. Surface waves are neither longitudinal nor transverse. In longitudinal and transverse waves, all the particles in the entire bulk of the medium move in a parallel and a perpendicular direction (respectively) relative to the direction of energy transport.

In a surface wave, it is only the particles at the surface of the medium that undergo the circular motion. The motion of particles tends to decrease as one proceeds further from the surface.

Surface Wave

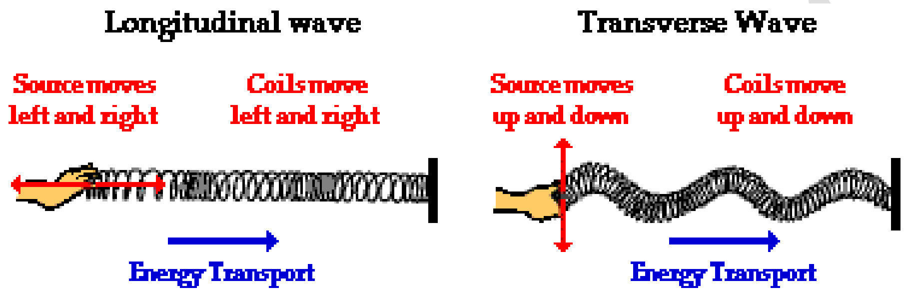


A surface wave is sometimes referred to as a circular wave since particles of the medium undergo a motion in a complete circle.

Any wave moving through a medium has a source. Somewhere along the medium, there was an initial displacement of one of the particles. For a slinky wave, it is usually the first coil that becomes displaced by the hand of a person. For a sound wave, it is usually the vibration of the vocal chords or a guitar string that sets the first particle of air in vibrational motion. At the location where the wave is introduced into the medium, the particles that are displaced from their equilibrium position always moves in the same direction as the source of the vibration. So if you wish to create a transverse wave in a slinky, then the first coil of the slinky must be displaced in a direction perpendicular to the entire slinky. Similarly, if you wish to create a longitudinal wave in a slinky, then the first coil of the slinky must be displaced in a direction parallel to the entire slinky.

Electromagnetic versus Mechanical Waves

Another way to categorize waves is on the basis of their ability or inability to transmit energy through a vacuum (i.e., empty space). Categorizing waves on this basis leads to two notable categories: electromagnetic waves and mechanical waves.



The subsequent direction of motion of individual particles of a medium is the same as the direction of vibration of the source of the disturbance.

An electromagnetic wave is a wave that is capable of transmitting its energy through a vacuum (i.e., empty space). Electromagnetic waves are produced by the vibration of charged particles. Electromagnetic waves that are produced on the sun subsequently travel to Earth through the vacuum of outer space. Were it not for the ability of electromagnetic waves to travel to through a vacuum, there would undoubtedly be no life on Earth. All light waves are examples of electromagnetic waves. Light waves are the topic of another unit at The Physics Classroom Tutorial. While the basic properties and behaviors of light will be discussed, the detailed nature of an electromagnetic wave is quite complicated and beyond the scope of The Physics Classroom Tutorial.

A mechanical wave is a wave that is not capable of transmitting its energy through a vacuum. Mechanical waves require a medium in order to transport their energy from one location to another. A sound wave is an example of a mechanical wave. Sound waves are incapable of traveling through a vacuum. Slinky waves, water waves, stadium waves, and jump rope wavesare other examples of mechanical waves; each requires some medium in order to exist. A slinky wave requires the coils of the slinky; a water wave requires water; a stadium wave requires fans in a stadium; and a jump rope wave requires a jump rope.

INTERFERENCE

Introduction

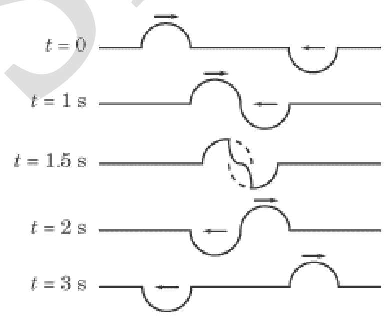
When two light waves of the same amplitude and the same frequency travelling in the same direction are superimposed upon each other in such a way that they support each other at some points and cancel each other at the other points. Such type phenomenon is called interference.

Types of Interference:

There are two types of Interference, Destructive Interference and constructive Interference.

Destructive Interference:

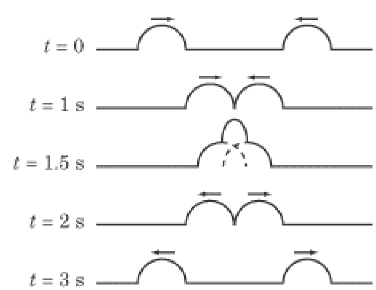
If two light waves cancel each other at a point, a dark fringe is produces. This type of interference is called Destructive interference.



Suppose one of the experimenters yanks the string downward, while the other pulls up by exactly the same amount. In this case, the total displacement when the pulses meet will be zero: this is called **destructive interference**. Don't be fooled by the name, though: neither wave is destroyed by this interference. After they pass by one another, they will continue just as they did before they met.

Constructive Interference:

If two waves meet at a point in which or such a way that they reinforce each other and bright fringes produced on the screen. This type of interference is called Constructive Interference.



On the other hand, if both experimenters send upward pulses down the string, the total displacement when they meet will be a pulse that's twice as big. This is called **constructive interference**.

Put difference of constructive interference is $0, \lambda, 2\lambda, 3\lambda, \ldots m\lambda$

Where $m = 0, \pm 1, \pm 2, \pm 3, \ldots$

And path difference of Destructive Interference is $0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots (m+\frac{1}{2})\lambda$

Condition of Interference

1. Source must be phase coherent.
2. The source light should be monochromatic.
3. The superposition principle must apply.

At all the places where two parts meets constructively brightness is produced and at all these places where two parts meets destructively darkness is produced.

The Principle of Linear Superposition

The principle of linear superposition states that the resultant wave is the sum of individual waves. For simplicity we will assume that all the waves we discuss a) have the same wavelength, b) have the same amplitude and c) are polarized so that their polarization directions are parallel. (See Figures 27.2 and 27.3 on pages 855 and 856 of your text.) Also for simplicity, we consider the interference of just two waves at a time. We also assume that the waves come from **coherent sources**; that is, the phase relation of the waves emitted by the sources remains constant with time.

When waves combine, the resultant wave depends on the phase difference of the component waves. When we were dealing with AC circuits we used a phase angle measured in radians to keep track of phase differences. In this chapter it will be more convenient express the phase difference of two waves in terms of wavelength.

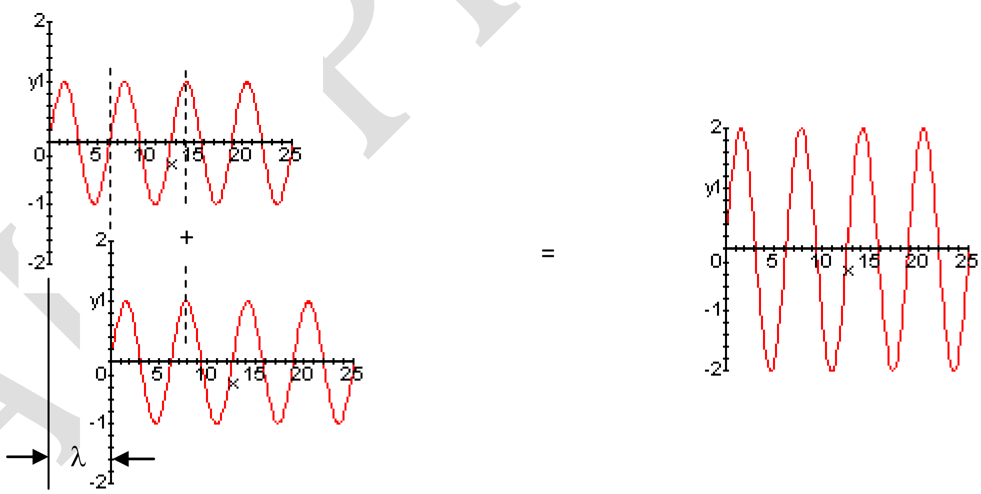


Figure: constructive interference

In the figure above (on the left) we see two waves. The bottom wave is one wavelength (λ) out of phase with the top wave. When the waves combine crests meet crests and valleys meet valleys. The resultant wave (on the right) has the same wavelength as the component wave but twice the amplitude. This is called **constructive interference**.

It should be clear that the same resultant wave is obtained if the phase difference of the component waves is 2λ, 3λ, 4λ or any integer multiple of λ:

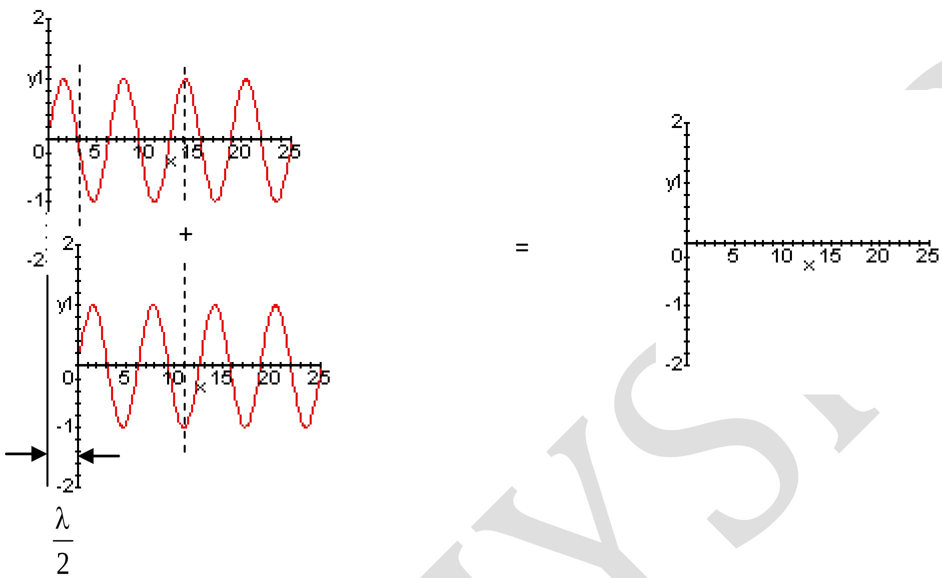


Figure: Destructive interference

Phase difference for constructive interference = $m\lambda$ (where *m* is an integer).

Now consider what happens when the waves are one-half wavelength out of phase.

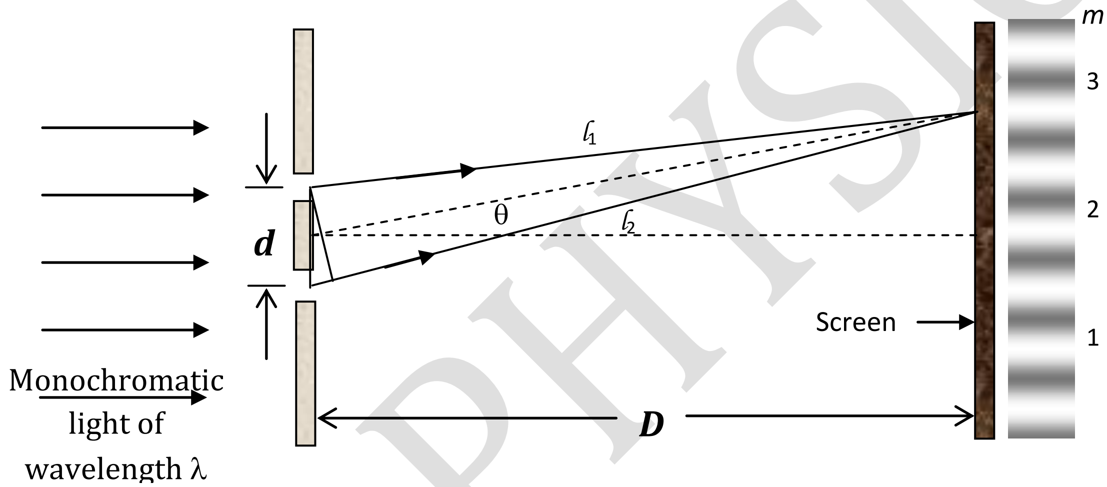
In the figure above (on the left) we see two waves. The bottom wave is one-half wavelength ($\lambda/2$) out of phase with the top wave. When the waves combine crests meet valleys and valleys meet crests. The resultant wave (on the right) has zero amplitude. This is called ***total destructive interference***.

It should be clear that the same resultant wave is obtained if the phase difference of the component waves is $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, $\frac{7}{2}\lambda$ *or any half odd integer multiple of* λ:

Phase difference for total destructive interference = $\left(m+\frac{1}{2}\right)\lambda$ (where *m* is an integer).

Young’s Double Slit Experiment

The first wave theory of light was proposed by Christian Huygens (1629 – 1695). It was not until 1801, however, that the English scientist Thomas Young (1773 – 1829) performed an experiment that demonstrated the wave nature of light. In Young’s double slit experiment monochromatic light illuminates two slits in an opaque barrier. The slits are separated by a distance *d*. Light from the slits falls on a screen a distance *D* away. It is assumed that $D >> d$. The image that appears on the screen is a set of light and dark bands called *interference fringes*. The pattern the fringes form is called *an interference pattern*. The following figure shows a typical double-slit diffraction pattern. The arrows mark the central bright fringe, which is taken to be bright fringe 0.

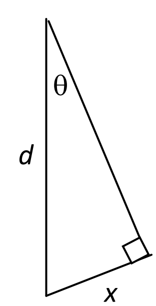


We can use geometry to locate the bright and dark fringes of the double-slit diffraction pattern.

Figure: Young’s Double Slit Experiment

In the above figure a light ray from the top slit and a light ray from the bottom slit come together on the screen to form a bright fringe. The angle θ shown in the above diagram can be used to locate the fringe on the screen. We want to find a formula for θ in terms of the slit separation *d*, wavelength λ and fringe number *m*.

The distances l_1 and l_2 are distance the top ray travels to the screen and the distance the bottom ray travels to the screen, respectively. A small triangle has been drawn next to the slits in such a way that its base is the path length difference $x = l_2 - l_1$ of the light rays. If $D >> d$ we may label this small triangle as follows:



We see that $\sin\theta = \frac{x}{d}$. For a bright fringe to appear on the screen, we must have constructive interference of the light waves at the point where the rays meet on the screen. According to the discussion, we must have $x = m\lambda$, where *m* is an integer.

Hence for **Bright fringes of a double slit:** $\sin\theta = m\frac{\lambda}{d}$ $\quad m = 0, \ \pm1, \ \pm2, \ \pm3\ldots$

Note that in the figure at the top of the page the rays locate the fringe corresponding to *m* = 2. The negative values of *m* locate bright fringes on the other side of the central bright fringe.

To locate the dark fringes in the double-slit diffraction pattern we only need note that total destructive interference of the light waves must occur at the point where the rays meet on the screen. In this case the path length difference of the rays has to be given by $x = \left(m + \tfrac{1}{2}\right)\lambda$, where *m* is an integer.

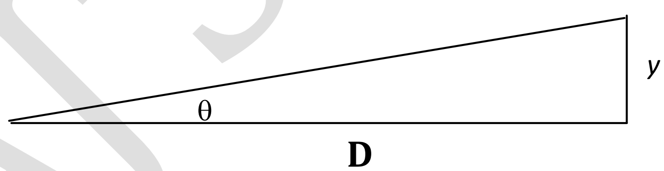
Hence for **Dark fringes of a double slit:** $\sin\theta = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{d}$ $\quad m = 0, \ \pm1, \ \pm2, \ \pm3\ldots$

Example

Light of wavelength 550 nm is incident on a pair of slits separated by a distance of 0.12 mm. A screen is placed 2.0 m behind the slits. Find the distance *y* on the screen between the central bright fringe and the second *dark* fringe above it.

Caution. The dark fringes are labeled differently from the bright fringes. The *first* dark fringe from the central bright fringe corresponds to *m* = 0 in the above formula. The *second* dark fringe from the central bright fringe corresponds to *m* = 1 (*not m* = 2).

$$\sin\theta = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{d}; \quad \sin\theta = \left(1 + \tfrac{1}{2}\right)\frac{5.5\times10^{-7}\ \text{m}}{1.2\times10^{-4}\ \text{m}}; \quad \sin\theta = 6.9\times10^{-3}; \quad \theta = 6.9\times10^{-3}\ \text{rad}$$



Since $y << D$, $\theta \approx \frac{y}{D}$ and $y \approx D\theta$: $y = (2.0\ \text{m})\,6.9\times10^{-3}\ \text{rad}$; $\boxed{y = 14\ \text{mm}}$

Thin-Film Interference

You have probably noticed that oil spots on the street show rainbow-like colors when viewed in daylight. This phenomenon is due to *thin-film interference*: light waves reflected from the street surface beneath the oil interfere with light rays reflected from the oil surface.

To understand thin-film interference we must first consider the phase relationship between light that is incident on a medium and light that is reflected from the medium.

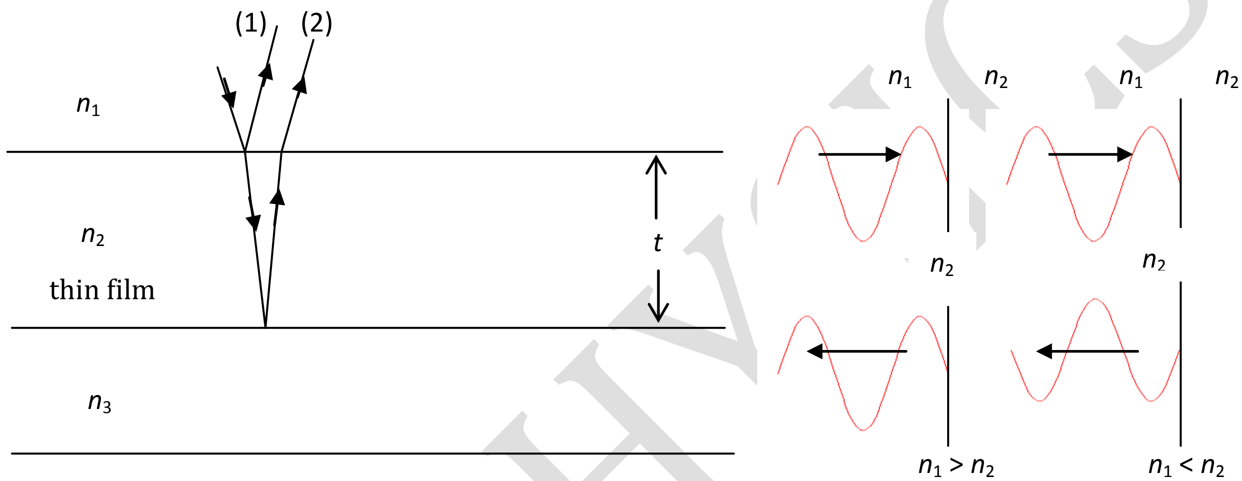


Figure:thin-film interference

In the figure on the left light is incident on the surface of a medium that has an index of refraction that is *less than* that of the incident medium. The reflected light has the same phase as the incident light. In the figure on the right light is incident on the surface of a medium that has an index of refraction that is *greater than* that of the incident medium. The reflected light *does not* have the same phase as the incident light; *it is one-half wavelength out of phase with the incident light*.

Now let us consider light rays that reflect from the surface of a thin film and the surface that is beneath the film.

In the figure above a monochromatic light ray in medium 1 is incident on the surface of medium 2. Part of the ray is refracted into the film and reflects at the bottom of the film from the surface of medium 3. This reflected ray emerges from the film as ray 2. Ray 1 is just that part of the incident ray that is reflected from the surface of medium 2. (Note that in the figure we assume $n_1 < n_2$.)

Whether rays (1) and (2) interfere constructively or destructively depends on two things: *i*) their path length difference (which occurs *entirely* within the film) and *ii*) their phase relationship, which depends on the relative indices of refraction of the three media. Since the path length difference of the rays occurs entirely within the thin film, we need the wavelength of the light *in the film*. Let f be the frequency of the light and v_{film} be the speed of light in the film. Then

$$c = f\lambda_{\text{vacuum}}; \quad v_{\text{film}} = f\lambda_{\text{film}}; \quad \frac{c}{v_{\text{film}}} = \frac{f\lambda_{\text{vacuum}}}{f\lambda_{\text{film}}}; \quad n = \frac{\lambda_{\text{vacuum}}}{\lambda_{\text{film}}};$$

$$\boxed{\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n}}$$

where n is the index of refraction of the film.

Let us now consider the following example.

Example

Crown glass in a camera lens is to be made "nonreflecting" by coating it with magnesium flouride (MgF_2) that has index of refraction 1.38. This coating is to prevent light of wavelength 550 nm from reflecting from the lens. Find the minimum nonzero thickness of the coating that is required.

Given, $n_1 = 1.00$, and $n_3 = 1.52$. $n_2 = 1.38$. Since $n_1 < n_2$ *and* $n_2 < n_3$ there is a half wavelength phase change for the ray reflected at the coating surface *and* the ray reflected at the glass surface. Hence the only possible phase difference in the reflected rays will be due to the path length difference, the distance the second ray travels in the coating. We will get destructive interference if this path length difference is $\frac{1}{2}\lambda, \ \frac{3}{2}\lambda, \ \frac{5}{2}\lambda, \ \ldots$, where λ is the wavelength of the light *in the coating*. For the minimum coating thickness we set $2t = \frac{1}{2}\lambda$. ($2t$ because the light ray in the film traverses it twice – once going down and again going up.)

$$\lambda = \frac{\lambda_{\text{vacuum}}}{n}; \quad \lambda = \frac{550 \text{ nm}}{1.38} = 398.6 \text{ nm}. \quad 2t = \frac{398.6 \text{ nm}}{2}; \quad \boxed{t = 99.6 \text{ nm}}$$

Remarks. The human eye is most sensative to light with wavelength 550 nm in air. This light is in the green part of the visible spectrum. (Can you give an explanation as to why the human eye is most sensative to green light?) Precision lenses (used in the more expensive cameras) are coated to prevent the loss of light by reflection. These lenses, when viewed in ordinary light, appear to have a purple or bluish tint. Why is this?

DIFFRACTION

Introduction

It is the phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wavelength of light.

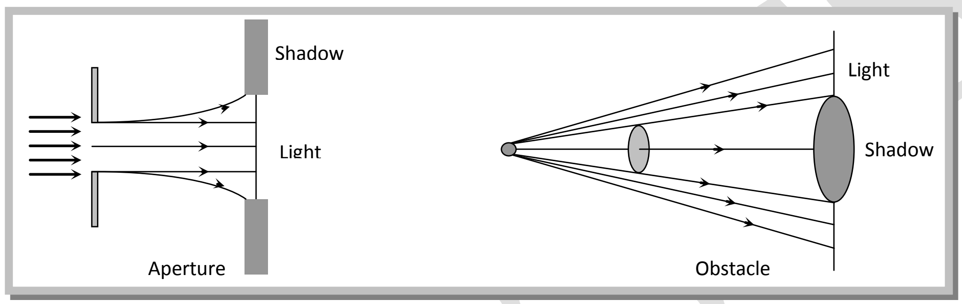


Figure: Diffraction of light

Note :Diffraction is the characteristic of all types of waves.

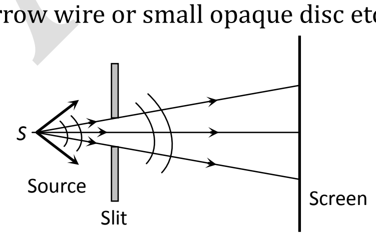
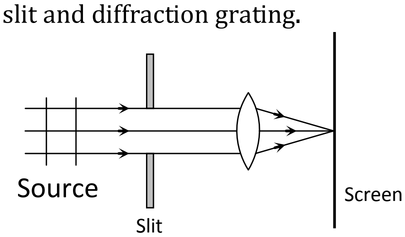
Greater the wavelength of wave, higher will be it's degree of diffraction.

Experimental study of diffraction was extended by Newton as well as Young.

Most systematic study carried out by Huygens on the basis of wave theory.

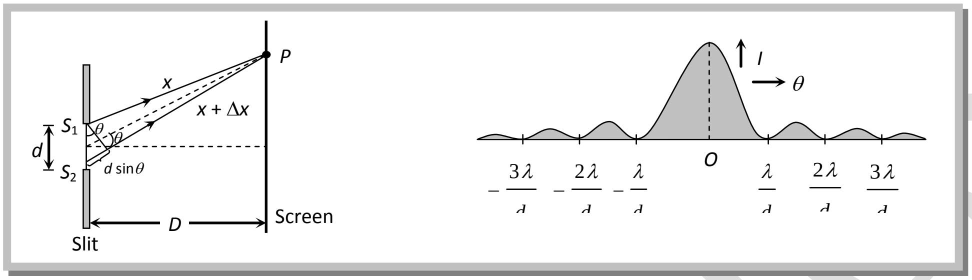
The minimum distance at which the observer should be from the obstacle to observe the diffraction of light of wavelength λ around the obstacle of size d is given by $x = \frac{d^2}{4\lambda}$.

Types of diffraction: The diffraction phenomenon is divided into two types

Fresnel diffraction	Fraunhofer diffraction
(i) If either source or screen or both are at finite distance from the diffracting device (obstacle or aperture), the diffraction is called Fresnel type. (ii) Common examples: Diffraction at a straight edge, narrow wire or small opaque disc etc. 	(i) In this case both source and screen are effectively at infinite distance from the diffracting device. (ii) Common examples: Diffraction at single slit, double slit and diffraction grating. 

Diffraction of light at a single slit:

In case of diffraction at a single slit, we get a central bright band with alternate bright (maxima) and dark (minima) bands of decreasing intensity as shown



(i) Width of central maxima $\beta_0 = \frac{2\lambda D}{d}$; and angular width $= \frac{2\lambda}{d}$

(ii) Minima occurs at a point on either side of the central maxima, such that the path difference between the waves from the two ends of the aperture is given by $\Delta = n\lambda$; where $n = 1, 2, 3 ....$

$$i.e.\ d \sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{d}$$

(iii) The secondary maxima occurs, where the path difference between the waves from the two ends of the aperture is given by $\Delta = (2n+1)\frac{\lambda}{2}$; where $n = 1, 2, 3 ....$

$$i.e.\ d \sin\theta = (2n+1)\frac{\lambda}{2} \Rightarrow \sin\theta = \frac{(2n+1)\lambda}{2d}$$

Comparison between interference and diffraction

Interference	Diffraction
Results due to the superposition of waves from two coherent sources.	Results due to the superposition of wavelets from different parts of same wave front. (single coherent source)
All fringes are of same width $\beta = \frac{\lambda D}{d}$	All secondary fringes are of same width but the central maximum is of double the width $\beta_0 = 2\beta = 2\frac{\lambda D}{d}$

Single-Slit Diffraction

Consider a monochromatic light wave with wavelength λ incident on a slit of width W as shown below. (The size of the slit is greatly exaggerated.)

A dashed line is drawn at the center of the slit, dividing it in half. Consider two parallel rays, one from the top half of the slit and the other from the bottom half of the slit. The distance x in the figure is the path length difference of the two rays. A converging lens focuses the two rays onto a distant screen. (The size of the lens is exaggerated; we assume that a thin lens is used and that each ray travels the same distance through the glass.) The small triangle drawn next to the slit may be labeled as follows:

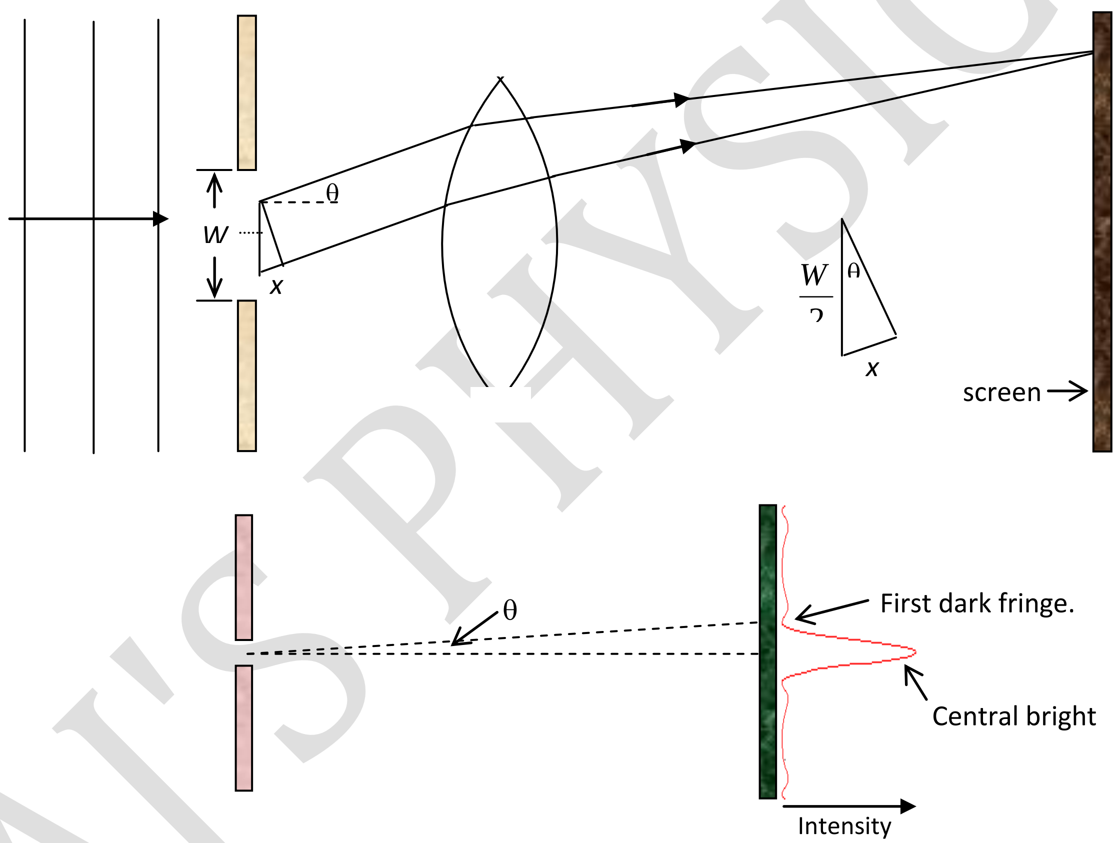


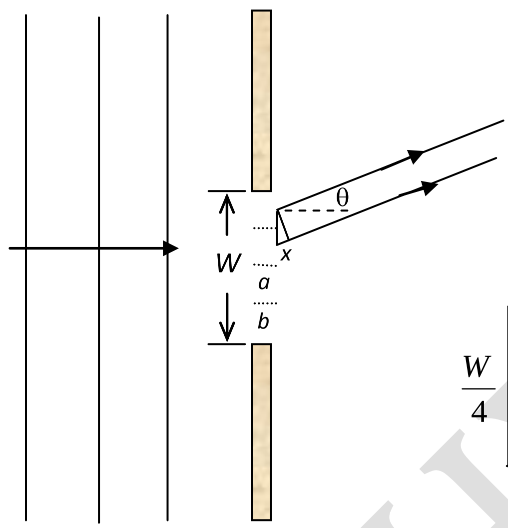
Figure: Single-Slit Diffraction

The single-slit diffraction pattern consists of a broad bright band in the center with alternating dark and bright fringes on either side. The center of the broad bright band lies at a point on the screen that is directly across from the slit. What we want to do is locate the dark fringes by finding the angle θ the line drawn from the slit to a dark fringe makes with respect the line drawn from the slit to the broad bright band.

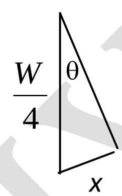
To locate the first dark fringe we set the path length difference *x* equal to $\lambda/2$.

$$\sin\theta = \frac{x}{(W/2)} = \frac{(\lambda/2)}{(W/2)}; \qquad \sin\theta = \frac{\lambda}{W} \leftarrow$$ This formula gives the angle for the first dark fringe in the single slit diffraction pattern.

To locate the second dark fringe we divide the slit into *four* regions.



Once again *x* is the path length difference between the two rays. The small triangle drawn next to the slit is now labelled as follows:



For destructive interference we once again set x = $\lambda/2$:

$$\sin\theta = \frac{x}{(W/4)} = \frac{(\lambda/2)}{(W/4)}; \qquad \sin\theta = 2\frac{\lambda}{W} \leftarrow$$ This formula gives the angle for the *second* dark fringe in the single slit diffraction pattern.

Question: How do the two rays from the regions labeled a and b interfere, if they are parallel to the rays just discussed? Why?

By continuing the above analysis we find a general formula for the dark fringes of the single-slit diffraction pattern:

Dark fringes for single-slit diffraction:

$$\boxed{\sin\theta = m\frac{\lambda}{W}} \qquad m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots$$

The positive values of m locate the centers of the dark fringes on one side of the central bright maximum; the negative values locate the centers of dark fringes on the other side.

Example:

A slit of width W is illuminated with red light of wavelength 650 nm. For what value of W will the first dark fringe for the red light be at θ = 15°?

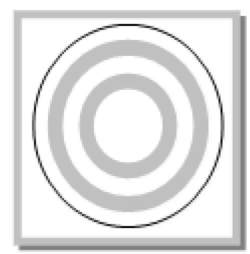
$$\sin\theta = m\frac{\lambda}{W}; \quad W = \frac{m\lambda}{\sin\theta}; \quad W = \frac{(1)6.5\times10^{-7}\ \text{m}}{\sin(15^\circ)}$$

$$W = 2.5\times10^{-6}\ \text{m} \quad \text{or} \quad \boxed{W = 2.5\ \mu\text{m}}$$

For the incident light to flare out by as much as ±15° the slit has to be very fine. Note that a fine human hair may be about 100 μm in diameter. Because the wavelength of light is so small when compared to everyday objects, the diffraction of light is difficult (but not impossible) to observe. This is why ray optics works so well in the applications of optics, such as the manufacture of lenses for glasses, binoculars and telescopes.

Diffraction and optical instruments:

The objective lens of optical instrument like telescope or microscope etc. acts like a circular aperture. Due to diffraction of light at a circular aperture, a converging lens cannot form a point image of an object rather it produces a brighter disc known as Airy disc surrounded by alternate dark and bright concentric rings.



The angular half width of Airy disc $= \theta = \frac{1.22\,\lambda}{D}$ (where D = aperture of lens)

The lateral width of the image $= f\theta$ (where f = focal length of the lens)

Note : Diffraction of light limits the ability of optical instruments to form clear images of objects when they are close to each other.

Diffraction grating:

Consists of large number of equally spaced parallel slits. If light is incident normally on a transmission grating, the diffraction of principle maxima (PM) is given by $d\sin\theta = n\lambda$; where d = distance between two consecutive slits and is called grating element.

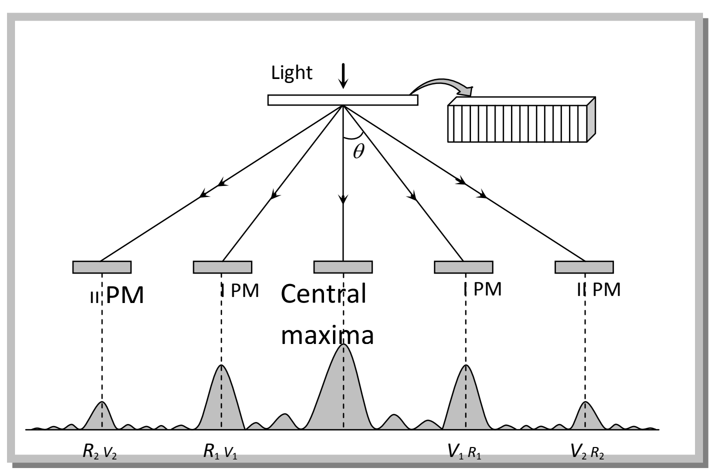
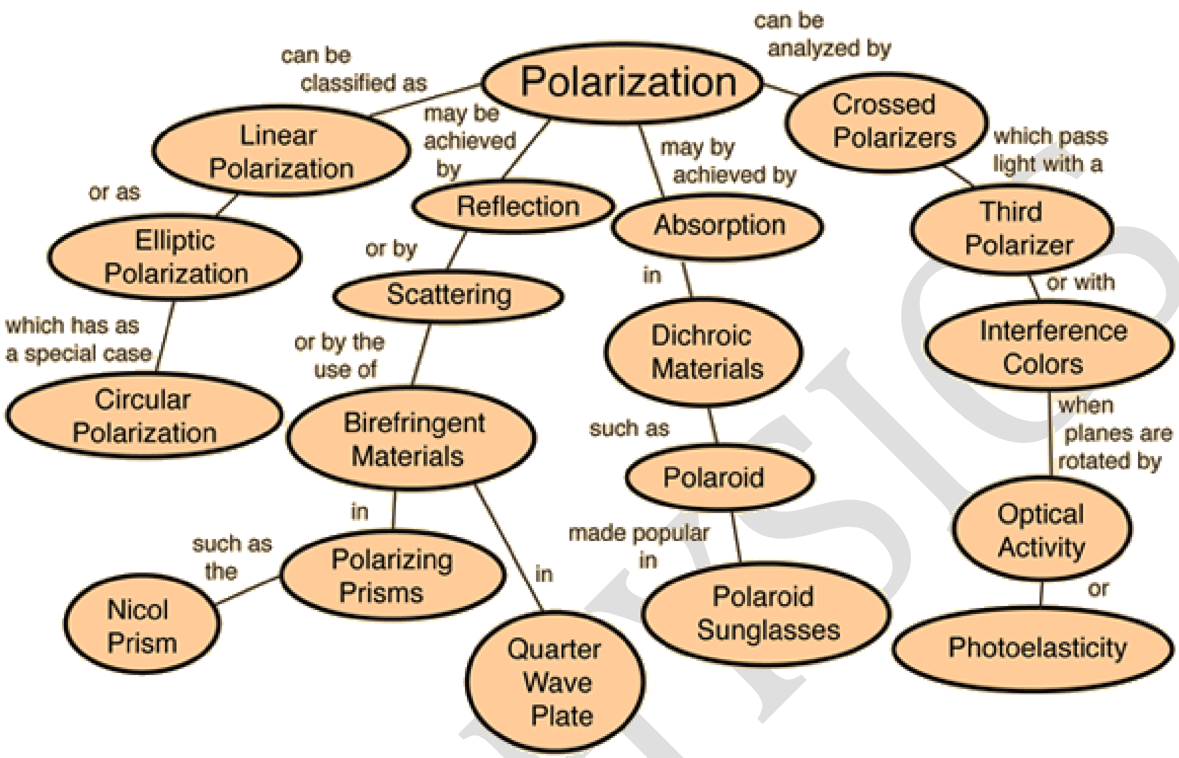


Figure: Diffraction grating

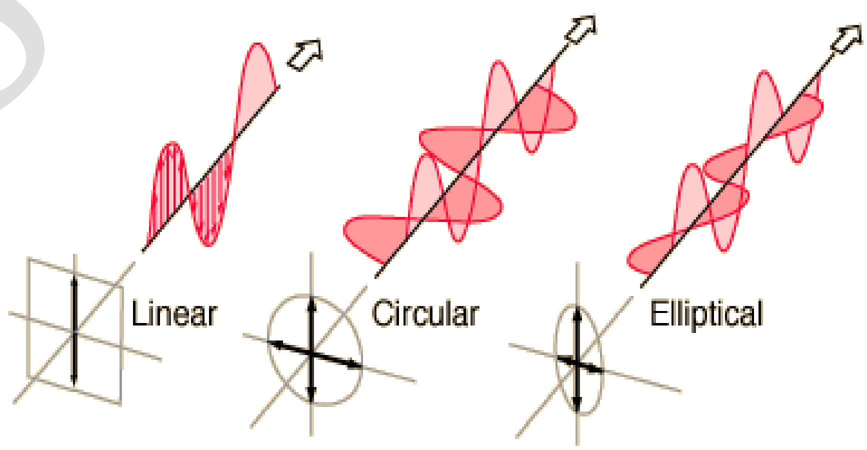
MODULE II

POLARIZATION



Introduction

Light in the form of a plane wave in space is said to be linearly polarized. Light is a transverse electromagnetic wave, but natural light is generally unpolarized, all planes of propagation being equally probable. If light is composed of two plane waves of equal amplitude by differing in phase by 90°, then the light is said to be circularly polarized. If two plane waves of differing amplitude are related in phase by 90°, or if the relative phase is other than 90° then the light is said to be elliptically polarized.



Classification of Polarization

Linear Polarization

A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave as illustrated.

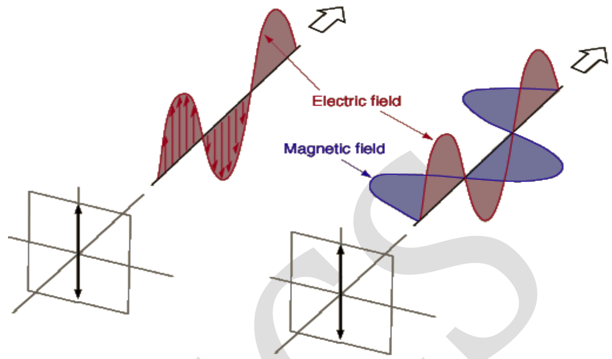


Figure: Linear Polarization

Circular Polarization

Circularly polarized light consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase. The light illustrated is right- circularly polarized. If light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized. If you could see the tip of the electric field vector, it would appear to be moving in a circle as it approached you. If while looking at the source, the electric vector of the light coming toward you appears to be rotating counter clockwise, the light is said to be right-circularly polarized.

If clockwise, then left-circularly polarized light. The electric field vector makes one complete revolution as the light advances one wavelength toward you. Another way of saying it is that if the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.

Circularly polarized light may be produced by passing linearly polarized light through a quarter-wave plate at an angle of 45° to the optic axis of the plate.

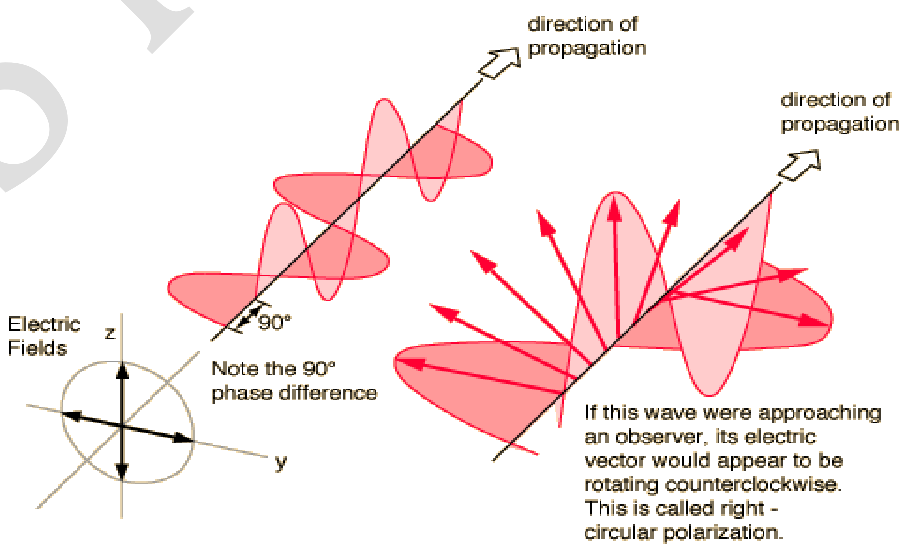


Figure: Circular Polarization

Elliptical Polarization

Elliptically polarized light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°. The illustration shows right- elliptically polarized light.

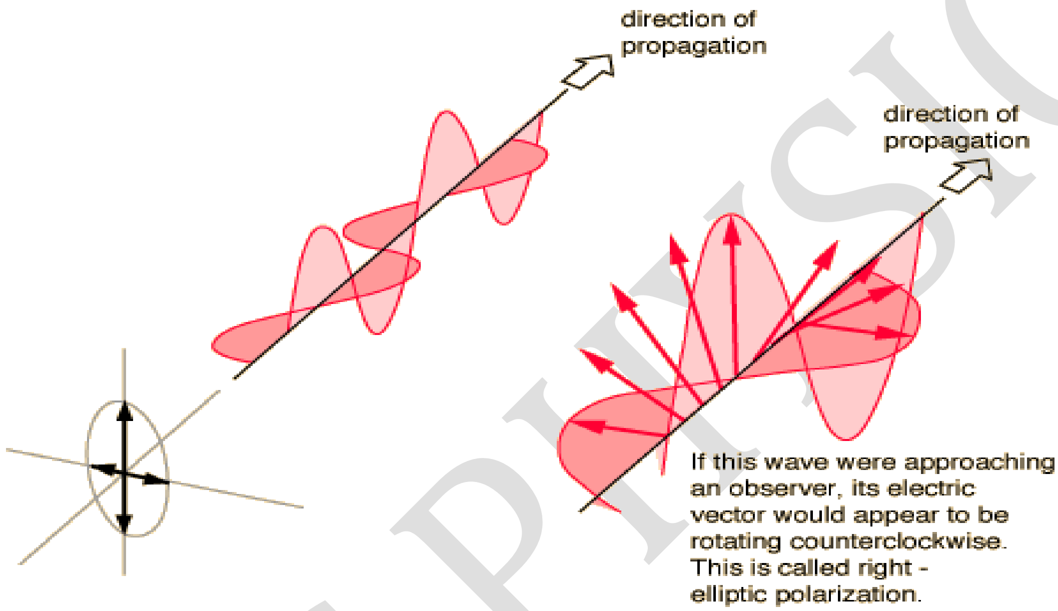


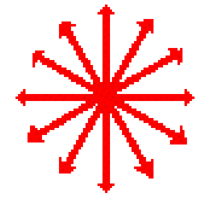
Figure: Elliptical Polarization

If the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers

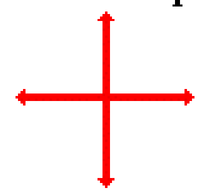
Methods of Polarization

A light wave is an electromagnetic wave that travels through the vacuum of outer space. Light waves are produced by vibrating electric charges. The nature of such electromagnetic waves is beyond the scope of The Physics Classroom Tutorial. For our purposes, it is sufficient to merely say that an electromagnetic wave is a transverse wave that has both an electric and a magnetic component.

A light wave is known to vibrate in a multitude of directions ...



... In general, a light wave can be thought of as vibrating in a vertical and in a horizontal plane.



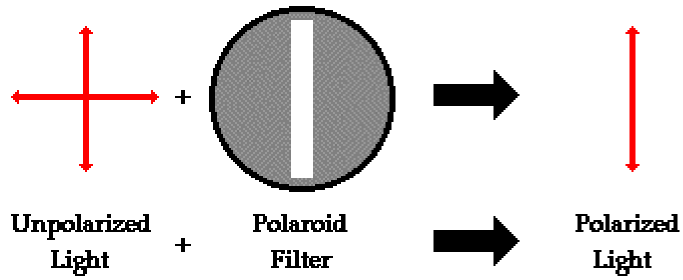
The transverse nature of an electromagnetic wave is quite different from any other type of wave. Let's suppose that we use the customary slinky to model the behavior of an electromagnetic wave. As an electromagnetic wave traveled towards you, then you would observe the vibrations of the slinky occurring in more than one plane of vibration. This is quite different than what you might notice if you were to look along a slinky and observe a slinky wave traveling towards you. Indeed, the coils of the slinky would be vibrating back and forth as the slinky approached; yet these vibrations would occur in a single plane of space. That is, the coils of the slinky might vibrate up and down or left and right. Yet regardless of their direction of vibration, they would be moving along the same linear direction as you sighted along the slinky. If a slinky wave were an electromagnetic wave, then the vibrations of the slinky would occur in multiple planes. Unlike a usual slinky wave, the electric and magnetic vibrations of an electromagnetic wave occur in numerous planes. A light wave that is vibrating in more than one plane is referred to as unpolarized light. Light emitted by the sun, by a lamp in the classroom, or by a candle flame is unpolarized light. Such light waves are created by electric charges that vibrate in a variety of directions, thus creating an electromagnetic wave that vibrates in a variety of directions. This concept of unpolarized light is rather difficult to visualize. In general, it is helpful to picture unpolarized light as a wave that has an average of half its vibrations in a horizontal plane and half of its vibrations in a vertical plane.

It is possible to transform unpolarized light into polarized light. Polarized light waves are light waves in which the vibrations occur in a single plane. The process of transforming unpolarized light into polarized light is known as polarization. There are a variety of methods of polarizing light. The four methods discussed on this page are:

- Polarization by Transmission
- Polarization by Reflection
- Polarization by Refraction
- Polarization by Scattering

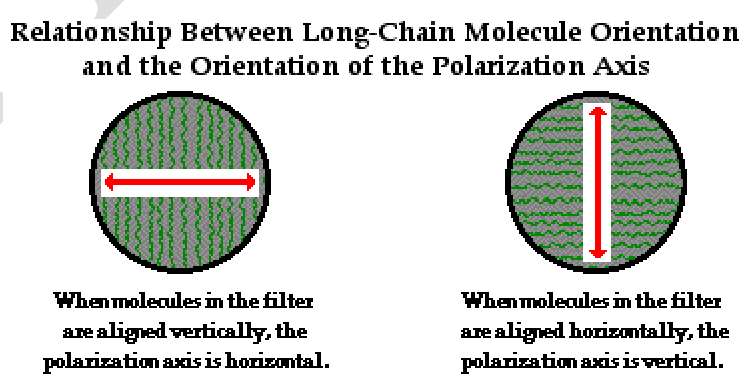
Polarization by Use of a Polaroid Filter

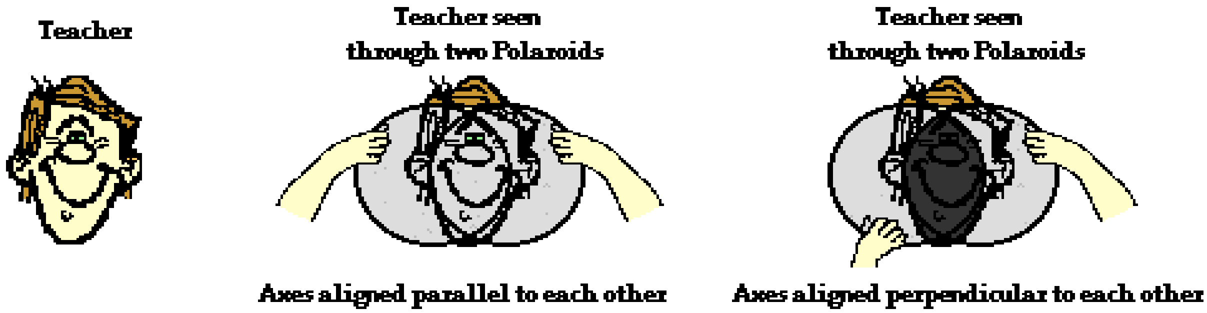
The most common method of polarization involves the use of a Polaroid filter. Polaroid filters are made of a special material that is capable of blocking one of the two planes of vibration of an electromagnetic wave. (Remember, the notion of two planes or directions of vibration is merely a simplification that helps us to visualize the wavelike nature of the electromagnetic wave.) In this sense, a Polaroid serves as a device that filters out one-half of the vibrations upon transmission of the light through the filter. When unpolarized light is transmitted through a Polaroid filter, it emerges with one-half the intensity and with vibrations in a single plane; it emerges as polarized light.



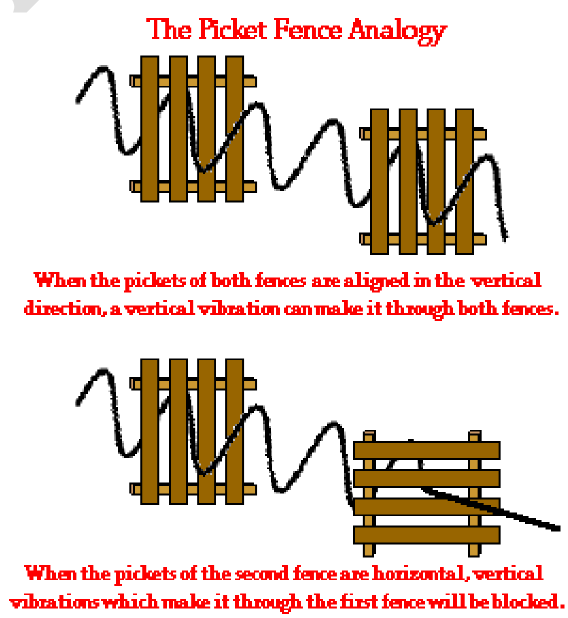
A Polaroid filter is able to polarize light because of the chemical composition of the filter material. The filter can be thought of as having long-chain molecules that are aligned within the filter in the same direction. During the fabrication of the filter, the long-chain molecules are stretched across the filter so that each molecule is (as much as possible) aligned in say the vertical direction. As unpolarized light strikes the filter, the portion of the waves vibrating in the vertical direction are absorbed by the filter. The general rule is that the electromagnetic vibrations that are in a direction parallel to the alignment of the molecules are absorbed.

The alignment of these molecules gives the filter a polarization axis. This polarization axis extends across the length of the filter and only allows vibrations of the electromagnetic wave that are parallel to the axis to pass through. Any vibrations that are perpendicular to the polarization axis are blocked by the filter. Thus, a Polaroid filter with its long-chain molecules aligned horizontally will have a polarization axis aligned vertically. Such a filter will block all horizontal vibrations and allow the vertical vibrations to be transmitted (see diagram above). On the other hand, a Polaroid filter with its long-chain molecules aligned vertically will have a polarization axis aligned horizontally; this filter will block all vertical vibrations and allow the horizontal vibrations to be transmitted.





Polarization of light by use of a Polaroid filter is often demonstrated in a Physics class through a variety of demonstrations. Filters are used to look through and view objects. The filter does not distort the shape or dimensions of the object; it merely serves to produce a dimmer image of the object since one-half of the light is blocked as it passed through the filter. A pair of filters is often placed back to back in order to view objects looking through two filters. By slowly rotating the second filter, an orientation can be found in which all the light from an object is blocked and the object can no longer be seen when viewed through two filters. What happened? In this demonstration, the light was polarized upon passage through the first filter; perhaps only vertical vibrations were able to pass through. These vertical vibrations were then blocked by the second filter since its polarization filter is aligned in a horizontal direction. While you are unable to see the axes on the filter, you will know when the axes are aligned perpendicular to each other because with this orientation, all light is blocked. So by use of two filters, one can completely block all of the light that is incident upon the set; this will only occur if the polarization axes are rotated such that they are perpendicular to each other.

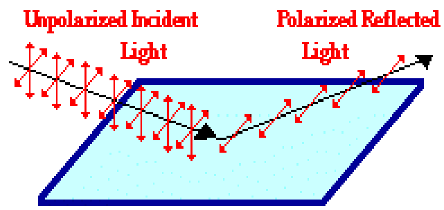


A picket-fence analogy is often used to explain how this dual-filter demonstration works. A picket fence can act as a polarizer by transforming an unpolarized wave in a rope into a wave that vibrates in a single plane. The spaces between the pickets of the fence will allow vibrations that are parallel to the spacings to pass through while blocking any vibrations that are perpendicular to the spacings. Obviously, a vertical vibration would not have the room to make it through a horizontal spacing. If two picket fences are oriented such that the pickets are both aligned vertically, then vertical vibrations will pass through both fences. On the other hand, if the pickets of the second fence are aligned horizontally, then the vertical vibrations that pass through the first fence will be blocked by the second fence.

In the same manner, two Polaroid filters oriented with their polarization axes perpendicular to each other will block all the light. Now that's a pretty cool observation that could never be explained by a particle view of light.

Polarization by Reflection

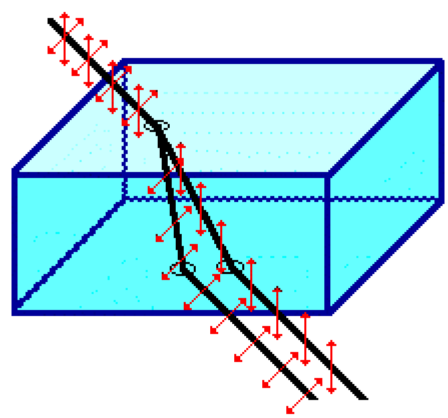
Unpolarized light can also undergo polarization by reflection off of nonmetallic surfaces. The extent to which polarization occurs is dependent upon the angle at which the light approaches the surface and upon the material that the surface is made of. Metallic surfaces reflect light with a variety of vibrational directions; such reflected light is unpolarized. However, nonmetallic surfaces such as asphalt roadways, snowfields and water reflect light such that there is a large concentration of vibrations in a plane parallel to the reflecting surface. A person viewing objects by means of light reflected off of nonmetallic surfaces will often perceive a glare if the extent of polarization is large. Fishermen are familiar with this glare since it prevents them from seeing fish that lie below the water. Light reflected off a lake is partially polarized in a direction parallel to the water's surface. Fishermen know that the use of glare-reducing sunglasses with the proper polarization axis allows for the blocking of this partially polarized light. By blocking the plane-polarized light, the glare is reduced and the fisherman can more easily see fish located under the water.



Reflection of light off of non-metallic surfaces results in some degree of polarization parallel to the surface.

Polarization by Refraction

Polarization can also occur by the refraction of light. Refraction occurs when a beam of light passes from one material into another material. At the surface of the two materials, the path of the beam changes its direction. The refracted beam acquires some degree of polarization. Most often, the polarization occurs in a plane perpendicular to the surface. The polarization of refracted light is often demonstrated in a Physics class using a unique crystal that serves as a double-refracting crystal. Iceland Spar, a rather rare form of the mineral calcite, refracts incident light into two different paths. The light is *split* into two beams upon entering the crystal. Subsequently, if an object is viewed by looking through an Iceland Spar crystal, two images will be seen. The two images are the result of the double refraction of light. Both refracted light beams are polarized - one in a direction parallel to the surface and the other in a direction perpendicular to the surface. Since these two refracted rays are polarized with a perpendicular orientation, a polarizing filter can be used to completely block one of the images. If the polarization axis of the filter is aligned perpendicular to the plane of polarized light, the light is completely blocked by the filter; meanwhile the second image is as bright as can be. And if the


The two refracted rays passing through the Iceland Spar crystal are polarized with perpendicular orientations.

filter is then turned 90-degrees in either direction, the second image reappears and the first image disappears. Now that's pretty neat observation that could never be observed if light did not exhibit any wavelike behavior.

Polarization by Scattering

Polarization also occurs when light is scattered while traveling through a medium. When light strikes the atoms of a material, it will often set the electrons of those atoms into vibration. The vibrating electrons then produce their own electromagnetic wave that is radiated outward in all directions. This newly generated wave strikes neighboring atoms, forcing their electrons into vibrations at the same original frequency. These vibrating electrons produce another electromagnetic wave that is once more radiated outward in all directions. This absorption and reemission of light waves causes the light to be scattered about the medium. (This process of scattering contributes to the blueness of our skies, a topic to be discussed later.)

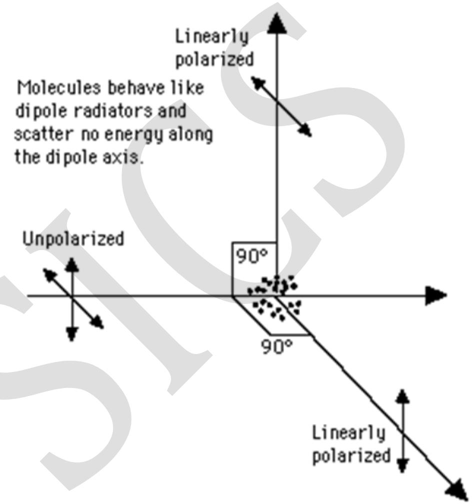


Figure: Polarization by Scattering

This scattered light is partially polarized. Polarization by scattering is observed as light passes through our atmosphere. The scattered light often produces a glare in the skies. Photographers know that this partial polarization of scattered light leads to photographs characterized by a *washed-out*sky. The problem can easily be corrected by the use of a Polaroid filter. As the filter is rotated, the partially polarized light is blocked and the glare is reduced. The photographic secret of capturing a vivid blue sky as the backdrop of a beautiful foreground lies in the physics of polarization and Polaroid filters.

Applications of Polarization

Polarization has a wealth of other applications besides their use in glare-reducing sunglasses. In industry, Polaroid filters are used to perform stress analysis tests on transparent plastics. As light passes through a plastic, each color of visible light is polarized with its own orientation. If such a plastic is placed between two polarizing plates, a colorful pattern is revealed. As the top plate is turned, the color pattern changes as new colors become blocked and the formerly blocked colors are transmitted. A common Physics demonstration involves placing a plastic protractor between two Polaroid plates and placing them on top of an overhead projector. It is known that structural stress in plastic is signified at locations where there is a large concentration of colored bands. This location of stress is usually the location where structural failure will most likely occur. Perhaps you wish that a more careful stress analysis were performed on the plastic case of the CD that you recently purchased.

Polarization is also used in the entertainment industry to produce and show 3-D movies. Three-dimensional movies are actually two movies being shown at the same time through two projectors. The two movies are filmed from two slightly different camera locations. Each individual movie is then projected from different sides of the audience onto a metal screen. The movies are projected through a polarizing filter. The polarizing filter used for the projector on the left may have its polarization axis aligned horizontally while the polarizing filter used for the projector on the right would have its polarization axis aligned vertically. Consequently, there are two slightly different movies being projected onto a screen. Each movie is cast by light that is polarized with an orientation perpendicular to the other movie. The audience then wears glasses that have two Polaroid filters. Each filter has a different polarization axis - one is horizontal and the other is vertical. The result of this arrangement of projectors and filters is that the left eye sees the movie that is projected from the right projector while the right eye sees the movie that is projected from the left projector. This gives the viewer a perception of depth.

Our model of the polarization of light provides some substantial support for the wavelike nature of light. It would be extremely difficult to explain polarization phenomenon using a particle view of light. Polarization would only occur with a transverse wave. For this reason, polarization is one more reason why scientists believe that light exhibits wavelike behaviour.

ELECTROMAGNETISM

Introduction

Study of Electricity in which electric charges are static i.e. not moving, is called electrostatics. Electric charge is characteristic developed in particle of material due to which it exert force on other such particles. It automatically accompanies the particle wherever it goes.

Key concepts:

Coulomb's Law of Force:- states that the force between two point charges at rest is dirctly porportional to the product of the magnitude of the charges, i.e., and is inversely proportional to the square of the distance between them i.e. . Thus, Coulomb's law in vector form becomes:

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$

Electric Field: When an electric charge is place at some point in space, this establishes everywhere a state of electric stress, which is called electric field. The space where charge influence can be felt, is called site of electric field. The electric field strength at a point is operationally defined as the force acting on a unit test chargeat that point

$$\vec{E} = \frac{\vec{F}}{q_\circ}$$

Electric Potential: The electrostatic potential at a point is the work done against the forces of the electric field in bringing unit positive test charge from a point at zero potential to the point.

Electric Dipole moment: - The product of the magnitude of teither charge of a dipole and the distance separating the two point charges.

Direct Current:-A steady flow of elctric charge carriers in one direction only.

Alternatining Current: - A current flowing in a circuit which reverses direction many time a second; it is caused by an alternating e.m.f. acting in a circuit and reversing many times a second

Ohm's Law:-States that the voltage across an arbitrary segment of an electric circuit equals the product of the resistance by the current**.**

Current Density: is the flow of current per unit area. Symbolized by J, it has a magnitude of i / A and is measured in amperes per square metre. Wires of different materials have different current densities for a given value of the electric field E ; for many materials, the current density is directly proportional to the electric field.

Gauss's Law: states that the electric flux across any closed surface is proportional to the net electric charge enclosed by the surface. The law implies that isolated electric charges exist and that like charges repel one another while unlike charges attract. Gauss's law for magnetism states that the magnetic flux across any closed surface is zero; this law is consistent with the observation that isolated magnetic poles (monopoles) do not exist.

Magnetic Field: A magnetic field is one of the the constituents of an electromagnetic field It si produced by current-carrying conductors, by moving charged particles and bodies, by magnetized bodies or by variable electirc field Its distinguishing feature is that it acts only on moving charged particles and bodies.

Magnetic Flux: The flux (Φ) of a magnetic field through a small plane surface is the product of the area of the surface and the component of the flux density (B) normal to the surface. If the plane is inclined at an angle (φ) to the direction of the magnetic field, and has an area (A), then

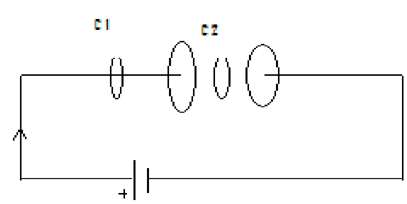
$\Phi = B\,A \sin\varphi$

Displacement current

British physicist James C. Maxwell gave final shape to all phenomenon connecting electricity and magnetism. He noticed an inconsistency in Ampere's Law connecting Electric current and magnetic fields and an asymmetry in the laws of electromagnetism. To remove this inconsistency he suggested the idea of Displacement current.

Displacement current

- Ampere's circuital law is $\oint_C \vec{B}.d\vec{l} = \mu_0 I$
- Where I is the conduction current across a surface whose periphery is the loop C. Let's consider a parallel plate capacitor shown by plates A and B which is being charged with a battery. During the time when capacitor is being charged, a current I flows through the connecting wires which varies with respect to time.
- This current will produce a magnetic field around the wires which can be detected using a compass



- Along with C1 the magnetic field is also observed at location C 2 though no current is threading through this loop.

Maxwell suggested that varying electric field within two disks produce this magnetic field. Hence this varying electric field is equivalent to a current. He called this Displacement current.

- Certain well-known physics experiments demonstrate the magnetic field produced by Maxwell's displacement current addresses the question of whether the displacement current acts as a source of magnetic field in the same way as a current in a wire would. Expression for this current is given in Maxwell's equation.

Maxwell's Equations

- Maxwell's equations are complete sum up of electromagnetism.
- It informs us the relationships and interdependence between electrical fields, magnetic fields, Electric current and the charge based on their time dependence.

Maxwell's First Equation - Gauss's Law in electrostatics:

It states that the total electric flux through any closed surface S is always equal to $1/\varepsilon_0$ times the net charge inside the surface i.e., $\oint_S \vec{E}.d\vec{S} = \frac{q}{\varepsilon_0}$

Maxwell's Second Equation – Gauss's Law in Magnetism:

It states that the total electric flux through any closed surface S is always equal to 0. i.e., $\oint_s \vec{B}.d\vec{S} = 0$

Maxwell's Third Equation - Faraday's Law of electromagnetic induction:

It states that the induced e.m.f. produced in a circuit is numerically equal to the rate of change of magnetic flux through it, i.e., $e = -\frac{d\phi B}{dt}$

The –ve sign shows that the induced e.m.f. produced opposes the rate of change of magnetic flux.

Since the e.m.f. can be defined as the line integral of electric field and ϕ_B can be expressed as the surface integral of the magnetic flux over a small area of the total surface S,

Hence the above equation can also be written as $\oint_C \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt} = -\frac{d}{dt}\oint_S \vec{B}.d\vec{S}$

Maxwell's Fourth Equation:

Maxwell-Ampere's Circuital Law It states that the line integral of magnetic field along a closed path is equal to μ_o times the total current (the sum of conduction current and displacement currents) threading through the surface bounded by the closed path.

$$\oint \vec{B}.d\vec{l} = \mu_0\left(I + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

lso be written in the following for

$$\oint \vec{B}.d\vec{l} = \mu_0\left[I + \varepsilon_0 \int_S \frac{d\vec{E}}{dt}.d\vec{S}\right]$$

Electromagnetic Waves

Waves

Wave is patterns of disturbances which propagates and carry energy with it.

- Visible examples of waves are water waves, and waves in a rope or a string.
- Sound wave is an example of energy propagation, which needs medium, molecules of the medium oscillate.
- Mechanical waves require the presence of a material medium in order to transport their energy from one location to another.
- Electromagnetic waves are waves which can travel through the vacuum of outer space as well as material. They do not need material medium.
- Sound waves are examples of mechanical waves while light waves are examples of electromagnetic waves.

Properties

- Electromagnetic waves are created by the vibration of an electric charge. This vibration creates a wave which has both an electric and a magnetic component. An electromagnetic wave transports its energy through a vacuum at a speed of 3.00×10^8 m/s (commonly known as "c"). The propagation of an electromagnetic wave through a material medium occurs at a net speed which is less than 3.00×10^8 m/s.

Source of Electromagnetic wave

- An accelerated charge generates electromagnetic waves. Oscillating charge is an example. An LC circuit is example of oscillating circuit.
- In the gap of capacitor charge do not exist.
- The oscillating charge does not propagate in vacuum. But for new magnetic field in the next location we need new charge.
- Since 1982 the question has been: Where does this new charge come from?
- Not from the upper conductor, because by definition, displacement current is not the flow of real charge. Not from somewhere to the left, because such charge would have to travel at the speed of light in a vacuum.
- Conventional electromagnetic theory says that the drift velocity of electric current is slower than the speed of light. Hence charge do not propagate and change in electric field act as electric current to produce varying magnetic field.
- Directions and natureElectromagnetic waves are transverse in the sense that associated electric and magnetic field vectors are both perpendicular to the direction of wave propagation. The Poynting vector defined by, **S** = **E** x **H** (W/m^2), indicates not only the magnitude of the energy flux density (energy flow rate per unit area per unit time, Watts/ m^2) but also the direction of energy flow. For simple electromagnetic waves, the Poynting vector is in the same direction as the wave vector, **k**.

Energy And Momentum In Electromagnetic Waves:

- **Generalization of momentum**
- Momentum is the quantity of translational invariance. As such, even fields as well as other things can have momentum, not just particles..
- **Momentum of mass less objects**
- Massless objects such as photons also carry momentum. The formula is:
- Where h is Planck's constant, λ is the $p = \frac{h}{\lambda} = \frac{E}{c}$
- Wavelength of the photon, E is the energy the photon carries and c is the speed of light.
- **Pressure by E/M waves**: since these waves carry momentum, they also can create force and pressure. This pressure for Sun rays is measured to be $7 \times 10^{-6}\ N/m^2$

Wave Equations

- Sinusoidal plane waves are one type of electromagnetic waves. Not all EM waves are sinusoidal plane waves, but all electromagnetic waves can be viewed as a linear superposition of sinusoidal plane waves traveling in arbitrary directions.
- A plane EM wave traveling in the x-direction is of the form
- $\mathbf{E(y,t) = E_{max} \cos(kx- \omega\ t+f)}$,
- $\mathbf{B(z,t) = B_{max} \cos(kx- \omega\ t+f)}$.
- E is the electric field vector and B is the magnetic field vector of the EM wave. K is **Propagation factor** given by 2Π/ λ.
- Speed of the wave is given by ω/k
- For electromagnetic waves E and B are always perpendicular to each other, and perpendicular to the direction of propagation. The direction of propagation is the direction of EXB.

Energy equations

We all know that if we stand in the sun we get hot. This occurs because we absorb electromagnetic radiation emitted by the Sun. So, radiation transport energy. The electric and magnetic fields in electromagnetic radiation are mutually perpendicular, and are also perpendicular to the direction of propagation (this is a unit vector). Furthermore, Equation can easily be transformed into the following relation between the electric and magnetic fields of an electromagnetic wave:

$$\mathbf{E} \times \mathbf{B} = \frac{E^2}{c}\hat{\mathbf{k}}.$$

Energy equations:

- **Energy conservation**
- We know that energy density of an electric field is given by
- whereas the energy density of a magnetic field satisfies
- This suggests that the energy density of a general electromagnetic field is
- u is the electromagnetic energy density,

Intensity of the wave is defined as the energy crossing per second per unit area perpendicular to the direction of e/m wave

Speed of electromagnetic wave:

In free space, velocity of electromagnetic waves is given by $C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

By this calculation $C = 3 \times 10^8\ ms^{-1}$

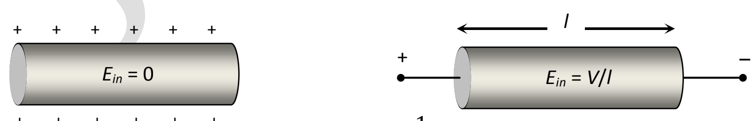
Where, μ_0 represents absolute permeability and ε_0 the absolute permittivity of the medium

$$V = \frac{1}{\sqrt{\mu\epsilon}}$$

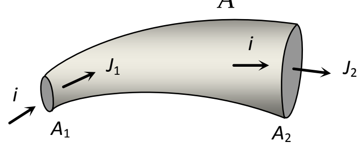
Where μ represents absolute permeability and ε the absolute permittivity of the medium.

Concepts

- *Human body, though has a large resistance of the order of kΩ (say 10 kΩ), is very sensitive to minute currents even as low as a few mA. Electrocution excites and disorders the nervous system of the body and hence one fails to control the activity of the body.*
- *1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross-section of the conductors.*
- *dc flows uniformly throughout the cross-section of conductor while ac mainly flows through the outer surface area of the conductor. This is known as skin effect.*
- *It is worth noting that electric field inside a charged conductor is zero, but it is non-zero inside a current carrying conductor and is given by $E = \frac{V}{l}$ where V = potential difference across the conductor and l = length of the conductor. Electric field outside the current carrying is zero.*
-



- *For a given conductor JA = i = constant so that $J \propto \frac{1}{A}$ i.e., $J_1 A_1 = J_2 A_2$; this is called equation of continuity*



- *If cross-section is constant, $I \propto J$ i.e. for a given cross-sectional area, greater the current density, larger will be current.*

MODULE III

QUANTUM MECHANICS

Introduction

In the previous chapters, we have learned fundamental laws of mechanics, vibrations and waves, special relativity, surface phenomena of liquid and Optics that are parts of classical physics theories. In the above courses, the physical quantities used are continuous, such as momentum, angular momentum and energies. These theories are valid in large scales, but not proper if they are employed in small scale like within atoms. In this chapter, we will discuss a new and important theory, called Quantum Mechanics, which is valid in a very small region but its large scale derivation could also give classical results.

It is known that quantum theory is a very important theory not only in physics, but also in many branches of science, such as life science and medical science.

The concept of quantum theory has great difference with classical physics. Even we can say, some concepts are opposite and contradicted (continuous and discrete for example)!

As a starting point to quantum theory, the particle's nature of light is usually explained in detail. This is because the quantum theory stemmed (originated) from the study of this part.

Blackbody radiation and Planck hypothesis

Historical recall: studies on light

- Without question, geometric optics is the earliest theory to describe the phenomenon of light and it is still used today. As you know, there are several basic laws and equations in geometric optics and they are still useful today in explaining the optical instrument. You might notice that light paths are all straight lines in geometric optics.

- In the early of 18th century, based on the existing knowledge of light at that time, Newton proposed his corpuscular theory of light to explain the light travels along a straight line in the space or uniform medium. In such a theory, light is regarded as a stream of particles that set up disturbances in the 'aether' (which is recognized as the medium of light propagation) of space. Newton's corpuscular theory is the first one to try to explain the nature of light but there exist many other interesting and beautiful effects that cannot be explained by Newton's corpuscular theory such as light bending around corners. Therefore it is not a correct theory.

- It is known that the Geometric optics is just a phenomenon, macroscopic and application study on light. That tight travels in a straight line in space or medium is just a physical model used to study light in geometric optics and it is the superficial phenomenon of light and it has nothing to do with the property of light.

- The wave nature of light is based on the experiments of interference and diffraction of light. Such a phenomenon can be explained by Huygen's principle and these phenomena put Newton's corpuscular theory of light into history.

- The complete wave theory of light was finally given by Maxwell (James Clerk) who showed that light formed the part of electromagnetic waves. The famous experiments to show light has wave property are Thomas Yang's double-slit interference experiment and Fraunhofer Single slit diffraction experiment. In 1801, Thomas Young did a double-slit interference experiment of light and showed that a wave theory was essential to interpret this type of phenomenon. And from then, Newton's corpuscular theory of light has been considered as a wrong theory for the explanation of the nature of light. Another famous experiment was done by Fraunhofer Single-slit diffraction. It should be also explained by wave theory of light.

- In the middle of 19th century, lights are recognized as part of electromagnetic spectrum and its space and time dependence follows the laws given in Maxwell equations which were considered as a complete theory for electromagnetic waves. Therefore, light belongs to electromagnetic waves and it does have the wave properties.

- Most physicists thought Physics skyscraper had been completely built and some minor problems like blackbody radiation and light speed would be soon found out using existing physics theory. However, Genius physicists thought that the blackbody radiation and light speed were the two patches of clouds in physics sky. In fact, the "minor" problems had shaken the "skyscraper" of classical physics.

- In order to explain the quantum nature of light, we have to understand how the following three problems to be solved: blackbody radiation, photoelectric effect and Compton Effect. Among the three problems, blackbody radiation is the most important one.

Basic concepts of blackbody radiation

Types of energy transmission are *conduction, convection and radiation*.

Conduction is transfer of heat energy by molecular vibrations not by actual motion of material. For example, if you hold one end of an iron rod and the other end of the rod is put on a flame, you will feel hot some time later. We can say that the heat energy reaches your hand by heat conduction.

Convection is transfer of heat by actual motion of material. The hot-air furnace, the hot-water heating system, and the flow of blood in the body are examples.

Radiation The heat reaching the earth from the sun cannot be transferred either by conduction or convection since the space between the earth and the sun has no material medium. The energy is carried by electromagnetic waves that do not require a material medium for propagation. The kind of heat transfer is called thermal radiation. Blackbody radiation problem was found in the research of thermal radiation.

Blackbody is defined as the body that can absorb all energies that fall on it. This is like a black hole. No lights or material can get away from it as long as it is trapped. A large cavity with a small hole on its wall can be taken as a blackbody.

Blackbody radiation: Any radiation that enters the hole is absorbed in the interior of the cavity, and the radiation emitted from the hole is called blackbody radiation.

Two successful laws

1. Stefan and Boltzmann's law: it is found that the radiation energy is proportional to the fourth power of the associated temperature.

$$M(T) = \sigma T^4 \qquad (1)$$

where $M(T)$ is radiation energy and actually it is the area under each curve, σ is called Stefan's constant determined by experiment and T is absolute temperature.

2. Wien's displacement law: the peak of the curve shifts towards longer wavelength as the temperature falls.

$$\lambda_{peak} T = b \qquad (2)$$

λ is the peak value of the curve $M_\lambda(T)$ and b is called Wein's constant. This law is quite useful while measuring the temperature of a blackbody with a very high temperature. You can see the example for how to measure the temperature on the surface of the sun.

• The above laws describe the blackbody radiation very well.

Problems exist in blackbody radiation and three formulas

• The problem existing in the relation is between the radiation power $M_\lambda(T)$ &the wavelength λ.

• Blackbody radiation has nothing to do with both the material used in the blackbody concave wall and the shape of the concave wall.

• Two typical blackbody radiation formulas: one is given by Rayleigh and Jeans and the other is given by Wein.

1. **Rayleigh and Jeans' formula:** In 1890, Rayleigh and Jeans obtained a formula using the classical electromagnetic (Maxwell) theory and the classical equipartition theorem of energy in thermotics. The formula is given by

$$M_\lambda(T) = C_1\lambda^{-4}T \qquad (3)$$

where $M_\lambda(T)$ is radiation power, C_1 is a constant number to be determined, λ is the wavelength of blackbody radiation, T is the absolute temperature. Rayleigh-Jeans formula was correct for very long wavelength in the far infrared but hopelessly wrong in the visible light and ultraviolet region. Maxwell's electromagnetic theory and thermodynamics are known as correct theory. The failure in explaining blackbody radiation puzzled physicists! It was regarded as *ultraviolet Catastrophe* (disaster).

2. **Wein's formula**: Later on in 1896, Wein derive another important formula using thermodynamics.

$$M_\lambda(T) = C_2\lambda^{-5}e^{-\frac{C_3}{\lambda T}} \qquad (4)$$

where C_2 and C_3 are constants to be determined. Unfortunately, this formula is only valid in the region of short wavelengths.

3. **Plank's empirical formula:** In 1900, after studying the above two formulas carefully, Planck proposed an empirical formula

$$M_\lambda(T) = 2\pi hc^2\lambda^{-5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \qquad (5)$$

where c is light velocity, k is Boltzmen's constant, e is the base of natural logarithms.

It is surprising that the experience formula can describe the curve of blackbody radiation exactly for all wavelengths.

Derivations from Planck's formula

1. Rayleigh and Jeans' formula;
   For very large wavelength, the Rayleigh-Jeans' formula can be obtained from Planck's formula;

2. Wein's formula;
   For smaller wavelength of blackbody radiation, the Wein's formula can be achieved also from Planck's experience formula;
3. Stefan and Boltzmann's law;
   Integrating Planck's formula with respect to wavelength, the Stefan and Boltzmann's law can be obtained as well.
4. Wien's displacement law.
   Finally, according to the basic mathematical theory and differentiating the Planck's formula with respect to wavelength, Wien's displacement law can also derived!

Planck's Hypotheses

1. The molecules and atoms composing the blackbody concave can be regarded as the linear harmonic oscillator with electrical charge;
2. The oscillators can only be in a special energy state. All these energies must be the integer multiples of a smallest energy ($\varepsilon_0 = h\nu$). Therefore the energies of the oscillators are $E = n\,h\nu$ with n = 1, 2, 3, …
3. derivation of Planck's fumula (omitted)
4. $h\nu$ was named photon by Einstein and

$$E = h\nu = \frac{hc}{\lambda} \tag{9.6}$$

is called *Planck-Einstein quantization law*. *h* is called Planck constant.

$$\begin{aligned} h &= 6.626\times10^{-34}\,J\cdot s \\ &= 4.136\times10^{-15}\,eV\cdot s \end{aligned} \tag{9.7}$$

The relation between joule (J) and Electron Volt (eV) is given by

$$\begin{aligned} 1eV &= 1.602\times10^{-19}\,J \\ 1J &= 6.242\times10^{18}\,eV \end{aligned} \tag{9.8}$$

Now we know that the electromagnetic waves are emitted in a quantum form, or wave package. But we still do not know whether they are absorbed in quantum form or move in space in a quantum form. For further details, see Photoelectric effect and Compton effect.

Example: Calculate the photon energies for the following types of electromagnetic radiation: (a) a 600kHz radio wave; (b) the 500nm green light; (c) a 0.1 nm X-rays.

Solution: (a) for the radio wave, we can use the Planck-Einstein law directly

$$E = h\nu = 4.136\times10^{-15}eV\cdot s\times600\times10^{3}Hz$$
$$= 2.48\times10^{-9}eV$$

As a homework, work out (b) and (c) using Planck-Einstein equation. After working out the problems, you will find that the higher frequency corresponds to the higher energy and the same for the shorter wavelength. The X-rays have quite high energy, so they have high power of penetration.

Planck associated the energy quanta only with the resonators in the cavity walls and Einstein extended them to the absorption of radiation in his explanation of the photoelectric effect.

The photoelectric effect

Basic investigation on photoelectric effect

The quantum nature of light had its origin in the theory of thermal radiation and was strongly reinforced by the discovery of the photoelectric effect.

In figure , a glass tube contains two electrodes of the same material, one of which is irradiated by light. The electrodes are connected to a battery and a sensitive current detector measures the current flow between them.

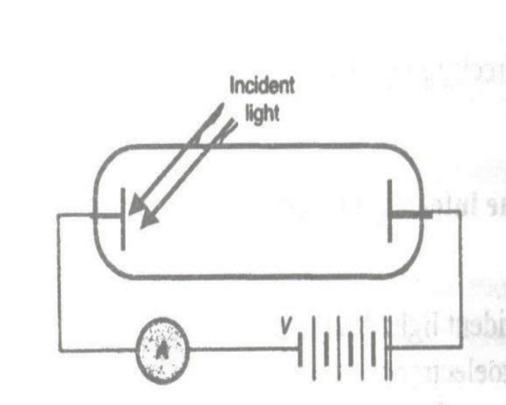


Figure: Apparatus to investigate the Photoelectric effect

The electrons in the electrodes can be ejected by the light and have a certain amount of kinetic energy.

Now we make some changes in the investigations:

1. the frequency and intensity of the light;
2. the electromotive force (e.m.f. or voltage);
3. the nature of electrode surface;

The current flow is a direct measure of the rate of emission of electrons from the irradiated electrode

It is found that:

(1). For a given electrode material, no photoemission exists at all below a certain frequency of the incident light. When the frequency increases, the emission begins at a certain frequency. The frequency is called threshold frequency of the material. The threshold frequency has to be measured in the existence of e.m.f. (electromotive force) as at such a case the photoelectrons have no kinetic energy to move from the cathode to anode. Different electrode material has different threshold frequency.

(2). The rate of electron emission is directly proportional to the intensity of the incident light.

That is: **Photoelectric current ∝ (is proportional to) the intensity of light**

(3) .Increasing the intensity of the incident light does not increase the kinetic energy of the

photoelectrons.

That is: **Intensity of light is not proportional to the kinetic energy of photoelectron**

However increasing the frequency of light does increase the kinetic energy of photoelectrons even for very low intensity levels.

That is: **Frequency of light ∝ kinetic energy of photoelectron**

(4). There is no measurable time delay between irradiating the electrode and the emission of photoelectrons, even when the light is of very low intensity. As soon as the electrode is irradiated, photoelectrons are ejected.

(5) The photoelectric current is deeply affected by the nature of the electrodes and chemical contamination of their surface.

It is found that the second and the fifth conclusions can be explained easily by classical theory of physics — Maxwell's electromagnetic theory of light, but the other three cases conflict with any reasonable interpretation of the classical theory of physics.

Problem investigation and Einstein's solution

1. **Three trouble cases in the photoelectric effect**

(a) The existence of a threshold frequency: Classical theory cannot explain the phenomenon as the light energy does not depend on the frequency of light. Light energies should depend on its intensity and the irradiating time.

(b) In third case, it is unbelievable that the kinetic energy of photoelectron does not depend on the intensity of incident light as the intensity indicates the light energy!

(c) No time delay in the photoelectric effect

Since the rate of energy supply to the electrode surface is proportional to the intensity of the light, we would expect to find a time delay in photoelectron emission for a very low intensity light beam. The delay would allow the light to deliver adequate energy to the electrode surface to cause the emission. Therefore "the no time delay phenomenon" puzzled us.

2. **Einstein's solution**

In 1905, Einstein solved the photoelectric effect problem by applying the Planck's hypothesis. He pointed out that Planck's quantization hypothesis applied not only to the emission of radiation by a material object but also to its transmission and its absorption by another material object. The light is not only electromagnetic waves but also a quantum. All the effects of photoelectric emission can be readily explained from the following assumptions:

(1) The photoemission of an electron from a cathode occurs when an electron absorbs a photon of the incident light;

(2) The photon energy is calculated by the Planck's quantum relationship: $E = h\nu$.

(3) The minimum energy is required to release an electron from the surface of the cathode. The minimum energy is the characteristic of the cathode material and the nature of its surface. It is called work function.

The equation for the photoelectric emission can be written out by supposing the photon energy is completely absorbed by the electron. After this absorption, the kinetic energy of the electron should have the energy of the photon. If this energy is greater than the work function of the material, the electron should become a photoelectron and jumps out of the material and probably have some kinetic energy.

Therefore we have the equation of photoelectric effect:

$$h\nu = A + \frac{1}{2}mv^2 \quad (9)$$

where $h\nu$ is photon energy, A is work function and is the photoelectron's kinetic energy.

Using this equation and Einstein's assumption, you should be able to explain all the results in the photoelectric effect: why does threshold frequency exist (problem)? why is the number of photoelectrons proportional to the light intensity? why does high intensity not mean high photoelectron energy (problem)? why is there no time delay (problem)? And also you should be able to solve the following problem.

Problem: Ultraviolet light of wavelength 150nm falls on a chromium electrode. Calculate the maximum kinetic energy and the corresponding velocity of the photoelectrons (the work function of chromium is 4.37eV).

The mass and momentum of photon

According to relativity, the particles with zero static mass are possibly existent. From the relativistic equation of energy-momentum,

$$E^2 = p^2c^2 + m_0^2c^4$$

when $m_0 = 0$, then $E = pc = mc^2 = h\nu$. So the mass of photon is

$$m_p = \frac{E}{c^2} = \frac{h\nu}{c^2} \quad (10)$$

the momentum of photon could also be found as

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \quad (11)$$

these formulas are valid for all possible particles with zero rest mass.

Compton Effect

A phenomenon called Compton scattering, first observed in 1924 by Compton, provides additional direct confirmation of the quantum nature of electromagnetic radiation. When X-rays impinges on matter, some of the radiation is scattered, just as the visible light falling on a rough surface undergoes diffuse reflection. Observation shows that some of the scattered radiation has smaller frequency and longer wavelength than the incident radiation, and that the change in wavelength depends on the angle through which the radiation is scattered. Specifically, if the scattered radiation emerges at an angle φ with the respect to the incident direction, and if λ and λ′ are the wavelength of the incident and scattered radiation, respectively, it is found that

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\varphi) = 2\frac{h}{m_0 c}\sin^2\left(\frac{\varphi}{2}\right) \qquad (12)$$

where m_0 *is the electron's rest mass.*

In figure 2, the electron is initially at rest with incident photon of wavelength λ and momentum *p*; scattered photon with longer wavelength λ′ and momentum *p*′ and recoiling electron with momentum *P*.

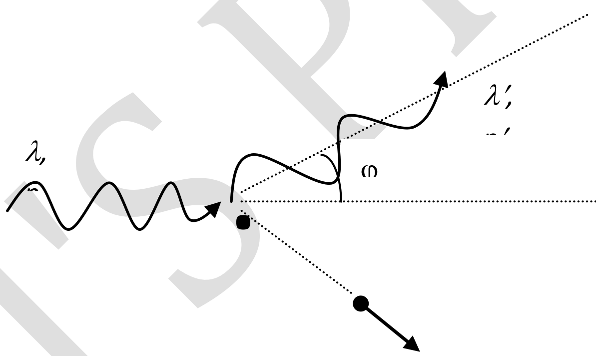


Fig. 2 Explanation of Compton effect model

The direction of the scattered photon makes an angle φ with that of the incident photon, and the angle between *p* and *p*′ is also φ.

$$\lambda_c = \frac{h}{m_0 c} = 0.00243 nm \qquad (13)$$

is called Compton wavelength. Therefore we have

$$\Delta\lambda = \lambda' - \lambda = 2\lambda_c \sin^2\left(\frac{\varphi}{2}\right) \qquad (14)$$

The duality of light

The concept that waves carrying energy may have a corpuscular (particle) aspect and that particles may have a wave aspect; which of the two models is more appropriate will depend on the properties the model is seeking to explain. For example, waves of electromagnetic radiation need to be visualized as particles, called photons to explain the photoelectric effect, for example. Now you may confuse the two properties of light and ask what the light actually is?

The fact is that the light shows the property of waves in its interference and diffraction and performances the particle property in blackbody radiation, photoelectric effect and Compton Effect. Till now we say that light has duality property.

We can say that light is wave when it is involved in its propagation only like interference and diffraction. This means that light interacts with itself. The light shows photon property when it interacts with other materials.

Line spectra and Energy quantization in atoms

The quantum hypothesis, used in the preceding section for the analysis of the photoelectric effect, also plays an important role in the understanding of atomic spectra.

Line spectra of Hydrogen atoms

- Hydrogen always gives a set of line spectra in the same position.

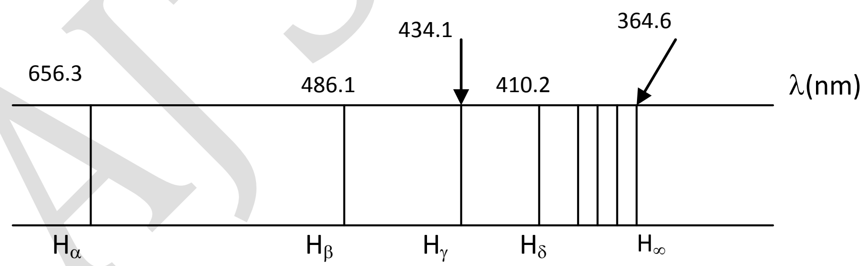


Figure: the Balmer series of atomic hydrogen

It is impossible to explain such a line spectrum phenomenon without using quantum theory. For many years, unsuccessful attempts were made to correlate the observed frequencies with those of a fundamental and its overtones (denoting other lines here). Finally, in 1885, Balmer found a simple formula that gave the frequencies of a group lines emitted by atomic hydrogen.

Under the proper conditions of excitation, atomic hydrogen may be made to emit the sequence of lines illustrated in Fig. 9.2. This sequence is called *series*. There is evidently a certain order in this spectrum and the lines become crowded more and more closely together as the limit of the series is approached. The line of Longest wavelength or lowest frequency, in the red, is known as H_α, the next, in the blue-green, as H_β, the third as H_γ, and so on.

Balmer found that the wavelength of these lines were given accurately by the simple formula

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad (15)$$

where λ is the wavelength, *R* is a constant called the Rydberg constant, and n may have the integral values 3, 4, 5, etc.* if λ is in meters,

$$R = 1.097 \times 10^7\, m^{-1} \qquad (16)$$

Substituting R and n = 3 into the above formula, one obtains the wavelength of the H_a-line: $\lambda = 656.3nm$

For n = 4, one obtains the wavelength of the H_β-line, etc. for n = ∞, one obtains the limit of the series, at λ= 364.6nm –shortest wavelength in the series.

Other series spectra for hydrogen have since been discovered. These are known, after their discoveries, as Lymann, Paschen, Brackett and Pfund series. The formulas for these are

Lymann series: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \quad n = 2,3,4,\Lambda$ (15a)

Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \quad n = 4,5,6,\Lambda$ (15b)

Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \quad n = 5,6,7,\Lambda$ (15c)

Pfund series: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \quad n = 6,7,8,\Lambda$ (15d)

The Lymann series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared. All these formulas can be generalized into one formula which is called the general Balmer series.

$$\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right), \qquad n = k+1, k+2, k+3, \Lambda \ ; \ k = 1,2,3,4,5 \qquad (17)$$

All the spectra of atomic hydrogen can be described by this simple formula. As no one can explain this formula, it was ever called ***Balmer formula puzzle***.

Bohr's atomic theory

Bohr's theory was not by any means the first attempt to understand the internal structure of atoms. Starting in 1906, Rutherford and his co-workers had performed experiments on the scattering of alpha Particles by thin metallic. These experiments showed that each atom contains a massive nucleus whose size is much smaller than overall size of the atom.

The atomic model of Rutherford:

The nucleus is surrounded by a swarm of electrons. To account for the fact, Rutherford postulated that the electrons *revolve* about the nucleus in orbits, more or less as the planets in the solar system revolve around the sun, but with electrical attraction providing the necessary centripetal force. This assumption, however, has two unfortunate consequences.

(1) Accelerated electron will emit electromagnetic waves; Its energy will be used up sometimes later and it would become a **dead atoms.** (A body moving in a circle is continuously accelerated toward the center of the circle and, according to classical electromagnetic theory, an accelerated electron radiates energy. The total energy of the electrons would therefore gradually decrease, their orbits would become smaller and smaller, and eventually they would spiral into the nucleus and come to rest.)

(2) The emitted frequency should be that of revolution and they should emit **continuous frequency.** (Furthermore, according to classical theory, the frequency of the electromagnetic waves emitted by a revolving electron is equal to the frequency of revolution. Their angular velocities would change continuously and they would emit a continuous spectrum (a mixture of frequencies), in contradiction to the line spectrum actually observed.

In order to solve the above contradictions, Bohr made his hypotheses:

(1) Static (or stable-orbit) postulate:

Faced with the dilemma, Bohr concluded that, in spite of the success of electromagnetic theory in explaining large scale phenomenon, it could not be applied to the processes on an atomic scale. He therefore postulated that an electron in an atom can revolve in certain stable orbits, each having a definite associated energy, without emitting radiation. The momentum *mvr* of the electron on the stable orbits is supposed to be equal to the integer multiple of h/2π. This condition may be stated as

$$mvr = n\frac{h}{2\pi} \qquad n = 1,2,3,\Lambda \tag{18}$$

where *n* is quantum number, this is the hypothesis of stable state and it is called the quantization condition of orbital angular momentum.

(2) Transition hypothesis

Bohr postulated that the radiation happens only at the transition of electron from one stable state to another stable state. The radiation frequency or the energy of the photon is equal to the difference of the energies corresponding to the two stable states.

$$h\nu = E_n - E_k \qquad \nu = \frac{E_n - E_k}{h} \tag{19}$$

(3) Corresponding principle

The new theory should come to the old theory under the limited conditions.

(4) Important conclusions

Another equation can be obtained by the electrostatic force of attraction between two charges and Newton's law:

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\frac{v^2}{r} \tag{20}$$

Solving the simultaneous equation of (9.5.5) and (9.5.7), we have

$$r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2, \qquad v_n = \frac{e^2}{2\varepsilon_0 h n} \qquad (n = 1,2,3,\Lambda\,) \tag{21}$$

So the total energy of the electron on the n^{th} orbit is

$$E_n = E_k + E_p = \frac{1}{2}mv_n^2 + \frac{-e^2}{4\pi\varepsilon_0 r_n} = -\frac{me^4}{8\varepsilon_0 h^2 n^2} \tag{22}$$

It is easy to see that all the energy in atoms should be discrete. When the electron transits from nth orbit to kth orbit, the frequency and wavelength can be calculated as

$$\begin{aligned} \nu &= \frac{E_n - E_k}{h} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \quad n > k \\ \frac{1}{\lambda} &= \frac{\nu}{c} = \frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{k^2} - \frac{1}{n^2}\right) = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \end{aligned} \tag{23}$$

where $R = \dfrac{me^4}{8\varepsilon_0^2 h^3 c}$ is called Rydberg constant.

It is found that the value of R is matched with experimental data very well. Till then, the puzzle of line spectra of atoms had been solved by Bohr since equation (10) is exactly the general Balmer formula.

When Bohr's theory met problems in explaining a little bit more complex atoms (He) or molecules (H_2), Bohr realized that his theory is full of contradictions as he used both quantum and classical theories. The problem was solved completely after De Broglie proposed that electron also should have the wave-particle duality. Since then, the proper theory describing the motion of the micro-particles, quantum mechanics, has been gradually established by many scientists.

De Broglie Wave

In the previous sections we traced the development of the quantum character of electromagnetic waves. Now we will turn to the consequences of the discovery that particles of classical physics also possess a wave nature. The first person to propose this idea was the French scientist Louis De Broglie.

De Broglie's result came from the study of relativity. He noted that the formula for the photon momentum can also be written in terms of wavelength

$$p = m_p c = \frac{h\nu}{c^2} c = \frac{h\nu}{c} = \frac{h}{\lambda}$$

If the relationship is true for massive particles as well as for photons, the view of matter and light would be much more unified.

De Broglie's point was the assumption that momentum-wavelength relation is true for both photons and massive particles.

So De Broglie wave equations are

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad E = h\nu \qquad (24a)$$

Where p is the momentum of particles, λ is the wavelength of particles. At first sight, to claim that a particle such as an electron has a wavelength seems somewhat absurd. The classical concept of an electron is a point particle of definite mass and charge, but De Broglie argued that the wavelength of the wave associated with an electron might be so small that it had not been previously noticed. If we wish to prove that an electron has a wave nature, we must perform an experiment in which electrons behave as waves.

De Broglie's wave speed is

$$V_d = \lambda\nu = \frac{h}{mv} \cdot \frac{E}{h} = \frac{mc^2}{mv} = \frac{c^2}{v} > c \qquad (24b)$$

V_d is phase speed that is hard to understand. The speed of mechanical waves is the same as their phase speed, but De Broglie's wave is not mechanical wave. The explanation of its physical properties will be given later. Leave it.

The Heisenberg Uncertainty principle

• In classical physics, there is no limitation for measuring physical quantities.

• Heisenberg (1927) proposed a principle that has come to be regarded as a basic postulate to the theory of quantum mechanics. It is called "uncertainty principle", and it limits the extent to which we can possess accurate knowledge about certain pairs of dynamical variables.

• Both momentum and position are vectors. When dealing with a real three dimensional situation, we take the uncertainties of the components of each vector in the same direction.

• Our sample calculation is restricted to the simplest interpretation of what we mean by uncertainty. A more elaborate statistical interpretation gives the lower limit of the uncertainty product as

$$a \sin\theta = m\lambda$$

for the first dark position, m = 1, the width of slit should be Δx. From Fig. 4, we have

$$\Delta p_x = p \cdot \sin\theta \approx \frac{h}{\lambda} \cdot \frac{\lambda}{\Delta x} \approx \frac{h}{\Delta x}$$

Consider other order diffractions, we have:

$$\Delta x \cdot \Delta p_x \geq h \qquad (25a)$$

But precise derivation gives

$$\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}. \qquad (25b)$$

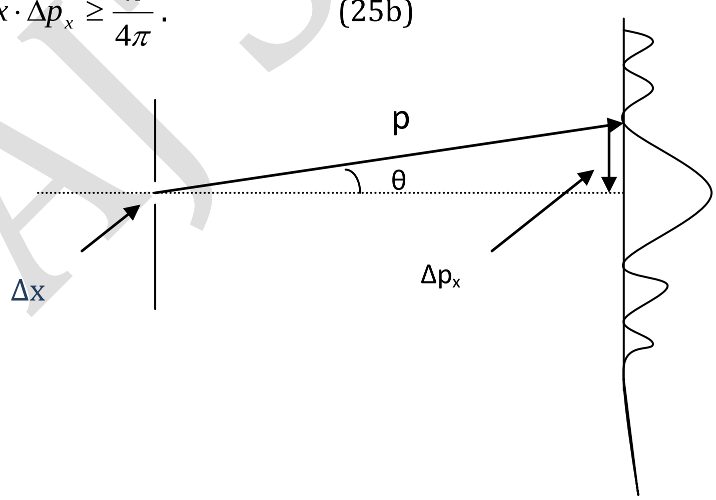


Figure: The electron single-slit diffraction

Another pair of important uncertainty is between time and energy, which can be derived by differentiating E with respective to p,

$$E = E_k + E_p = \frac{p_x^2}{2m} + E_p(x)$$

$$\frac{dE}{dp_x} = \frac{p_x}{m} + 0 = v_x \Rightarrow \Delta E = v_x \cdot \Delta p_x$$

$$\Delta E \cdot \Delta t = (\Delta t \cdot v) \cdot \Delta p = \Delta x \cdot \Delta p \geq h \qquad (25c)$$

Example: Suppose the velocities of an electron and of a rifle bullet of mass 0.03 kg are each measured with an uncertainty of $\Delta v = 10^{-3}$ ms^{-1}. What are the minimum uncertainties in their positions according to the uncertainty principle?

Solution: using $\Delta p_X = m\,\Delta v$, *for each,* the minimum position uncertainty satisfies $\Delta x\, m\, \Delta v = h$. For the electron, $m = 9.11´10^{-31}$kg. So

$$\Delta x = \frac{h}{m\Delta v} = \frac{6.626\times10^{-34}}{9.11\times10^{-31}\times10^{-3}} = 0.727m,$$

and for bullet,

$$\Delta x = \frac{h}{m\Delta v} = \frac{6.626\times10^{-34}}{0.03\times10^{-3}} = 3.5\times10^{-30}\,m$$

We can see from the proceeding example that classical theory is still useful and accuracy in the macro-cases, such as bullet. However you have to use quantum theory in the micro-world as uncertainty principle has to be used, such as electrons in atoms.

We can briefly review how quantum dynamics differs from classical dynamics. Classically, both the momentum and position of a point particle can be determined to whatever degree of accuracy that the measuring apparatus permits. However, from the viewpoint of quantum mechanics, the product of the momentum and position uncertainties must be at least as large as h. Therefore they could not be measured simultaneously. Why?

In order to understand the uncertainty principle, consider the following thought experiment. Suppose you wish to measure the position and momentum of an electron with a powerful microscope. In order for you to see the electron and thus determine its location, at least one photon must bounce off the electron and pass through the microscope to your eye.

When the photon strikes the electron, it transfers some of its energy and momentum to the electron. Thus, in the process of attempting to locate the electron very accurately, we have caused a rather large uncertainty in its momentum.

In other words, *the measurement procedure of itself limits the accuracy to which we can determine position and momentum simultaneously*.

Let us analyze the collision between the photon and the electron by first noting that the incoming photon has a momentum of h/l. As a result of this collision, the photon transfers part or all of its momentum to the electron. Thus the uncertainty in the electron's momentum after the collision is at least as great as the momentum of the incoming photon.

That is, $\Delta p = h/\lambda$, or $\lambda\ \Delta p = h$. Since the light has wave properties, we would expect the uncertainty in the position of the electron to be on the order of one wavelength of the light being used to view it, because the diffraction effects. So $\Delta x = \lambda$ and we also have the uncertainty relation. On the other hand, according to the diffraction theory, the width of slit cannot be smaller as the wavelength we used in order to observe the first order dark fringe i.e. $\Delta x \geq \lambda$. So we also have $\Delta x\ \Delta p \geq h$.

Schrödinger equation

Schrödinger equation in Quantum mechanic is as important as the Newton equations in Classical physics. The difference is that in Newton's mechanics, the physical quantities, like coordinates and velocities, but in quantum mechanics, the particles are described by a function of coordinates and time. The function has to be a solution of a Schrödinger equation.

The interpretation of wave function

It is known that the functions have been used to describe the mechanic waves. The magnitude of the value of a wave function means the energy and the position of a particle in a particular moment. We also use a function to describe the motion of particles in quantum mechanics but it has different meanings. The function is called wave function.

The symbol usually used for this wave function is Ψ, and it is, in general, a function of all space coordinates and time. Just as the wave function for mechanical waves on a string provides a complete description of the motion, the wave function $\Psi(x,y,z,t)$ for a particle contains all the information that can be known about the particle.

Two questions immediately arise. First, what is the meaning of the wave function Y for a particle? Second, how is Y determined for any given physical situation? Our answers to both these questions must be qualitative and incomplete.

Answer to question 1: The wave function describes the distribution of the particle in space. It is related to the probability of finding the particle in each of various regions; the particle is most likely to be found in regions where Y is large, and so on. If the particle has a charge, the wave function can be used to find the charge density at any point in space. In addition, from Y one can calculate the average position of the particle, its average velocity, and dynamic quantities such as momentum, energy, and angular momentum. The required techniques are far beyond the scope of this discussion, but they are well established and no longer subject to any reasonable doubt.

The answer to the second question is that the wave function must be one of a set of solutions of a certain differential equations called *Schrödinger equation, developed by* Schrödinger in 1925. One can set up a Schrödinger equation for any given physical situation, such as electron in hydrogen atom; the functions that are solutions of this equation represent the various possible physical states of the system. These solutions are usually a series of functions corresponding to specific energy levels.

The precise interpretation of the wave function was given by M Born in 1926. He pointed out a statistical explanation for the wave functions which are given below:

(1) the wave function Ψ which is the solution of Schrödinger equation, is *a probability wave function.* $|\Psi|^2 = \Psi \cdot \Psi^*$ is the density of probability, $|\Psi(x)|^2 dx$ denotes the probability of the particle in question which can be found near the point *x*, at the interval of *dx*;

(2) the wave functions have to be satisfied with the standard conditions of single value, continuity and finite. The physical meaning of the conditions is that at a particular moment, the probabilities of finding the particle in a particular point in space should be unique, finite (less than 1) and that the distribution of the probabilities of the appearance of the particle in space should be continuous.

(3) the wave functions should be normalized,

$$\int |\Psi(\vec{r},t)|^2 d^3r = 1 \qquad (9.8.1)$$

This is called normalizing condition. It means that at a particular time, the probability of finding the particle in the whole space is equal to 1.

(4) The wave function is satisfied with the superposition theorem. This means that if y_1 and y_2 are the possible states of the particle, and their combination state, $c_{1\psi_1} + c_2\psi_2$, is also a possible state of the particle.

Schrödinger's time dependent wave equation:

As stated before, Schrödinger equation in quantum mechanics is as important as Newton's laws in classical mechanics. He shared the Nobel prize in 1933 with Dirac who established the relativistic quantum mechanics while Schrödinger built up the non-relativistic quantum mechanics.

Schrödinger equation cannot be derived from classical theory, so it is regarded as one of several fundamental hypotheses in quantum mechanics.

The five fundamental postulates in quantum mechanics are given below:

- A state of micro-system can be completely described by a wave function;
- Physical quantities can be represented by the linear and Hermitian operators that have complete set of eigen-functions.
- Fundamental quantization condition, the commutator relation between coordinate and momentum.
- The variation of wave function with time satisfies Schrodinger equation.
- Identical principle: the systematic state is unchanged if two identical particles are swapped in the system.

The general form of Schrödinger equation is widely used in quantum mechanics is

$$i\eta\frac{\partial\Psi(\vec{r},t)}{\partial t} = \hat{H}\Psi(\vec{r},t) \qquad (26)$$

Where i is the imaginary unit, η is called universal constant which is equal to the Planck's constant h divided by 2π; H is the Hamiltonian of the system and Ψ is the wave function for the particle concerned.

In most cases, the wave function Ψ can be separated into the product of two functions which are those of coordinates and time respectively, that is

$$\Psi(\vec{r},t) = \psi(\vec{r})e^{-i\frac{Et}{\eta}} \qquad (27)$$

The Schrödinger equation for the stationary state could be obtained:

$$\hat{H}\psi(\vec{r}) = E\psi(\vec{r}) \qquad (28)$$

Where E is the total energy of the system or the particle.

Solving the above equation, the wave function can be obtained and the energy corresponding to the wave function could be also achieved. Classically, the Hamiltonian H is defined as the summation of kinetic and potential energies:

$$H = E_k + E_p = \frac{P^2}{2m} + V(\vec{r}, t) \tag{29}$$

The potential energy in quantum mechanics does not change with time in most cases. Therefore it is just a function of coordinates x, y, and z. The Hamiltonian H in quantum theory is an operator which could be easily obtained by the following substitutions:

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \qquad P \rightarrow -i\hbar\nabla \tag{30}$$

Where ∇ is Laplace operator which is expressed as

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \tag{31}$$

therefore,

$$P_x \rightarrow -i\hbar\frac{\partial}{\partial x}, \quad P_y \rightarrow -i\hbar\frac{\partial}{\partial y}, \quad P_z \rightarrow -i\hbar\frac{\partial}{\partial z}$$

When the Hamiltonian does not contain time variable, it gives

$$\begin{aligned} \hat{H} &= \frac{P^2}{2m} + V(r) = -\frac{\hbar^2}{2m}\nabla^2 + V(r) \\ &= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(r) \end{aligned} \tag{32}$$

Substituting the above formula into (4), we have

$$\left[-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(r)\right]\psi(x, y, z) = E\psi(x, y, z) \tag{33}$$

Rearrange this formula, we have

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi+\frac{2m}{\eta^2}(E-V)\psi=0$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{2m}{\eta^2}(E-V)\psi=0 \qquad (34)$$

APPLICATION OF QUANTUM MECHANICS

One-dimensional infinite deep potential well

As a simple explicit example of the calculation of discrete energy levels of a particle in quantum mechanics, we consider the one-dimensional motion of a particle that is restricted by reflecting walls that terminate the region of a constant potential energy.

It is supposed that the V(x) = 0 in the well but becomes infinity while at x = 0 and *L*. Therefore, the particle in the well cannot reach the perfectly rigid walls. So there is no probability to find the particle at x =0, *L*. According to the interpretation of wave functions, they should be equal to zero at x =0, *L*.

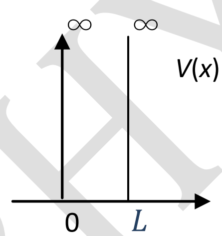


Figure: One-dimensional square well potential with perfectly rigid walls.

According to the interpretation of wave functions, they should be equal to zero at x =0, *L*.

These conditions are called boundary conditions. Let's derive the Schrödinger equation in the system. Generally,

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{2m}{\eta^2}(E-V)\psi=0$$

But now only one dimension and and V=0, so we have

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\eta^2}E\psi=0\Rightarrow\frac{d^2\psi}{dx^2}+k^2\psi=0 \qquad (37)$$

This differential equation having the same form with the equation of SHM. The solution can be written as

$$\psi(x) = A\sin kx + B\cos kx \tag{38}$$

with
$$k = \left(\frac{2mE}{\eta^2}\right)^{1/2} \tag{39}$$

Application of the boundary condition at x =0 and L gives

$$\psi(0) = 0 \quad \Rightarrow \quad B\cos k0 = 0$$

$$\psi(L) = 0 \quad \Rightarrow \quad A\sin kL + B\cos kL = 0$$

Solving these equations, we obtain

$$A\sin kL = 0 \quad \& \quad B = 0$$

Now we do not want A to be zero, since this would give physically uninteresting solution y =0 everywhere. There is only one possible solution that is given as

$$\sin kL = 0, \quad \Rightarrow \quad kL = n\pi \qquad n = 1,2,3,\Lambda \tag{40}$$

So the wave function for the system is

$$\psi(x) = A\sin\frac{n\pi}{L}x \qquad n = 1,2,3,\Lambda \tag{41}$$

It is easy to see that the solution is satisfied with the standard conditions of wave functions and the energies of the particle in the system can be easily found by the above two equations (39) and (40):

$$\frac{n\pi}{L} = \left(\frac{2mE}{\eta^2}\right)^{1/2} \Rightarrow E = \frac{\pi^2\eta^2 n^2}{2mL^2} \qquad n = 1,2,3,\Lambda \tag{42}$$

It is evident that n = 0 gives physically uninteresting result y = 0 and that solutions for negative values of n are not linearly independent of those for positive n. the constants A and B can easily be chosen in each case so that the normal functions have to be normalized by.

$$\int_0^L \psi^*(x)\psi(x) = A^2\frac{L}{2} = 1 \tag{43}$$

So the one-dimensional stationary wave function in solid wells is

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \qquad (44)$$

And its energy is quantized,

$$E = \frac{\pi^2 \eta^2 n^2}{2mL^2} \qquad n = 1,2,3,\Lambda \qquad (45)$$

Discussion:

1. Zero-point energy: when the temperature is at absolute zero degree 0°K, the energy of the system is called the zero point energy. From above equations, we know that the zero energy in the quantum system is not zero. Generally, the lowest energy in a quantum system is called the zero point energy. In the system considered, the zero point energy is

$$\varepsilon_0 = E_1 = \frac{\pi^2 \eta^2}{2mL^2} \qquad (46)$$

This energy becomes obvious only when $mL^2 \sim \eta^2$. When $mL^2 >> \eta^2$, the zero-point energy could be regarded as zero and the classical phenomenon will be appeared.

2. Energy intervals

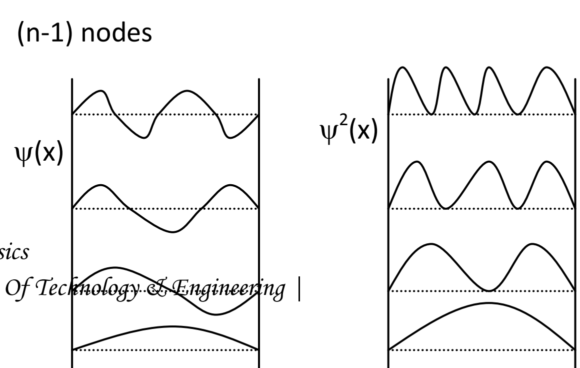
Comparing the interval of two immediate energy levels with the value of one these two energies, we have

$$\frac{E_{n+1} - E_n}{E_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{2n+1}{n^2} \qquad (47)$$

If we study the limiting case of the system, i.e. n →∞ or very large, the above result should be 2/n and approaches zero while n is very large. Therefore, the energy levels in such a case can be considered continuous and come to Classical physics.

3. Distributions of probabilities of the particle appearance in the solid wall well

n = 1, the biggest probability is in the middle of the well, but it is zero in the middle while n =2.



$x = 0$ $x = L$ $x = 0$ $x = L$

Figure: the wave shape and distribution of the density of the probability of the particle appearance.

When n is very large, the probability of the particle appearance will become almost equal at any point and come to the classical results.

The phenomenon within well can also be explained by standing wave theory.

$$L = n \cdot \frac{\lambda}{2} = n \cdot \frac{h}{2p} \Rightarrow p = \frac{nh}{2L} = \frac{n\pi\hbar}{L} \qquad (48)$$

where λ should be the wavelength of De Broglie wave function in the well and the energy in the infinite deep well could be easily found using the particle momentum found above:

$$E = \frac{p^2}{2m} = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad (49)$$

This is the simplest example in quantum mechanics. Students should fully understand all the concepts involved in the example.

Potential Barrier-Tunnelling Effect

This effect cannot be understood classically. When the kinetic energy of a particle is smaller than the potential barrier in front of it, it still have some probability to penetrate the barrier. This phenomenon is called tunneling effect. In quantum mechanics, when the energy of the particle is higher than the potential barrier, the particle still has some probabilities to be reflected back at any position of the barrier.

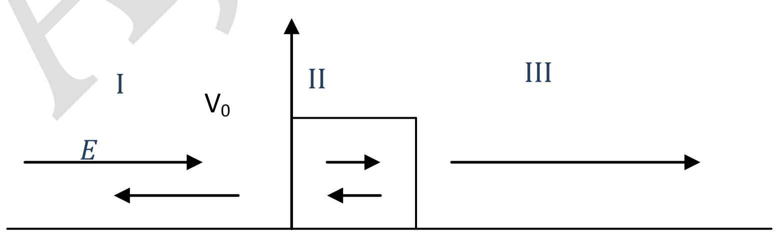


Figure: The tunnel effect

If you would like to solve the problem quantum-mechanically, you have to solve the Schrödinger equations at the three regions in Figure and the standard and boundary conditions of wave function have also to be used to solve them. This case is a little bit more complex than previous one.

$$V(x) = \begin{cases} V_0 & 0 \le \mathrm{x} \le \mathrm{a} \\ 0 & x < 0, x > a \end{cases} \qquad (50)$$

Based on the potential in different regions, the Schrödinger equations at the three regions in above figure could be written as $\frac{d^2\psi}{dx^2} + k_1^2\psi = 0$ (I & III) $(x < 0, x > \mathrm{a})$ (51)

$$\frac{d^2\psi}{dx^2} + k_2^2\psi = 0 \quad \text{(II)} \qquad (0 \le x \le \mathrm{a}) \qquad (52)$$

where $$k_1^2 = \frac{2mE}{\eta^2}, \qquad k_2^2 = \frac{2m(E - V_0)}{\eta^2} \qquad (53)$$

The concept of atomic structure in quantum mechanics

We consider one-electron atoms only, Hydrogen or hydrogen-like atoms. For hydrogen atom, a central proton holds the relatively light electron within a region of space whose dimension is of order of 0.1nm.

As we know, the one-dimensional example has one quantum number. But for an atomic quantum system, it is three dimensional, so we have three quantum numbers to determine the state of the system.

In order to set up the Schrödinger equation of the hydrogen system, we need to find potential energy for the system. It is known that such a system contain a proton in the center of the atom and an electron revolving around the proton. The proton has positive charge and the electron has negative charge.

If we assume that the proton should be much heavier than the electron and the proton is taken as stationary, the electron moves in the electrical field of the proton. The electrostatic potential energy in such a system can be written easily as

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r} \qquad (54)$$

Of course this formula is obtained in the electrically central force system, the proton is located at r = 0 and the electron is on the spherical surface with radius *r*. The Schrödinger equation of the atomic system is

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{8\pi^2 m}{h^2}\left(E+\frac{e^2}{4\pi\varepsilon_0 r}\right)\psi=0 \qquad (55)$$

Completely solving the problem (using spherical coordinates, $\Psi = \Psi(r,\theta,\varphi)$), the wave function should have the form $\Psi = \Psi_{nlm}(r,\theta,\varphi)$. Therefore, solving the Schrödinger equation, we have three quantum numbers. Of course we have another one for electron spin.

Four quantum numbers:

1. Energy quantization --- principal quantum number

The first one is called principal quantum number *n*. The energy of the system depends on this only while the system has a spherical symmetry. The energy is

$$E_n=-\frac{me^4}{8\varepsilon_0^2 h^2}\cdot\frac{Z^2}{n^2} \qquad n=1,2,3,\Lambda \qquad (56)$$

where Z is the number of protons in hydrogen-like atoms.

2. **Angular momentum quantization**

When the symmetry of the system is not high enough, the angular momentum is not in the ground state (*l* = 0) but in a particularly higher state. For a given *n*, angular momentum can be taken from 0 to *n* -1 different state. In this case, the angular momentum has been quantized

$$L=\sqrt{l(l+1)}\,\frac{h}{2\pi} \qquad l=0,1,2,3,\Lambda\ ,(n-1) \qquad (57)$$

l is called *angular quantum number* and taken from 0 to (*n* −1). The corresponding atomic states are called *s, p , d, f,* … states respectively.

3. Space quantization --- magnetic quantum number

when atom is in a magnetic field, the angular momentum will point different directions in the space and then has different projection value on the direction of the magnetic field. This phenomenon is called space quantization. Its quantum number, called *magnetic quantum number*, denoted by *m*, has values from –*l* to *l.*

$$L_z=m\frac{h}{2\pi} \qquad m=-l,-l+1,\Lambda\ ,-1,0,1,2,\Lambda\ ,l \qquad (58)$$

m is called magnetic quantum number. Hence, even we have the same angular momentum, it still has (2*l*+1) different states which have different orientation in the space.

4. Spin quantization ---- spin quantum number

The electron has two spin state generally, its spin angular momentum is defined as

$$L_s = \sqrt{s(s+1)}\frac{h}{2\pi} \qquad (59)$$

S is called *spin quantum number*. For electron, proton, neutron, s is equal to ½.

5. The total number of electronic states for a given main quantum number *n*

The possible number of electronic states in an atom in E_n state should be:

(1) when *n* is given, the angular momentum can be changed from 1 to n-1;

(2) for each *l*, magnetic quantum number *m* (l_Z) *can be from –l to +l including zero, this is* (2*l*+1);

(3) for each *nlm state, there are* ***two*** *spin states.*

Therefore the total number of electronic states for a given n should be:

$$Z_n = \sum_{l=0}^{n-1} 2(2l+1) = 2n^2 \qquad (60)$$

This explains why in the first shell of an atom, two electrons can be hold, and at the second shell, eight electrons could be hold and so on.

6. Electronic energy levels in and out atom

The energy levels in an atom is given by

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\cdot\frac{Z^2}{n^2} \qquad n = 1,2,3,\Lambda$$

The lowest energy in the Hydrogen-like atoms is *n* = 1, for hydrogen atom, the ground energy is

$$E_1 = -\frac{me^4}{8\varepsilon_0^2 h^2} = -13.6eV \qquad (61)$$

The other energies are called excited energies and their corresponding states are called excited states. As the excited state energies are inversely proportional to n^2, the excited energy levels can be easily obtained.

When the principal quantum number is very large, the energy approaches zero. From above we can see that from n=6 to n = infinity, all the corresponding energy levels are packed between –0.38eV and zero. Therefore, at the very large region of n, the discrete energy levels can be recognized as continuous!

B. Atom with multiple electrons:

In 1925, Pauli arrived at a principle from his experiments. This principle says that in atom it is impossible for two or more electrons to stay in one electronic state. This principle is called Pauli Exclusion Principle.

The distribution of the electrons in an atom has rules. They are determined by the energies of the electronic state.

In addition to the principal quantum number, the angular momentum number is also quite important. For any principal number, different angular momentum number l has different name. It is given by

$l =$ 0, 1, 2, 3, 4, 5, … s, p, d, f, g, h, …

$2(2l+1) =$ 2, 6, 10, 14, 18, …

The ground-state configuration of many of the elements can be written down simply from knowledge of the order in which the energies of the shells increase. This order can be inferred from spectroscopic evidence and is given as follows:

1*s*,2*s*,2p,3*s*,3*p*,[4*s*,3*d*],4*p*,[5*s*,4*d*],5*p*,[6*s*,4*f*,5*d*],6*p*,[7*s*,5*f*,6*d*]

The brackets enclose shells that have so nearly the same energy that they are not always filled in sequence. Those shell energies are close together because the increase in *n* and the decrease in *l* tend to compensate each other; thus the 4*s*, which has a higher energy than 3*d* state in hydrogen, is depressed by the penetration caused by its low angular momentum.

The *s* shell in each bracket is always filled first, although it can lose one or both of its electrons as the other shells in the bracket fill up. Apart from the brackets (*parenthesis and braces*), there are no deviations from the indicated order of filling.

Because of time, we have to terminate quantum theory here. However, quantum theory is far more complicated than we introduced above. For hydrogen, its eigenvalue equations can be written as

$$\begin{aligned} \hat{H}\psi_{nlm}(r,\theta,\varphi) &= E_n\psi_{nlm}(r,\theta,\varphi) \\ \hat{L}^2\psi_{nlm}(r,\theta,\varphi) &= l(l+1)\eta^2\psi_{nlm}(r,\theta,\varphi) \\ \hat{L}_z\psi_{nlm}(r,\theta,\varphi) &= m\eta\psi_{nlm}(r,\theta,\varphi) \end{aligned} \qquad (62)$$

APPENDIXES

Universal physical constants		
Name	**Symbol**	**Value (Unit)**
1. Acceleration due to gravity:	g	= 9.81 m/s^2
2. Atomic mass unit:	amu	= 1.660 X 10^{-24} g
3. Avogadro's number:	N	= 6.023 X 10^{-23}g /mol
4. Bohr magneton (magnetic moment):	β	= 9.273 X 10^{-24} A m^2
5. Boltzmann's constant:	k	= 1.380 X 10^{-23} J/K= 8.620 X 10^{-5} eV/k
6. Electron rest mass:	m_e	= 9.109 X 10^{-31} Kg
7. Electronic charge	e	= 1.602 X 10^{-19} C
8. Faraday's constant	F	= 9.649 X 10^{4} C/mol
9. Gas constant	R	= 8.314 J/mol/K
10. Mass of proton	m_p	= 1.673 X 10^{-24} g
11. Mass of electron	m_e	= 9.108 X 10^{-28} g
12. Planck's constant	h	= 6.626 X 10^{-34} Js
13. Permeability of free space	μ_0	= 1.257 X 10^{-6} H/m
14. Velocity of light in free space	c	= 2.998 X 10^{8} m/s
15. Permittivity of free space	ε_0	= 8.854 X 10^{-12} F/m
16. Volume of 1kg mole of ideal gas at N.T.P.	V	= 22.41 m^3
17. Magnetic constant	μ_i	= 1.2566 X 10^{-8} H/cm
18. Gravitation constants	G	= 6.670 X 10^{-1}
19. Radius of electron	r_e	= 2.81777 X 10^{-15} m.

Physical Constants

DENSITY
1. Water → 1000 Kgm^{-3}
2. Copper → 8900 Kgm^{-3}
3. Steel → 7800 Kgm^{-3}
4. Brass → 8600 Kgm^{-3}
5. Iron → 7500 Kgm^{-3}

YOUNGS MODULUS
1 .Box wood → 1×1010 Nm^{-2}
2. Teak wood → 1.7×1010 Nm^{-2}
3 .Wrought iron and steel → 20×1010 Nm^{-2}

RIGIDITY MODULUS
1. Aluminium → 2.5×1010 Nm^{-2}
2. Brass → 3.5 to 3.4×1010 Nm^{-2}
3. Cast iron → 5.0×1010 Nm^{-2}
4. Copper → 3.4 to 3.6×1010 Nm^{-2}
5. Steel(Cast) → 7.6×1010 Nm^{-2}
6. Steel(Mild) → 8.9×1010 Nm^{-2}

THERMAL CONDUCTIVITY
1. Card board → 0.04 $Wm^{-1}k^{-1}$
2. Ebonite → 0.7 $Wm^{-1}k^{-1}$
3. Glass → 1 $Wm^{-1}k^{-1}$
4. Wood & Rubber → 0.15 $Wm^{-1}k^{-1}$

SPECIFIC HEAT CAPACITY
1. Brass → 913 $JKg^{-1}K^{-1}$
2. Copper → 385 $JKg^{-1}K^{-1}$
3. Water → 4186 $JKg^{-1}K^{-1}$

BAND GAP
1. Germanium → 0.67 eV
2. Silicon → 1.12 eV

WAVELENGTH
Sodium Vapour Lamp → 5893 A^0
Mercury vapour lamp
1. Red → 6234 A^0
2. Yellow I → 5791 A^0
3. yellow ii → 5770 A^0
4. Green → 5461 A^0
5. Blueish green → 4916 A^0
6. Blue → 4358 A^0
7. Violet I → 4078 A^0
8. Violet ii → 4047 A^0

COMPRESSIBILITY
1. Water → 4.59×10-10 m^2N^{-1}
2. Castor oil → 4.7×10-10 m^2N^{-1}
3. Kerosene → 7.5×10-10 m^2N^{-1}

TEMPERATURE CO-EFFICIENT OF RESISTANCE
1. Aluminium → 0.0043 per ◦C
2. Brass → 0.001 to 0.002 per ◦C
3. Copper → 0.0039 per ◦C

COEFFICIENT OF VISCOSITY (AT ROOM TEMP.)
1. Water → 0.00081 Nsm^{-2}
2. Kerosene → 0.002 Nsm^{-2}
3. Glycerin → 0.3094 Nsm^{-2}

Conversion factors	
1. 1 atm = 0.101325 X 10^6 N/m^2 = 760 mm Hg = 10^5 Pascal	21. 1 Gauss = 10^{-4} Wb/m^2
2. 1° = 0.01745 rad	22. 1 gram-calorie = 4.185 J
3. 1 Å = 10^{-10} m = 0.1 nm	23. 1 Henry = 1 V-s/A
4. 1 A.hr = 3.6 Kc	24. 1 Hertz = 1 cycle/s
5. 1 ampere = 1 C/s	25. 1 HP = 0.7457 KW
6. 1 Btu = 1.05506 kJ	26. 1 Kgf/mm^2 = 9.81 MN/m^2
7. 1 Btu/lb = 2326 J/Kg	27. 1 KSi/in = 1.1 MN/m^3/2
8. 1 Btu/ft^3 = 37.2589 KJ/m^3	28. 1 lb/Cu.ft = 16.01 kg/m^3
9. 1 bar = 10^{-1} MPa	29. 1 lumen = 0.0016 W (at 0.55 m)
10. 1 Calorie = 4.18 J	30. 1 Newton = 1 kg.m/s^2
11. 1 Coulomb = 1 A-s	31. 1 Oersted = 79.6 A/m
12. 1 Debye = 3.33 X 10^{-30} cm	32. 1 Poise = 0.1 PaS
13. 1 Dyne/Cm = 10^{-3} N/m	33. 1 PSi = 6.89 KN/m^2
14. 1 dyne/cm^2 = 0.1 N/m^2	34. T°C = (T + 273.15) K
15. 1 erg = 10^{-7} J	35. T°F = 5/9 (T + 459.67) K
16. 1 erg/cm = 10^{-5} J/m	36. 1 torr (mm of Hg) = 133.3 N/m^2
17. 1 eV = 1.602 10^{-19} J	37. 1 TR = 3024 KCal
18. 1 eV/entity = 96.49 KJ/mol	38. 1 Watt = 1 Joule/sec
19. 1 eV/particle = 96.49 KJ/mol	39. TR = 3024 Kcal/hg
20. 1 farad = 1 C/V	40. 1 Joule = 10^7 ergs
	41. 1 K-cal = 4.18 kJ

Common Exponent

Symbol	Name	Name of ten	Factor
Y	yatta		10^{24}
Z	zetta		10^{21}
E	exa		10^{18}
p	peta		10^{15}
T	tera	trillion	10^{12}
G	giga	billion	10^9
M	mega	million	10^6
k	kilo	thousand	10^3
h	hecto	hundred	10^2
da	deca	ten	10^1
d	deci	tenth	10^{-1}
c	centi	hundredth	10^{-2}
m	milli	thousandth	10^{-3}
μ	micro	millionth	10^{-6}
n	nano	billionth	10^{-9}
p	pico	trillionth	10^{-12}
f	femto		10^{-15}
a	atto		10^{-18}
z	zepto		10^{-21}
y	yocto		10^{-24}

Formulas in current electricity (Direct Current)

1	Electric Current	i=q/t	"q" is charge passing in normal direction through a cross section of conductor in time "t"
2	Drift velocity V_d with Electric field	$V_d = \frac{-e\vec{E}\tau}{m}$	e is charge and m is mass on electron, E is electric field, τ is relaxation time.
3	Current I with Drift velocity V_d	$I = n\,e\,A\,V_d$	n is number density with of free electrons, A is area of cross section.
4	Mobility of charge "μ"	$\mu = V_d / E = \frac{q\tau}{m}$	
5	Mobility and drift velocity	$V_d = \mu_e E$	
6	Resistance, P.D., and Current	R = V / I	V Potential Difference, I Current .
7	Resistance R with specific Res.	$R = \rho \frac{l}{A}$	*l* is length of conductor and A is area of cross section
8	Specific Resistance, ρ	$\rho = R \frac{A}{l}$	
9	Resistivity with electrons	$\rho = \frac{m}{ne^2\tau}$	
10	Current density J	$\vec{J} = I / \vec{A}$	I is current, J current density, A is area of cross section
11	Conductance G	G = 1/R	
12	Conductivity σ	$\sigma = 1/\rho$	ρ is specific resistance
13	Microscopic form of Ohms Law	$J = \sigma E$	E is electric field
14	Temperature coefficient of Resistance α	$\alpha = \frac{Rt - R_0}{R_0 \; X \; t}$	R_o is resistance at 0_o C. R_t is resistance at t^o and "t" is temperature difference.
15	Resistances in series	$R = R_1+R_2+R_3$	Same current through all resistances (circuit Current
16	Resistances in parallel	$1/R_e = 1/R_1 + 1/R_2 + 1/R_3$	Same P.D. across each resistance (V of cell)
17	In a circuit with a cell	V = E - Ir	V is terminal potential difference
18	n Cells of emf E in series	Emf = nE	
19	Resistance of n cells in series	nr + R	r is internal resistance of one cell, R external Resistance
20	Current in circuit with n cells in series	$I = \frac{nE}{R+nr}$	r is internal resistance of one cell, R external Resistance
21	n cells in parallel, then emf	emf = E	
22	n cells in parallel, resistance	R + r/n	R external resistance, r internal resistance
23	Cells in mixed group, condition for maximum current	$R = \frac{nr}{m}$	n is number of cells in one row, m is number of rows. r is internal resistance, R external resis.
24	Internal resistance of a cell	$r = (\frac{E-V}{V}) \times R$	E is emf, V is terminal Potential difference, R is external resistance.
25	Power of a circuit	$P = I.V = I^2R = V^2/R$	
26	Energy consumed	E = I.V.ΔT	ΔT is time duration
27	Kirchoff Law (junction rule)	$\Sigma i = 0$	Sum of currents at junction is zero.
28	Kirchoff Law (Loop rule)	$\Sigma V = 0$	In a loop sum of all p.d.*s* is Zero

Brief History of Indian Nobel Laureates in PHYSICS

The Nobel Prize is the most respected award the world over and here is a list of those Indians who have won this award and made the country proud.

1. Sir C.V. Raman (7 November 1888 – 21 November 1970)

Nobel Prize for Physics (1930) Chandrasekhara Venkata Raman was born on 7th Nov. 1888 in Thiruvanaikkaval, in the Trichy district of Tamil Nadu. He finished school by the age of eleven and by then he had already read the popular lectures of Tyndall, Faraday and Helmoltz. He acquired his BA degree from the Presidency College, Madras, where he carried out original research in the college laboratory, publishing the results in the philosophical magazine. Then went to Calcutta and while he was there, he made enormous contributions to vibration, sound, musical instruments, ultrasonics, diffraction, photo electricity, colloidal particles, X-ray diffraction, magnetron, dielectrics, and the celebrated "RAMAN" effect which fetched him the Noble Prize in 1930. He was the first Asian scientist to win the Nobel Prize. The Raman effect occurs when a ray of incident light excites a molecule in the sample, which subsequently scatters the light. While most of this scattered light is of the same wavelength as the incident light.

2. Dr. Hargobind Khorana (9 January 1922 – 9 November 2011)

Nobel Prize for Medicine and Physiology (1968) Dr. Hargobind Khorana was born on 9th January 1922 at Raipur, Punjab (now in Pakistan). Dr. Khorana was responsible for producing the first man-made gene in his laboratory in the early seventies. This historic invention won him the Nobel Prize for Medicine in 1968 sharing it with Marshall Nuremberg and Robert Holley for interpreting the genetic code and analyzing its function in protein synthesis. They all independently made contributions to the understanding of the genetic code and how it works in the cell. They established that this mother of all codes, the biological language common to all living organisms, is spelled out in three-letter words: each set of three nucleotides codes for a specific amino acid.

3. Dr. Subramaniam Chandrasekar (19 October 1910 – 21 August 1995)

Nobel Prize for physics (1983) Subramaniam Chandrashekhar was born on October 19, 1910 in Lahore, India (later part of Pakistan). He attended Presidency College from 1925 to 1930, following in the footsteps of his famous uncle, Sir C. V. Raman. His work spanned over the understanding of the rotation of planets, stars, white dwarfs, neutron stars, black holes, galaxies, and clusters of galaxies. He won the Nobel Prize in 1983 for his theoretical work on stars and their evolution.

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