# TEMPLECITY INSTITUTE OF TECHNOLGY AND ENGINEERING (TITE)
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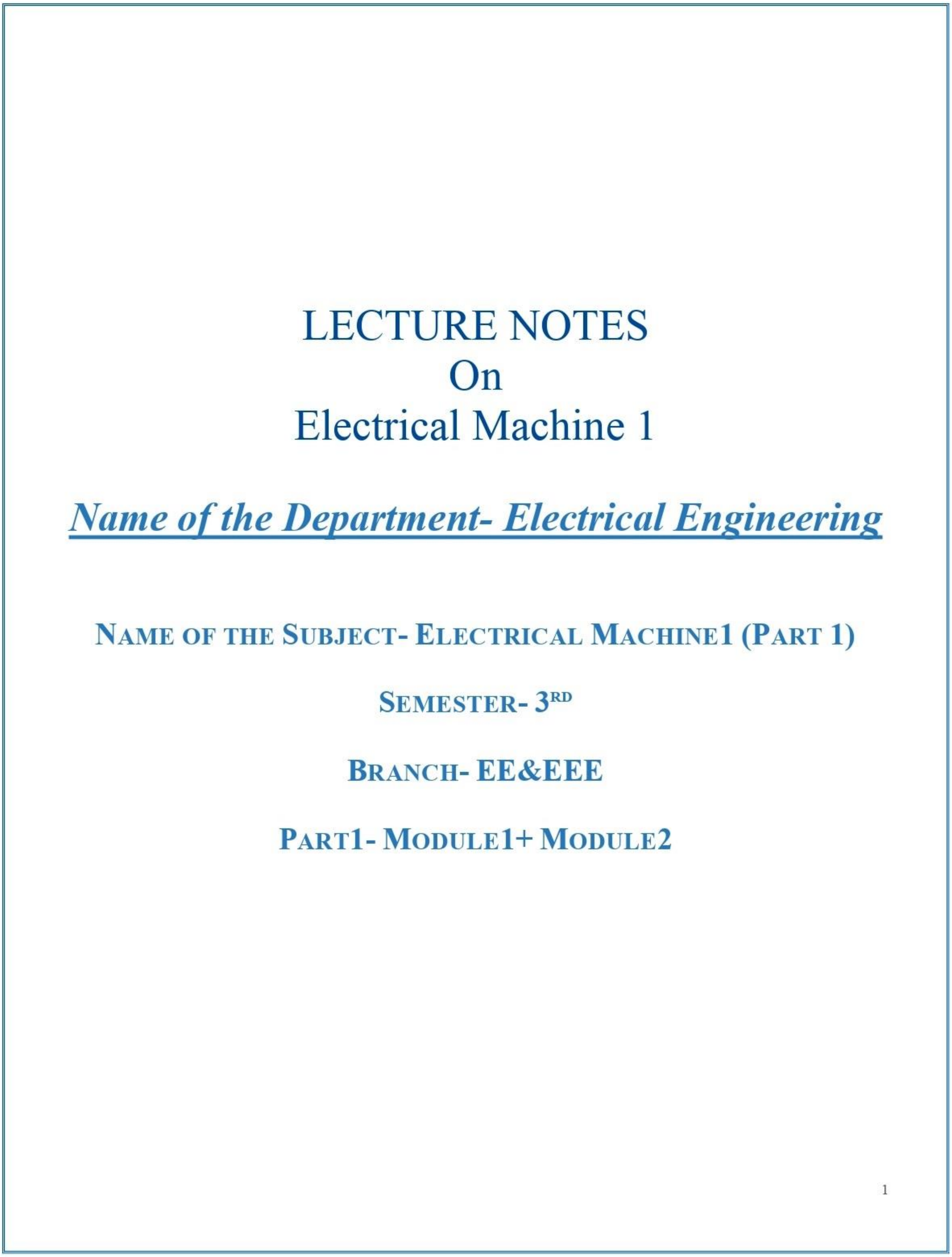


# LECTURE NOTES
# On
# Electrical Machine 1

## *Name of the Department- Electrical Engineering*

**NAME OF THE SUBJECT- ELECTRICAL MACHINE1 (PART 1)**

**SEMESTER- 3RD**

**BRANCH- EE&EEE**

**PART1- MODULE1+ MODULE2**

# MODULE-I

DC GENERATOR

## TOPICS

Electromechanical Energy conversion, forces and torque in magnetic field systems – energy balance, energy and force in a singly excited magnetic field system, determination of magnetic force, co-energy, multi excited magnetic field systems.

DC Generators – Principle of operation, Action of commutator, constructional features, armature windings, lap and wave windings, simplex and multiplex windings, use of laminated armature, E. M.F. Equation,

Methods of Excitation: separately excited and self -excited generators, build-up of E.M.F., critical field resistance and critical speed , causes for failure to self-excite and remedial measures, Armature reaction: Cross magnetizing and demagnetizing AT/pole, compensating winding, commutation, reactance voltage, methods of improving commutation

Load characteristics of shunt, series and compound generators, parallel operation of DC generators, use of equalizer bar and cross connection of field windings, load sharing.

[Topics are arranged as per above sequence]

## 1.1 Electromechanical-Energy-Conversion Principles

The electromechanical-energy-conversion process takes place through the medium of the electric or magnetic field of the conversion device of which the structures depend on their respective functions.

- Transducers: microphone, pickup, sensor, loudspeaker
- Force producing devices: solenoid, relay, and electromagnet
- Continuous energy conversion equipment: motor, generator

## 1.2 Forces and Torques in Magnetic Field Systems

The Lorentz Force Law gives the force $F$ on a particle of charge $q$ in the presence of electric and magnetic fields.

$$F= q\left(E+v\times B\right)$$

*Where,* $F$ : newtons, $q$: coulombs, $E$: volts/meter, $B$ : telsas, $v$: meters/second

> In a pure electric-field system,

$$F = qE$$

> In pure magnetic-field systems, $F= q\left(v\times B\right)$

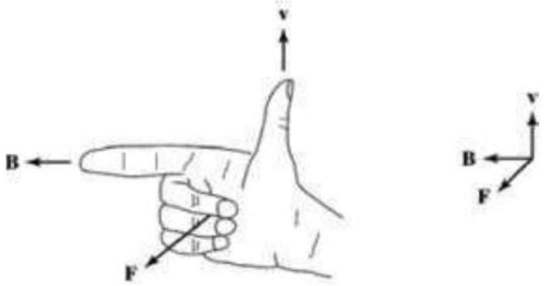


Fig 1.1 Right-hand rule for $F = q\left(v\times B\right)$

> For situations where large numbers of charged particles are in motion,

$$F_v = \rho\left(E+v\times B\right)$$
$$J = \rho v$$

$$F_v = J \times B$$

$\rho$ (charge density): coulombs/m$^3$, $F_v$ (force density): newtons/m$^3$, $J = \rho v$ (current density): amperes/m$^2$.

Most electromechanical-energy-conversion devices contain magnetic material.

- Forces act directly on the magnetic material of these devices which are constructed of rigid, non-deforming structures.
- The performance of these devices is typically determined by the net force, or torque, acting on the moving component. It is rarely necessary to calculate the details of the internal force distribution.
- Just as a compass needle tries to align with the earth's magnetic field, the two sets of fields associated with the rotor and the stator of rotating machinery attempt to align, and torque is associated with their displacement from alignment.
    - In a motor, the stator magnetic field rotates ahead of that of the rotor, pulling on it and performing work.
    - For a generator, the rotor does the work on the stator.

The Energy Method

\> Based on the principle of conservation of energy: energy is neither created nor destroyed; it is merely changed in form.

\> Fig. 1.2 shows a magnetic-field-based electromechanical-energy-conversion device.

- A lossless magnetic-energy-storage system with two terminals
- The electric terminal has two terminal variables: $e$ (voltage), $i$ (current).
- The mechanical terminal has two terminal variables: $f_{\text{fld}}$ (force), $x$ (position)
- The loss mechanism is separated from the energy-storage mechanism.

– Electrical losses: ohmic losses...

– Mechanical losses: friction, windage...

> Fig. 1.3: a simple force-producing device with a single coil forming the electric terminal, and a movable plunger serving as the mechanical terminal.

- The interaction between the electric and mechanical terminals, i.e. the electromechanical energy conversion, occurs through the medium of the magnetic stored energy.

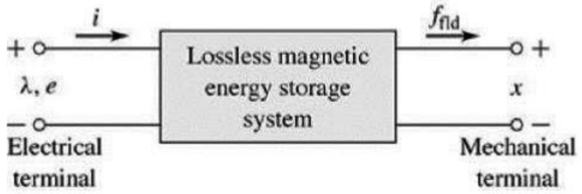


**Fig 1.2 Schematic diagram of magnetic-field electromechanical-energy-conversion device**

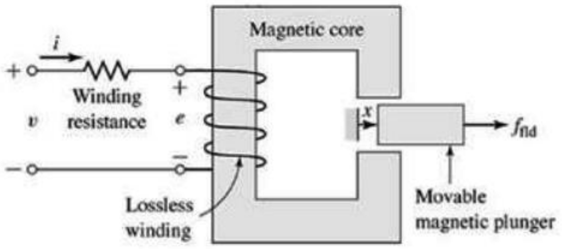


**Fig. 1.3 Schematic diagram of simple force-producing device**

- $W_{\text{fld}}$ : the stored energy in the magnetic field

$$\frac{dW_{\text{fld}}}{dt} = ei - f_{\text{fld}}\frac{dx}{dt}$$

$$e = \frac{d\lambda}{dt}$$

$$dW_{\text{fld}} = i\,d\lambda - f_{\text{fld}}\,dx$$

- From the above equation force can be solved as a function of the flux λ and the mechanical terminal position $x$.

- The above equations form the basis for the energy method

## 1.3 Energy Balance

Consider the electromechanical systems whose predominant energy-storage mechanism is in magnetic fields. For motor action, the energy transfer can be accounted as

$$\begin{pmatrix}\text{Energy input}\\ \text{form electric}\\ \text{sources}\end{pmatrix}=\begin{pmatrix}\text{Mechanical}\\ \text{energy}\\ \text{output}\end{pmatrix}+\begin{pmatrix}\text{Increase in energy}\\ \text{stored in magnetic}\\ \text{field}\end{pmatrix}+\begin{pmatrix}\text{Energy}\\ \text{converted}\\ \text{into heat}\end{pmatrix}$$

The ability to identify a lossless-energy-storage system is the essence of the energy method.

> This is done mathematically as part of the modeling process.

> For the lossless magnetic-energy-storage system of Fig. 1.2 can be rearranged and gives

$$dW_{\text{elec}} = dW_{\text{mech}} + dW_{\text{fld}}$$

where

$dW_{\text{elec}} = i d\lambda$ = differential electric energy input

$dW_{\text{mech}} = f_{\text{fld}} dx$ = differential mechanical energy output

$dW_{\text{fld}}$ = differential change in magnetic stored energy

>Here $e$ is the voltage induced in the electric terminals by the changing magnetic stored energy. It is through this reaction voltage that the external electric circuit supplies power to the coupling magnetic field and hence to the mechanical output terminals.

$$dW_{\text{elec}} = ei\ dt$$

> The basic energy-conversion process is one involving the coupling field and its action and reaction on the electric and mechanical systems.

> Combining above two equation –

$$dW_{\text{elec}} = ei\ dt = dW_{\text{mech}} + dW_{\text{fld}}$$

## 1.4 Energy in Singly-Excited Magnetic Field Systems

In energy-conversion systems the magnetic circuits have air gaps between the stationary and moving members in which considerable energy is stored in the magnetic field.

> This field acts as the energy-conversion medium, and its energy is the reservoir between the

electric and mechanical system.

Fig. 1.4 shows an electromagnetic relay schematically. The predominant energy storage occurs in the air gap, and the properties of the magnetic circuit are determined by the dimensions of the air gap.

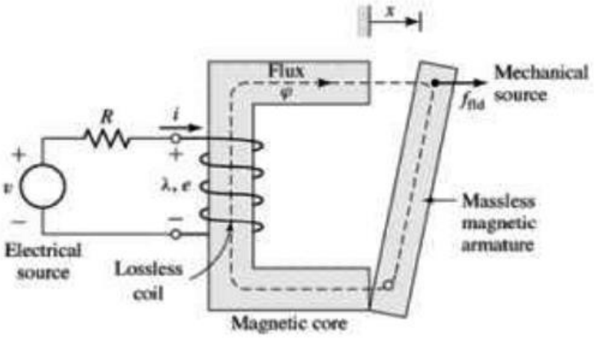


**Fig.1.4 Schematic of an electromagnetic relay**

$$\lambda = L(x)i$$
$$dW_{\mathrm{mech}} = f_{\mathrm{fld}}dx$$
$$dW_{\mathrm{fld}} = id\lambda - f_{\mathrm{fld}}dx$$

$W_{\mathrm{fld}}$ is uniquely specified by the values of $\lambda$ and $x$. Therefore, $\lambda$ and $x$ are referred to as state variables.

Since the magnetic energy storage is lossless, it is conservative system. $W_{\mathrm{fld}}$ is the same regardless of how $\lambda$ and $x$ are brought to their final values. Fig 1.5 shows where tow separate the paths.

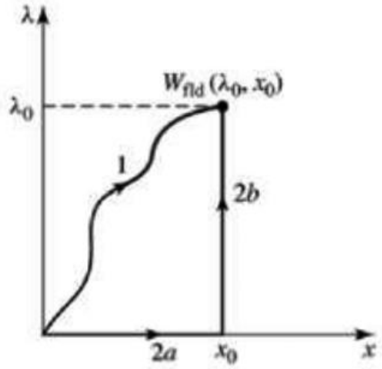


**Fig. 1.5 Integration paths for $W_{\mathrm{fld}}$**

On path 2a, $d\lambda$=0 and $f_{\mathrm{fld}}$=0. Thus $df_{\mathrm{fld}}$=0 on path 2a. On path 2b, $dx$= 0. Therefore the following equation can be written

$$W_{\text{fld}}(\lambda_0, x_0) = \int_0^{\lambda_0} i(\lambda, x_0)\, d\lambda$$

For a linear system in which $\lambda$ is proportional to $i$ the equation will change and can be written as-

$$W_{\text{fld}}(\lambda, x) = \int_0^{\lambda} i(\lambda', x)\, d\lambda' = \int_0^{\lambda} \frac{\lambda'}{L(x)} d\lambda' = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

$V$: the volume of the magnetic field

$$W_{\text{fld}} = \int_V \left( \int_0^B H.dB' \right) dV$$

$If\ B = \mu H$, then

$$W_{\text{fld}} = \int_V \left( \frac{B^2}{2\mu} \right) dV$$

## 1.5 Determination of Magnetic Force and Torque form Energy

The magnetic stored energy is a state function, determined uniquely by the values of the $W_{\text{fld}}$ independent state variables $\lambda$ and $x$.

$$dW_{\text{fld}}(\lambda, x) = i\,d\lambda - f_{\text{fld}}\,dx$$

$$dW_{\text{fld}}(\lambda, x) = \left.\frac{\partial W_{\text{fld}}}{\partial \lambda}\right|_x d\lambda + \left.\frac{\partial W_{\text{fld}}}{\partial x}\right|_\lambda dx$$

$$i = \left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial \lambda}\right|_x$$

$$\boxed{f_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x}\right|_\lambda}$$

## 1.6 Energy and Co-energy:

It is that energy from which the force can be obtained directly as a function of the current. The selection of energy or co-energy as the state function is purely a matter of convenience.

The co-energy $W'_{\text{fld}}(i, x)$ is defined as a function of I and x such that

$$W'_{\mathrm{fld}}(i\,,x) = i\lambda - W_{\mathrm{fld}}(\lambda\,,x)$$

$$d(i\lambda) = id\lambda + \lambda di$$

$$dW'_{\mathrm{fld}}(i\,,x) = d(i\lambda) - dW_{fld}(\lambda,x)$$

$$\boxed{dW'_{\mathrm{fld}}(i\,,x) = \lambda\,di + f_{\mathrm{fld}}dx}$$

From the above equation co-energy $W'_{\mathrm{fld}}(i,x)$ can be seen to be a state function of the two independent variables *i* and *x*.

$$dW'_{\mathrm{fld}}(i\,,x) = \left.\frac{\partial W'_{fld}}{\partial i}\right|_x di + \left.\frac{\partial W'_{fld}}{\partial x}\right|_i dx$$

$$\lambda = \left.\frac{\partial W'_{\mathrm{fld}}(i,x)}{\partial i}\right|_x$$

$$\boxed{f_{\mathrm{fld}} = \left.\frac{\partial W'_{\mathrm{fld}}(i,x)}{\partial x}\right|_i}$$

For a system with a rotating mechanical displacement,

$$W'_{\mathrm{fld}}(i,\theta) = \int_0^i \lambda(i',\theta)\,di'$$

$$T_{\mathrm{fld}} = \left.\frac{\partial W'_{\mathrm{fld}}(i\,,\theta)}{\partial\theta}\right|_i$$

If the system is magnetically linear,

$$W'_{\mathrm{fld}}(i\,,\theta) = \frac{1}{2}L(\theta)i^2$$

$$T_{\mathrm{fld}} = \frac{i^2}{2}\frac{dL(\theta)}{d\theta}$$

In field-theory terms, for soft magnetic materials

$$W'_{\mathrm{fld}} = \int_V\left(\int_0^{H_0} B\cdot dH\right)dV$$

$$W'_{\mathrm{fld}} = \int_V \frac{\mu H^2}{2}\,dV$$

For permanent-magnet (hard) materials

$$W'_{\text{fld}} = \int_V \left( \int_{H_o}^{H_0} B \cdot dH \right) dV$$

For a magnetically-linear system, the energy and co-energy (densities) are numerically equal:

$$\frac{1}{2}\lambda^2 / L = \frac{1}{2}Li^2, \ \frac{1}{2}B^2 / \mu = \frac{1}{2}\mu H^2$$

For a nonlinear system in which $\lambda$ and $i$ or $B$ and $H$ are not linearly proportional, the two functions are not even numerically equal.

$$W_{\text{fld}} + W'_{\text{fld}} = \lambda i$$

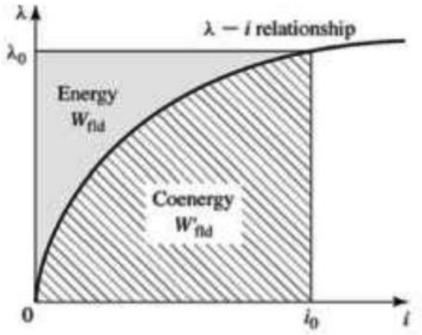


**Fig.1.6 Graphical interpretation of energy and co-energy in a singly-excited system**

## 1.7 Multiply-Excited Magnetic Field Systems

Many electromechanical devices have multiple electrical terminals.

> Measurement systems: torque proportional to two electric signals; power as the product of voltage and current.

> Energy conversion devices: multiply-excited magnetic field system.

> A simple system with two electrical terminals and one mechanical terminal:

Three independent variables: $\{\theta, \lambda_1, \lambda_2\}$, $\{\theta, i_1, i_2\}$, $\{\theta, \lambda_1, i_2\}$, or $\{\theta, i_1, \lambda_2\}$.

$$dW_{\text{fld}}(\lambda_1, \lambda_2, \theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - T_{\text{fld}} d\theta$$

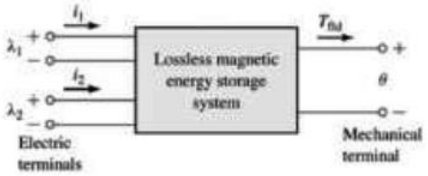


**Fig. 1.7 Multiply-excited magnetic energy storage system**

$$i_1 = \left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}\right|_{\lambda_2, \theta}$$

$$i_2 = \left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}\right|_{\lambda_1, \theta}$$

$$\boxed{T_{\text{fld}} = -\left.\frac{\partial W_{\text{fld}}(\lambda_1, \lambda_2, \theta)}{\partial \theta}\right|_{\lambda_1, \lambda_2}}$$

To find $W_{\text{fld}}$, use the path of integration as shown in Fig 1.8.

$$W_{\text{fld}}(\lambda_{1_0}, \lambda_{2_0}, \theta_0) = \int_0^{\lambda_{2_0}} i_2(\lambda_1 = 0, \lambda_2, \theta = \theta_0)\, d\lambda_2 + \int_0^{\lambda_{1_0}} i_1(\lambda_1, \lambda_2 = \lambda_{2_0}, \theta = \theta_0)\, d\lambda_1$$

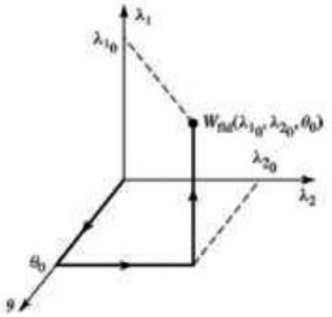


**Fig. 1.8 Integration path to obtain $W_{\text{fld}}(\lambda_{10}, \lambda_{20}, \theta_0)$**

> In a magnetically-linear system,

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$
$$\lambda_2 = L_{21} i_1 + L_{22} i_2$$
$$L_{12} = L_{21}$$

Note that $L_{ij} = L_{ij}(\theta)$

$$
\begin{aligned}
i_1 &= \frac{L_{22}\lambda_1 - L_{12}\lambda_2}{D} \\
i_2 &= \frac{-L_{21}\lambda_1 + L_{11}\lambda_2}{D} \\
D &= L_{11}L_{22} - L_{12}L_{21}
\end{aligned}
$$

The energy for this linear system is

$$
\begin{aligned}
W_{\text{fld}}\left(\lambda_{1_0},\lambda_{2_0},\theta_0\right) &= \int_0^{\lambda_{2_0}} \frac{L_{11}(\theta_0)\lambda_2}{D(\theta_0)}\,d\lambda_2 + \int_0^{\lambda_{1_0}} \frac{\left(L_{22}(\theta_0)\lambda_1 - L_{12}(\theta_0)\lambda_{2_0}\right)}{D(\theta_0)}\,d\lambda_1 \\
&= \frac{1}{2D(\theta_0)}L_{11}(\theta_0)\lambda_{2_0}^2 + \frac{1}{2D(\theta_0)}L_{22}(\theta_0)\lambda_{1_0}^2 - \frac{L_{12}(\theta_0)}{D(\theta_0)}\lambda_{1_0}\lambda_{2_0}
\end{aligned}
$$

Co-energy function for a system with two windings can be defined as

$$
\begin{aligned}
W'_{\text{fld}}\left(i_1,i_2,\theta\right) &= \lambda_1 i_1 + \lambda_2 i_2 - W_{\text{fld}} \\
dW'_{\text{fld}}\left(i_1,i_2,\theta\right) &= \lambda_1 di_1 + \lambda_2 di_2 + T_{\text{fld}}d \\
\lambda_1 &= \left.\frac{\partial W_{\text{fld}}\left(i_1,i_2,\theta\right)}{\partial i_1}\right|_{i_2,\theta} \\
\lambda_2 &= \left.\frac{\partial W_{\text{fld}}\left(i_1,i_2,\theta\right)}{\partial i_2}\right|_{i_1,\theta}
\end{aligned}
$$

$$
\boxed{T_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}\left(i_1,i_2,\theta\right)}{\partial \theta}\right|_{i_1,i_2}}
$$

$$
W'_{\text{fld}}\left(i_1,i_2,\theta_0\right) = \int_0^{i_{2_0}} \lambda_2\left(i_1 = 0, i_2, \theta = \theta_0\right)di_2 + \int_0^{\lambda_{1_0}} \lambda_1\left(i_1, i_2 = i_{2_0}, \theta = \theta_0\right)di_1
$$

For a linear system

$$
\begin{aligned}
W'_{\text{fld}}\left(i_1,i_2,\theta_0\right) &= \frac{1}{2}L_{11}(\theta)i_1^2 + \frac{1}{2}L_{22}(\theta)i_2^2 + L_{12}(\theta)i_1 i_2 \\
T_{\text{fld}} = \left.\frac{\partial W'_{\text{fld}}\left(i_1,i_2,\theta_0\right)}{\partial \theta}\right|_{i_1,i_2} &= \frac{i_1^2}{2}\frac{dL_{11}(\theta)}{d\theta} + \frac{i_2^2}{2}\frac{dL_{22}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta}
\end{aligned}
$$

- Note that the co-energy function is a relatively simple function of displacement.
- The use of a co-energy function of the terminal currents simplifies the determination of torque or force.

- Systems with more than two electrical terminals are handled in analogous fashion.

## DC Generators

### 1.8 Principle of operation of DC Generator

A D.C generator as shown in figure below the armature be driven by a prime mover in the clock wise direction and the stator field is excited to produce the field poles as shown. There will be induced voltage in each armature conductor. The direction of the induced voltage can be determined by applying *Fleming's right hand rule.* All the conductors under the influence of North Pole will have ⊗ directed induced voltage, while the conductors under the influence of South Pole will have ⊙ induced voltage in them. For a loaded generator the direction of the armature current will be same as that of the induced voltages. Thus ⊗ and ⊙ also represent the direction of the currents in the conductors. We know, a current carrying conductor placed in a magnetic field experiences force, the direction of which can be obtained by applying *Fleming's left hand rule.* Applying this rule to the armature conductors in fig 1.9, the rotor experiences a torque ($T_e$) in the counter clockwise direction (i.e., opposite to the direction of rotation) known as back torque. For steady speed operation back torque is equal to the machines input torque ($T_{pm}$) i.e. the torque supplied by prime mover.

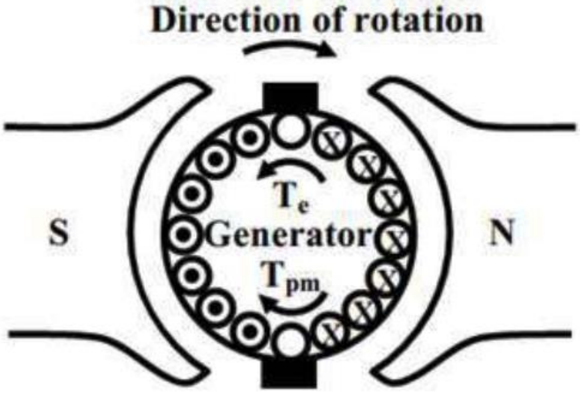


**Fig. 1.9 Action of DC generator**

### 1.9 Action of Commutator

In DC machines the current in each wire of the armature is actually alternating, and hence a device is required to convert the alternating current generated in the DC generator by electromagnetic induction into direct current. or at the armature of a DC motor to convert the input direct current into alternating current at appropriate times, as illustrated in Fig. 1.10.

DC generator: induced AC *emf* is converted to DC voltage;

DC motor: input direct current is converted to alternating current in the armature at appropriate times to produce a unidirectional torque. The commutator consists of insulated copper segments mounted on an insulated tube. Armature coils are connected in series through the commutator segments. Two brushes are pressed to the commutator to permit current flow. The brushes are placed in the neutral zone, where the magnetic field is close to zero, to reduce arcing. The *commutator* switches the current from one rotor coil to the adjacent coil. The switching requires the interruption of the coil current. The sudden interruption of an inductive current generates high voltages. The high voltage produces flashover and arcing between the commutator segment and the brush.

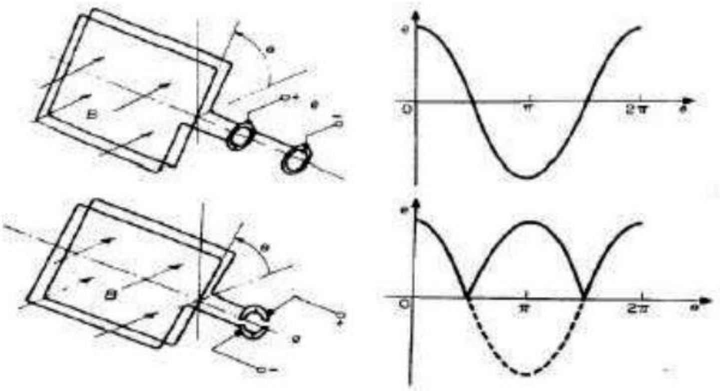


**Fig. 1.10Action of Commutator**

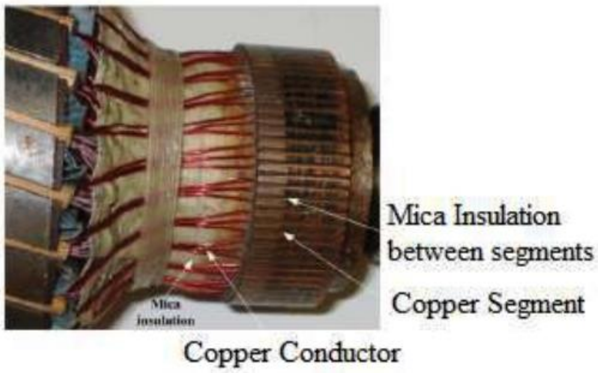


**Fig. 1.11 Mechanical view of commutator**

## 1.10 Constructional Features

The stator of the dc machine has poles, which are excited by dc current to produce magnetic fields. In the neutral zone, in the middle between the poles, commutating poles or interpoles are placed to reduce sparking of the commutator due to armature reaction. The commutating poles are supplied by dc current. Compensating windings are mounted on the main poles. Field poles are mounted on an iron core that provides a closed magnetic circuit. The motor housing supports the iron core, the brushes and the bearings. The rotor has a ring-shaped laminated iron core with slots. Coils with several turns are placed in the slots. The distance between the two legs of the coil is about 180 electric degrees for full pitch. The coils are connected in series through the commutator segments. The ends of each coil are connected to a commutator segment.

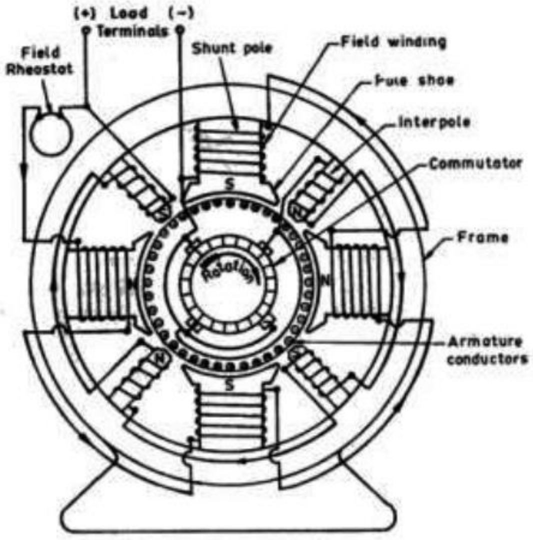


Fig. 1.12 Cross sectional view of DC Machine

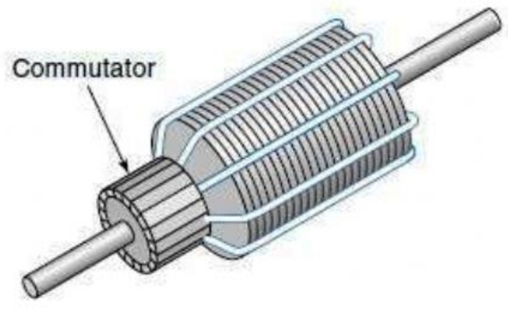


**Fig. 1.13 Armature of DC Machine**

## 1.11 Armature Winding

DC machines armature consists of armature conductors. The conductors distributed in slots provided on the periphery of the armature is called armature winding. Depending on the way in which the coils are interconnected at the commutator end of the armature, the windings can be classified as lap and wave windings. Further they can be classified as simplex and multiplex.

### 1.11.1 Coil Span/Coil Pitch:

It represents the span of the coil. For full pitched winding, the span is $180^0$ electrical or number of slots per pole. Coil pitch can be represented in terms of electrical degrees, slots or conductor. A full pitched coil leads to maximum voltage per coil.

### 1.11.2 Back Pitch *(Yb)*:

It is the distance measured in between the two coil sides of the same coil at the back end of the armature, the commutator end being the front end of armature. It can be represented in terms of number of slots or coil sides. Back pitch also represents the span of coil.

### 1.11.3 Front Pitch (*Yf*):

The distance between the two coil sides of two different coils connected in series at the front end of the armature is called front pitch.

## 1.12 Lap Winding

Lap winding is suitable for low voltage high current machines because of more number of parallel paths. The number of parallel path in lap winding is equal to number of poles.

$$A=P$$

Equalizing rings are connected in lap winding.

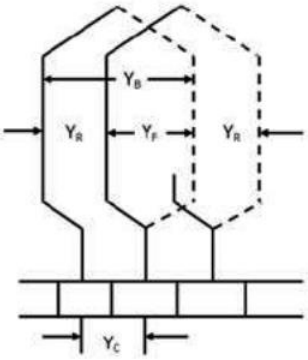


Fig. 1.14 Winding diagram of lap winding

## 1.13Wave Winding

Wave winding is used for high voltage low current machines. In case of wave winding, the number of parallel path (A) = 2 irrespective of number of poles. Each path will have  conductors connected in series.

Equalizing rings are not required in wave winding.

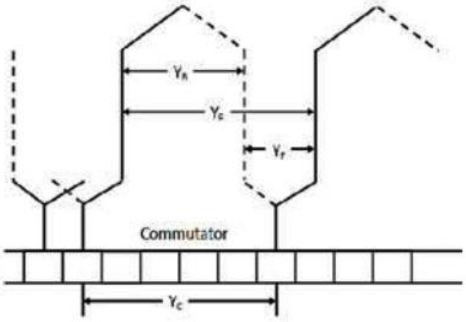


Fig. 1.15 Winding diagram of wave winding

## 1.14 Simplex and Multiplex Winding

Fig 1.14 and Fig. 1.15 shows simplex lap and simplex wave winding.

The degree of multiplicity of a multiplex winding indicates the relative number of parallel paths with respect to the number of parallel paths in the corresponding simplex winding. For example a duplex lap or wave winding is a lap or wave winding having twice as many as parallel paths as a simplex lap or wave winding respectively. The winding can be triplex or quadruplex winding in similar manner.

## 1.15 Use of Laminated Armature

The armature winding of DC machine should be laminated to reduce eddy current losses. The armature body (rotor) rotates in the field magnetic field. Thus in the core of the armature voltage induced which in turn causes current to flow in the body. This current is known as eddy current. This current causes loss and thus heat will be generated. This loss depends on the amount of current flow. To reduce the amount of current flow the resistance of the body should be increased. Thus using lamination the resistance of the path through which current flows will be increased. The amount of eddy current will be reduced and thus eddy current loss can be minimized.

## 1.16 EMF Equation

Let $\phi$ = flux per pole in weber

Z = number of armature conductors = Number of slots X conductors per slot.

P = Number of poles; A= Number of parallel paths in armature.

A= P for lap wound armature; A=2 for wave wound armature

N = speed of armature in rpm; E = induced emf in each parallel path.

Average emf generated/conductor in one revolution = $\frac{d\phi}{dt}$

Flux cut by a conductor in one revolution = $d\phi = P\phi$ weber.

Since Number of revolutions/second= $\frac{N}{60}$

Time taken for one revolution = $dt = \frac{60}{N}$ seconds

EMF generated/conductor = $\frac{d\phi}{dt} = \frac{P\phi}{\frac{60}{N}} = \frac{P\phi N}{60}$

Since each path has $\frac{Z}{A}$ conductors in series,

EMF generated in each path is $E = \frac{P\phi N}{60} \times \frac{Z}{A}$

$$\boxed{E = \frac{P\phi ZN}{60A}}$$

## 1.17 Methods of excitation

DC machines are excited in two ways-

### 1.17.1Separate excitation:

When the field winding is connected to an external source to produce field flux. According to the type of excitation this machines are called separately excited dc machine.

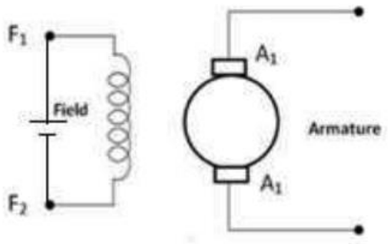


**Fig. 1.16 Schematic diagram of separately excited dc machine**

### 1.17.2 Self-excitation:

When the field winding is connected with the armature to produce field flux. A self-excited machine requires residual magnetism for operation. According to the type of excitation this machines are called self-excited dc machine.

Depending on the type of field winding connection DC machines can be classified as:

#### 1.17.2.1 Shunt machine:

The field winding consisting of large number of turns of thin wire is usually excited in parallel with armature circuit and hence the name shunt field winding. This winding will be having more resistance and hence carries less current.

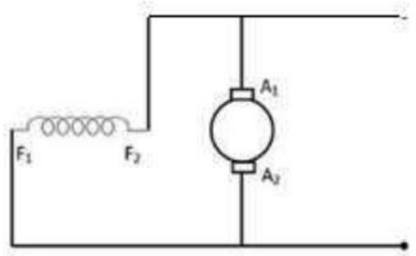


**Fig. 1.17 Schematic diagram of dc shunt machine**

#### 1.17.2.2 Series machine:

The field winding has a few turns of thick wire and is connected in series with armature.

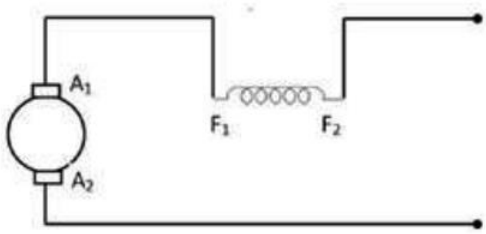
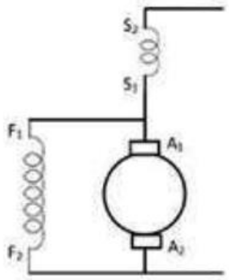


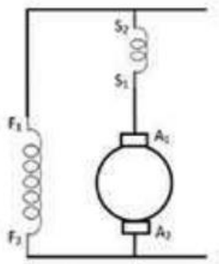
**Fig.1.18 Schematic diagram of dc series machine**

### 1.17.2.3 Compound machine:

Compound wound machine comprises of both series and shunt windings and can be either short shunt or long shunt, cumulative, differential or flat compounded.



A



B

**Fig. 1.19 A A Schematic diagram of short -shunt compound machine**

**Fig.1.19 B Schematic diagram of long-shunt compound machine**

## 1.18 Build-up of E.M.F

When the armature is rotating with armature open circuited, an emf is induced in the armature because of the residual flux. When the field winding is connected with the armature, a current flows through the field winding ( in case of shunt field winding, field current flows even on No-load and in case of series field winding only with load) and produces additional flux. This additional flux along with the residual flux generates higher voltage. This higher voltage circulates more current to generate further higher voltage. This is a cumulative process till the saturation is attained.

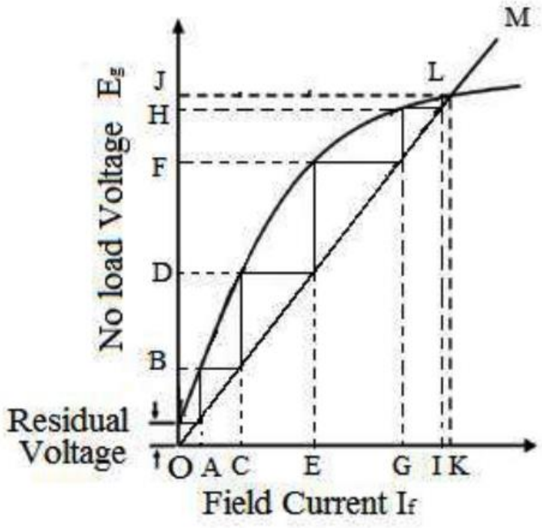


**Fig. 1.20 Process of voltage build-up in DC generator**

Here OM is the field resistance curve in Fig. 1.20. Initially there will be residual voltage which will create OA field current. This field current will increase the existing magnetic field and the induced voltage will increase up-to OB. This OB voltage will further applied to the field winding and increase the field current to OC. This process will continue upto the point L where the emf curve intersect with field resistance and finally the induced voltage will be OJ. This way voltage builds-up in dc generator.

## 1.19 Critical Resistance:-

The voltage to which it builds is decided by the resistance of the field winding as shown in the figure 1.21. If field circuit resistance is increased such that the resistance line does not cut OCC like 'OP' in the figure1.21, then the machine will fail to build up voltage to the rated value. The slope of the air gap line drawn as a tangent (OQ) to the initial linear portion of the curve represents the maximum resistance that the field circuit can have beyond which the machine fails to build up voltage. This value of field circuit resistance is called critical field resistance. The field circuit is generally designed to have a resistance value less than this so that the machine builds up the voltage to the rated value.

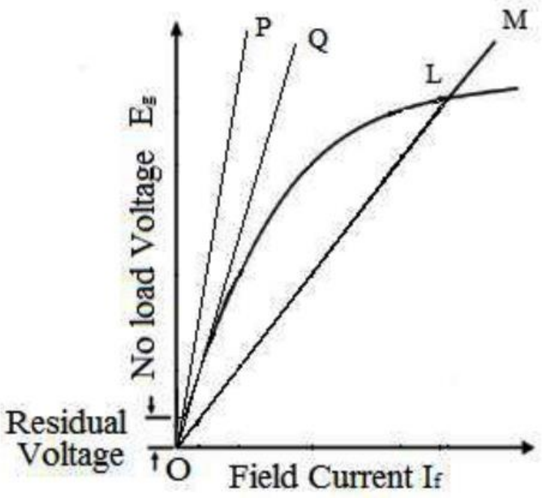
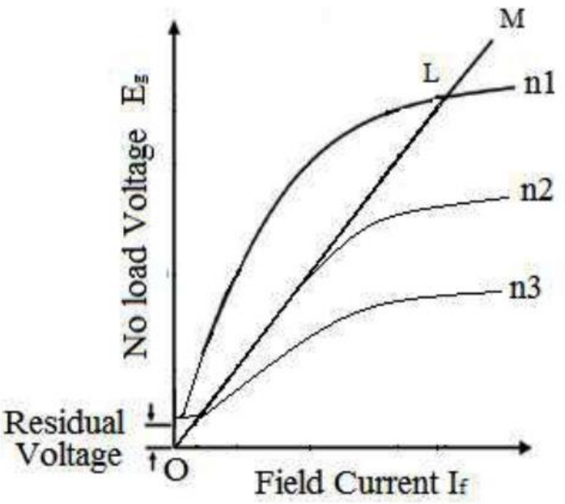


**Fig. 1.21 Field current vs No-load voltage for different field resistances**

Critical field resistance is defined as the maximum field circuit resistance for a given speed with which the shunt generator would excite. The shunt generator will build up voltage only if field circuit resistance is less than critical field resistance.

## 1.20 Critical Speed:

Voltage of a dc generator is proportional to its speed. Thus when speed will be reduced then the induced voltage will reduced. There can be such situation occur when the speed will be so low that the existing field winding resistance voltage bulid up will not occur. The speed of the generator can be lowered upto a certain level. This minimum value of the speed of the generator for which the generator can excite is called critical speed. It can also define as that speed of a generator for which the existing field resistance of generator becomes its critical field resistance.



connected to the dual flux.

**Fig. 1.22 Field current vs No-load voltage for different speed**

In the above figure it is showing that when speed of the generator changes from n1 to n2 and then n3 emf production changes accordingly. Here n1>n2>n3. For speed n3 voltage build-up is not possible. The speed n2 is the critical speed. As shown in the figure at speed n2 generator field resistance become its critical field resistance.

## 1.21 Causes for failure to self-excite and its remedial

i. The field poles may not have residual magnetism. Then the generator will fail to excite.

   Then to restore residual magnetism field winding should be connected to an external dc voltage source. This is called flashing of field.

ii. When the direction of rotation is not proper such that flux produced by the field current reinforces the residual magnetism.

   The rotation of the machine has to be reversed.

iii. The field winding resistance is more than critical resistance then the machine will fail to excite.

   The field winding resistance should be less than critical field resistance.

iv. When the speed of the machine is less than critical speed.

   The machine’s speed should be more than critical speed.

v. If the field winding connections are such that newly generated field flux is working in opposite to the existing residual magnetism. Then the generator will fail to excite.

Then the field winding connection should be reversed.

## 1.22Armature Reaction

The action of magnetic field set up by armature current on the distribution of flux under main poles of a DC machine is called armature reaction.

When the armature of a DC machines carries current, the distributed armature winding produces its own mmf. The machine air gap is now acted upon by the resultant mmf distribution caused by the interaction of field ampere turns ($AT_f$) and armature ampere turns ($AT_a$). As a result the air gap flux density gets distorted.

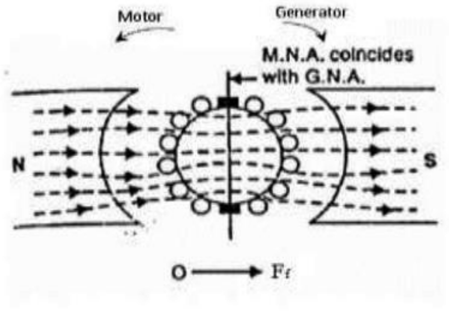


Fig. a

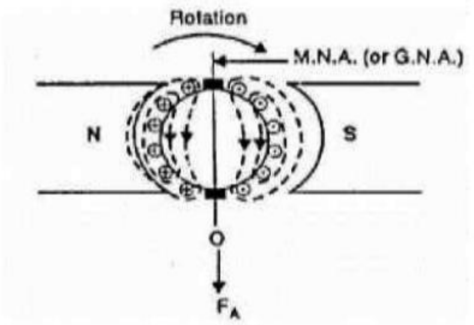


Fig. b

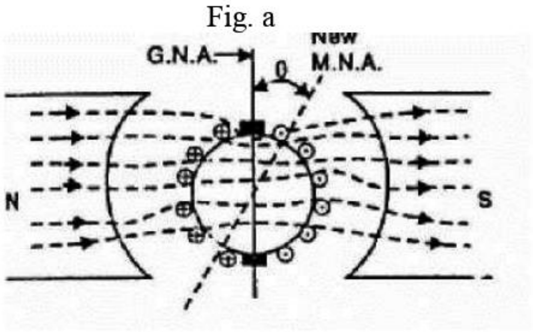


Fig. c

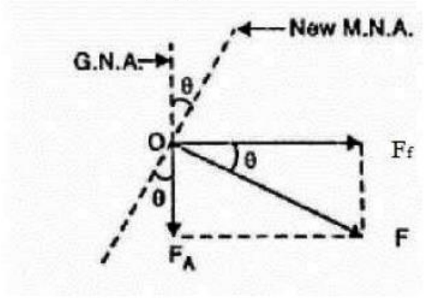


Fig. d

Figure a shows a two pole machine with single equivalent conductor in each slot and the main field mmf ($F_f$) acting alone. The axis of the main poles is called the direct axis (d-axis) and the interpolar axis is called quadrature axis (q-axis). It can be seen from the Figure b that armature mmf ($F_a$) is along the interpolar axis. $F_a$ which is at $90^0$ to the main field axis is known as cross magnetizing mmf.

Figure c shows the practical condition in which a DC machine operates when both the Field flux and armature flux are existing. Because of both fluxes are acting simultaneously, there is a shift in brush axis and crowding of flux lines at the trailing pole tip and flux lines are weakened or thinned at the leading pole tip. (The pole tip which is first met in the direction of rotation by the armature conductor is leading pole tip and the other is trailing pole tip).

If the iron in the magnetic circuit is assumed unsaturated, the net flux/pole remains unaffected by the armature reaction though the air gap flux density distribution gets distorted. If the main pole excitation is such that the iron is in the saturated region of magnetization (practical case) the increase in flux density at one end of the poles caused by armature reaction is less than the decrease at the other end, so that there is a net reduction in the flux/pole. This is called the demagnetizing effect. Thus it can be summarized that the nature of armature reaction in a DC machine is

1. Cross magnetizing with its axis along the q-axis.
2. It causes no change in flux/pole if the iron is unsaturated but causes reduction in flux/pole in the presence of iron saturation. This is termed as demagnetizing effect. The resultant mmf 'F' is shown in figure d.

## 1.22.1Cross Magnetizing Ampere Turns/pole($AT_c$)

If the brush is shifted by an angle θ as shown in figure 1.23 then the conductors lying in between the angles BOC and DOA are carrying the current in such a way that the direction of the flux is downwards i.e., at right angles to the main flux. This results is the distortion in the main flux. Hence, these conductors are called cross magnetizing or distorting ampere conductors.

$$\text{Total armature conductors/pole} = \frac{Z}{P}$$

$$\text{Demagnetizing conductors / pole} = Z\frac{2\theta}{360}$$

Therefore cross magnetizing conductors/pole= $\frac{Z}{P} - Z\frac{2\theta}{360}$

Cross magnetizing ampere turns/pole $AT_c$= $\frac{ZI_a}{a}\left(\frac{1}{2P} - \frac{\theta}{360}\right)$

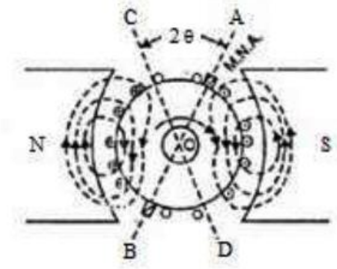


**Fig. 1.23 Cross-magnetizing ampere conductors**

### 1.22.2 Demagnetizing Ampere Turns /pole ($AT_d$)

The exact conductors which produce demagnetizing effect are shown in Fig 1.24, Where the brush axis is given a forward lead of θ so as to lie along the new axis of M.N.A. The flux produced by the current carrying conductors lying in between the angles AOC and BOD is such that, it opposes the main flux and hence they are called as demagnetizing armature conductors.

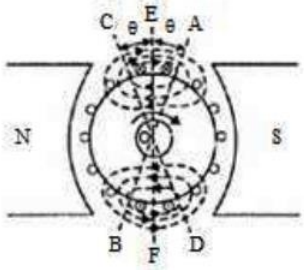


**Fig. 1.24 Demagnetizing ampere conductors**

Z= total no of armature conductors

Current in each armature conductors= $\frac{I_a}{a}$

θ =Forward lead in mechanical or angular deg.

Total no of armature conductors in between angles AOC & BOD = $\frac{4\theta}{360}Z$

Demagnetizing amp turns/poles $AT_d = \frac{Z\theta I_a}{360a}$

## 1.23 Compensating Winding

Due to armature reaction flux density wave get distorted and reduced. Due to distortion of flux wave the peak flux density increases to such a high value that it creates high induced emf. If this emf is higher than the breakdown voltage across adjacent segments, a spark over could result which can easily spread over the whole commutator, and there will be a ring of fire, resulting in the complete short circuit of the armature.

To protect armature from such adverse condition armature reaction must be neutralized. To neutralize the armature reaction ampere-turns by compensating winding placed in the slots cut out in pole face such that the axis of the winding coincides with the brush axis as shown figure 1.25.

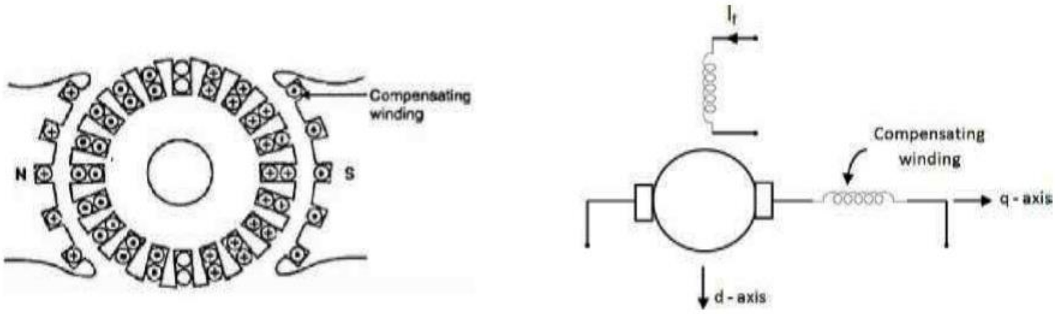


**Fig. 1.25 Compensating conductors in field poles and the connection of compensating conductors with armature**

The compensating windings neutralize the armature mmf directly under the pole which is the major portion because in the interpole region the air gap will be large. Compensating windings are connected in series with armature so that it will create mmf proportional to armature mmf.

The number of ampere-turns required in the compensating windings is given by

$$AT_c = \text{Total Armature Ampere Turns} \times \frac{\text{Pole Arc}}{\text{Pole Pitch}}$$

## 1.24 Commutation

The process of reversal of current in the short circuited armature coil is called 'Commutation'. This process of reversal takes place when coil is passing through the interpolar axis (q-axis), the coil is short circuited through commutator segments and brush.

The process of commutation of coil ‘CD’ is shown Fig. 1.26. In sub figure ‘c’ coil ‘CD’ carries 20A current from left to right and is about to be short circuited in figure ‘d’ brush has moved by a small width and the brush current supplied by the coil are as shown. In figure ‘e’ coil ‘CD’ carries no current as the brush is at the middle of the short circuit period and the brush current in supplied by coil ‘AB’ and coil ‘EF’. In sub figure ‘f’ the coil ‘CD’ which was carrying current from left to right carries current from right to left. In sub fig ‘g’ spark is shown which is due to the reactance voltage. As the coil is embedded in the armature slots, which has high permeability, the coil possess appreciable amount of self inductance. The current is changed from +20 to –20. So due to self inductance and variation in the current from +20 to –20, a voltage is induced in the coil which is given by L dI/dt. This emf opposes the change in current in coil ‘CD’ thus sparking occurs.

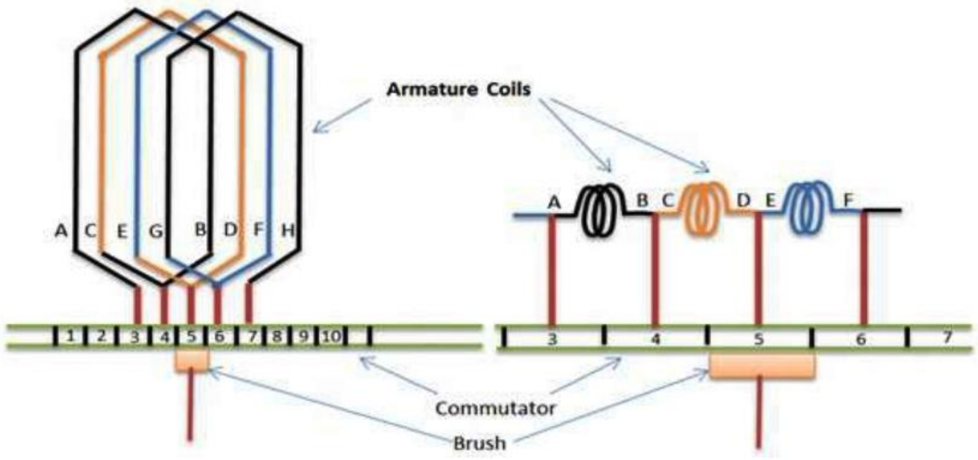


Fig a

Fig b

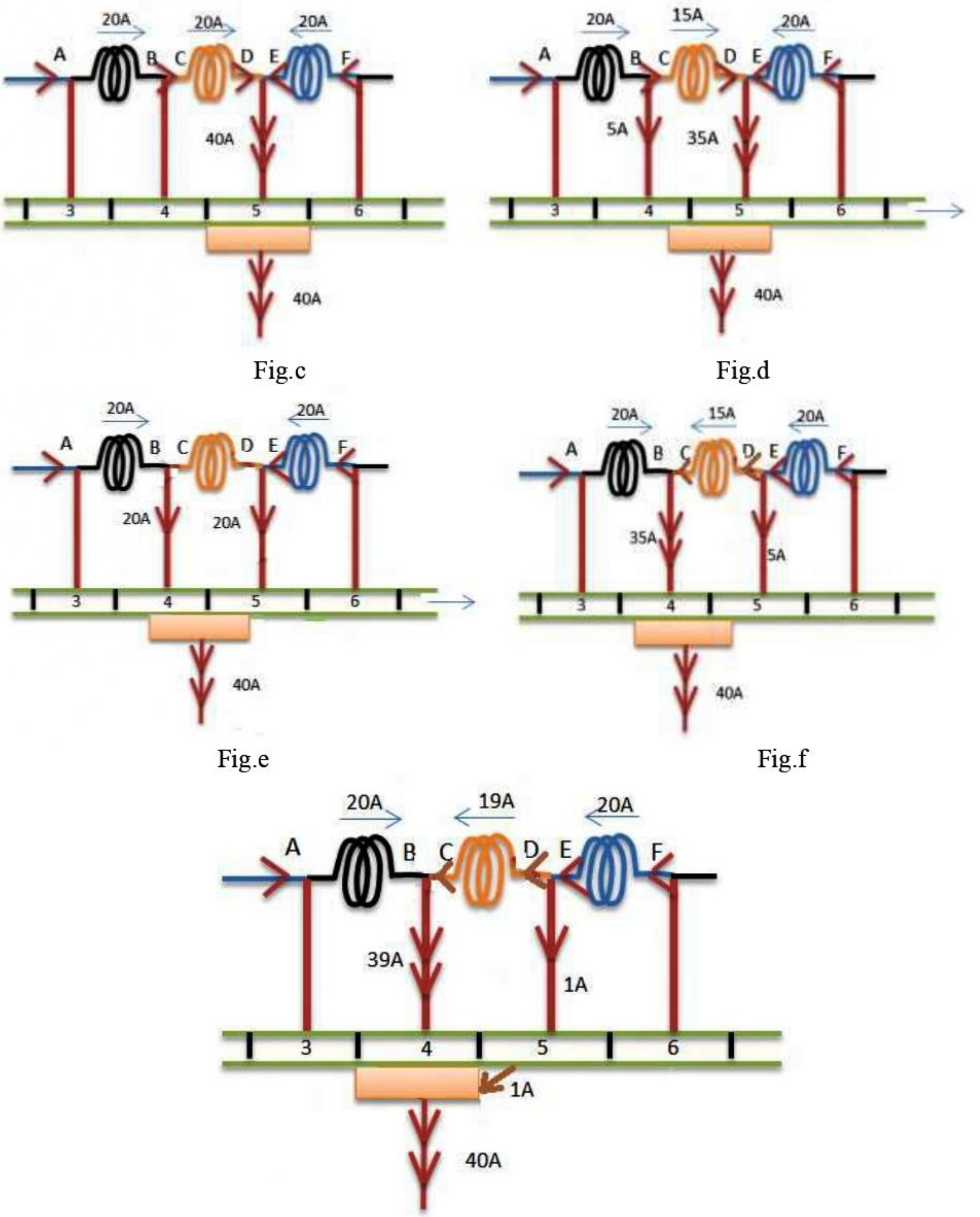


Fig.c

Fig.d

Fig.e

Fig.f

Fig.g

**Fig.1.26 a-g shows the process of commutation**

## 1.25 Reactance Voltage

During commutation sparking occurs in the commutator segment and brush due to presence of reactance voltage. This voltage is generated due to change of current in the commutating coil for its self-inductance and also due to mutual inductance of the adjacent coils. This voltage is called reactance voltage and according to Lenz's law this induced voltage oppose its cause of production. Here the cause of production is the change in current in the coil under commutation. Thus the commutation becomes poorer.

Reactance voltage = co-efficient of self-inductance X rate of change of current= $L\frac{di}{dt}$

Time of short circuit = $T_c$ = (time required by commutator to move a distance equal to the circumferential thickness of brush)–(one mica insulating strip) = Time of commutation

Let Wb= brush width in cm

$W_i$ = width of mica insulation in cm

$V_c$ = peripheral velocity of commutator segments in cm/sec.

Then $T_c = \frac{W_b - W_i}{V_c}$ sec

Total change in current = I - (-I) = 2I

Therefore self-induced or reactance voltage $= L\frac{2I}{T_c}$ for linear commutation

$=1.11L\frac{2I}{T_c}$ for sinusoidal commutation

If brush width is given in terms of commutator segments, then commutator velocity should be converted in terms of commutator segments/seconds.

## 1.26 Method of Improving Commutation

Commutation can be improved in two ways by (i) Resistance commutation

(ii) E.M.F commutation.

### 1.26.1Resistance Commutation

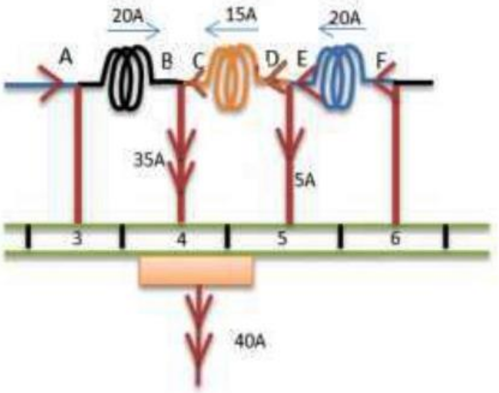


Fig. 1.27

In this method the resistance of the brushes are increased by changing then from copper brush to carbon brush. From the above figure 1.27 it is seen that when current '20A' from coil 'EF' reaches the commutator segment '5', it has two parallel paths opened to it. The first path is straight from bar '5' to the brush and the other is via short circuited coil 'CD' to bar '4' and then to brush. If copper brushes are used the current will follow the first path because of its low contact resistance. But when carbon brushes having high resistance are used, then current '20A' will prefer the second path because the resistance $r_1$ of first path will increase due to reducing area of contact with bar '5' and the resistance $r_2$ of second path decreases due to increasing area of contact with bar '4'. Hence carbon brushes help in obtaining sparkles commutation. Also, carbon brushes lubricate and polish commutator. But, because of high resistance the brush contact drop increases and the commutator has to be made larger to dissipate the heat due to loss. Carbon brushes require larger brush holders because of lower current density.

### 1.26.2 E.M.F commutation:

To neutralize sparking caused by reactance voltage in this method an emf is producied whichacts in opposite direction to that of reactance voltage, so that the reactance voltage is completely eliminated. The neutralization of emf may be done in two ways (i) by giving brush a forward lead sufficient enough to bring the short circuited coil under the influence of next pole of opposite polarity or (ii) by using interpoles or compoles. The second method is commonly employed.

### 1.26.3 Interpoles or Compoles

These are small poles fixed to the yoke and placed in between the main poles as shown in figure 1.28. They are wound with few turns of heavy gauge copper wire and are connected in series with the armature so that they carry full armature current. Their polarity in case of generator is that of the main pole ahead in the direction of rotation. Their polarity in case of motor is that of the main pole behind in the direction of rotation.

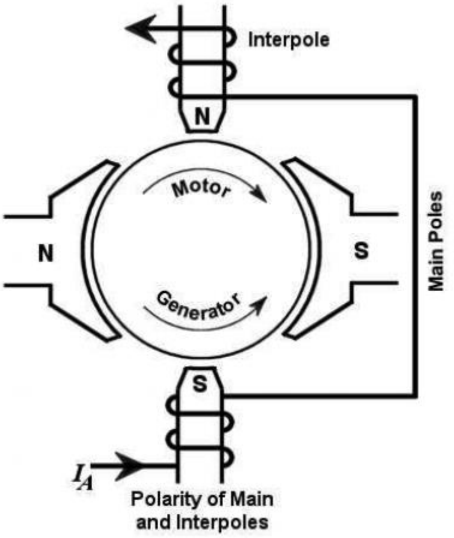


Fig. 1.28 Inter-poles of DC machines

The function of interpoles is (i) to induce an emf which is equal and opposite to that of the reactance voltage. Interpoles neutralize the cross magnetizing effect of armature reaction.

## 1.27 DC Machines Characteristics

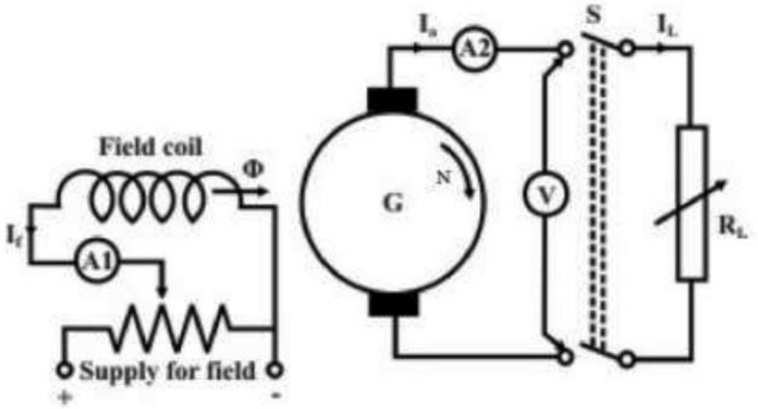
There main characteristics of dc machines are

i. No-load characteristics or Open Circuit Characteristics

ii. Load characteristics

### 1.27.1No-load characteristics or Open Circuit Characteristics of separately excited generator

In this type of generator field winding is excited from a separate source, hence field current is independent of armature terminal voltage as shown on figure 1.29. The generator is driven by a prime mover at rated speed, say N rpm. With switch *S* in opened condition, field is excited via a potential divider connection from a separate d.c source and field current is gradually increased. The field current will establish the flux per pole $\phi$. The voltmeter V connected across the armature terminals of the machine will record the generated emf ($E_g = \frac{P\phi ZN}{60A} = k\phi N$). As field current is increased, $E_g$ will increase. $E_g$ versus $I_f$ plot at speed $N_1, N_2, N_3$ is shown in figure below. Where $N_1 > N_2 > N_3$.



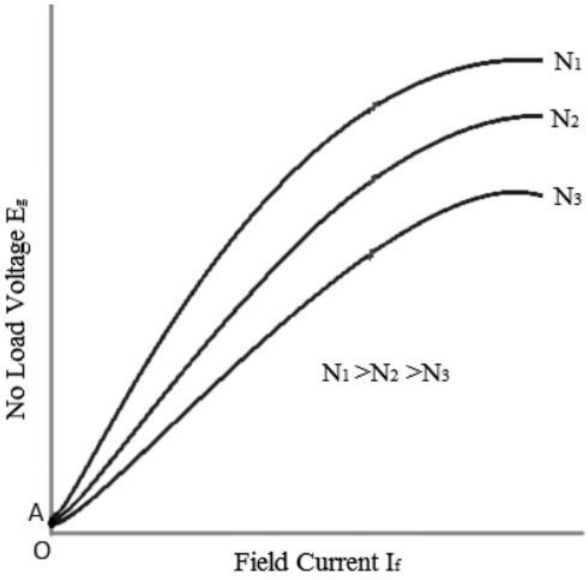


Fig. 1.29 Open circuit characteristics of DC separately excited generator

### 1.27.2Load characteristic of separately excited generator

Load characteristic is the characteristics in between terminal voltage $V_T$ with load current $I_L$ of a generator with constant speed and constant field current. For $I_L = 0$, $V_T = E_g$ should be the first point on the load characteristic. With increase of load current the terminal voltage will drop due to armature resistance and reaction drop. In the figure below the rated load current shown by point A. Hence the load characteristic will be drooping in nature as shown in figure 1.30.

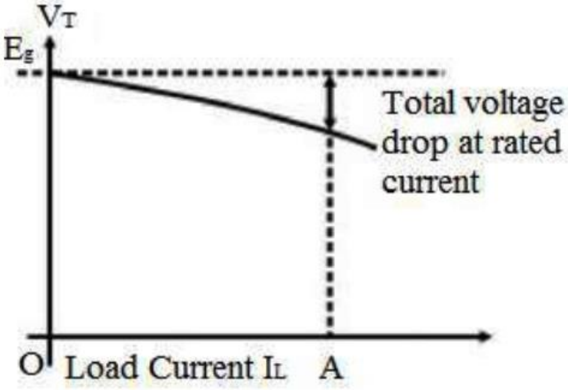


Fig 1.30 Load characteristics of DC separately excited generator

### 1.27.3 Characteristics of a shunt generator

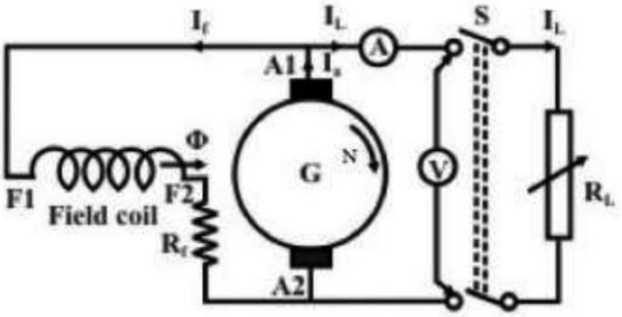


Fig 1.31 Connection diagram to obtain no-load and load characteristic

To obtain OCC of DC shunt generator the above circuit will be used with switch S kept open in fig. 1.31. As this machine is self-excited thus there is no need to use separate dc source for producing field current. The voltage build up process is described earlier. The open nature of the OCC will be similar to separately excited machine.

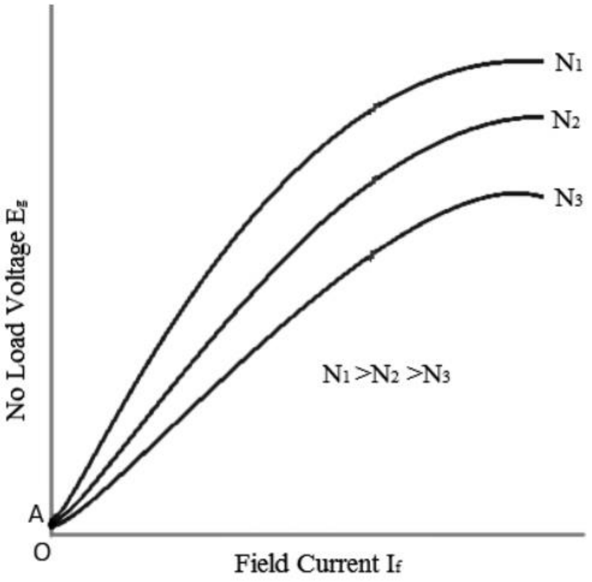


Fig. 1.32 Open circuit characteristics of DC shunt generator

### 1.27.4 Load characteristic of shunt generator

With switch S in open condition in fig. 1.31, the generator is practically under no load condition as field current is pretty small. The voltmeter reading will be $E_g$ as shown in figures below. In other words, $V_T = E_g$, $I_L = 0$ is the first point in the load characteristic. To load the machine S is closed and the load resistances decreased so that it delivers load current $I_L$. Unlike separately excited motor, here $I_L \neq I_a$. In fact, for shunt generator, $I_a = I_L - I_f$. So increase of $I_L$ will mean increase of $I_a$ as well. The drop in the terminal voltage will be caused by the usual $I_a r_a$ drop, brush voltage drop and armature reaction effect. Apart from these, in shunt generator, as terminal voltage decreases, field current hence $\phi$ also decreases causing additional drop in terminal voltage. Remember in shunt generator, field current is decided by the terminal voltage by virtue of its parallel connection with the armature. Figure 1.33 gives the plot of terminal voltage versus load current which is called the load characteristic.

As the load resistance is decreased (load current increased), the terminal voltage drops until point B is reached. If load resistance is further decreased, the load current increases momentarily. This momentary increase in load current produces more armature reaction thus causing a reduction in the terminal voltage and field current. The net reduction in terminal voltage is so large that the load current decreases and the characteristic turns back. In case the machine is short circuited, the curve terminates at point H. Here OH is the load current due to the voltage generated by residual flux.

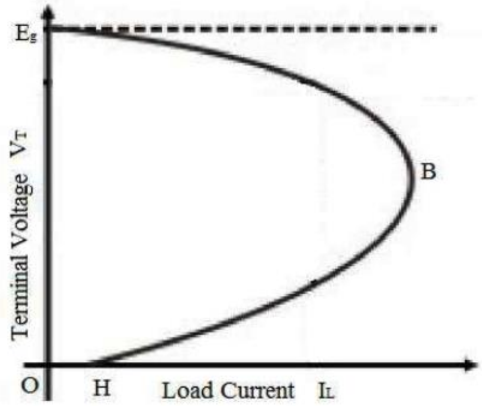


**Fig. 1.33 Load characteristics of DC shunt generator**

## 1.27.5 Compound generator

As introduced earlier, compound machines have both series and shunt field coils. Series field coil may be connected in such a way that the mmf produced by it aids the shunt field mmf-then the machine is said to be cumulative compound machine, otherwise if the series field mmf acts in opposition with the shunt field mmf – then the machine is said to be differential compound machine.

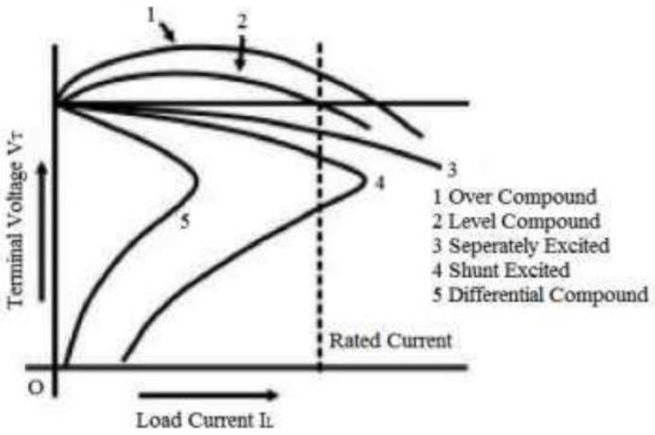


**Fig. 1.34 Load characteristics of DC compound generator**

In a compound generator, series field coil current is load dependent. Therefore, for a cumulatively compound generator, with the increase of load, flux per pole increases. This in turn increases the generated emf and terminal voltage. Unlike a shunt motor, depending on the strength of the series field mmf, terminal voltage at full load current may be same or more than the no load voltage. When the terminal voltage at rated current is same that at no load condition, then it is called a level compound generator. If however, terminal voltage at rated current is more than the voltage at no load, it is called a over compound generator. The load characteristic of a cumulative compound generator will naturally be above the load characteristic of a shunt generator as depicted in figure 1.34. At load current higher than the rated current, terminal voltage starts decreasing due to saturation, armature reaction effect and more drop in armature and series field resistances.

## 1.28 Parallel Operation of DC Generator

### 1.28.1 Advantages of DC generator operating in parallel

In a dc power plant, power is usually supplied from several generators of small ratings connected in parallel instead of from one large generator. This is due to the following reasons:

#### a. *Continuity of service:*

If a single large generator is used in the power plant, then in case of its breakdown, the whole plant will be shut down. However, if power is supplied from a number of small units operating in parallel, then in case of failure of one unit, the continuity of supply can be maintained by other healthy units.

### b. *Efficiency:*

Generators run most efficiently when loaded to their rated capacity. Therefore, when load demand on power plant decreases, one or more generators can be shut down and the remaining units can be efficiently loaded.

### c. *Maintenance and repair:*

Generators generally require routine-maintenance and repair. Therefore, if generators are operated in parallel, the routine or emergency operations can be performed by isolating the affected generator while load is being supplied by other units. This leads to both safety and economy.

### d. *Increasing plant capacity:*

In the modern world of increasing population, the use of electricity is continuously increasing. When added capacity is required, the new unit can be simply paralleled with the old units.

### e. *Non-availability of single large unit:*

In many situations, a single unit of desired large capacity may not be available. In that case a number of smaller units can be operated in parallel to meet the load requirement. Generally a single large unit is more expensive.

## 1.28.2 Connecting Shunt Generators in Parallel:

The generators in a power plant are connected in parallel through bus-bars. The bus-bars are heavy thick copper bars and they act as +ve and -ve terminals. The positive terminals of the generators are connected to the +ve side of bus-bars and negative terminals to the negative side of bus-bars. Fig. 1.35 shown shunt generator 1 connected to the bus-bars and supplying load. When the load on the power plant increases beyond the capacity of this generator, the second shunt generator 2 is connected in parallel with the first to meet the increased load demand.

### *The procedure for paralleling generator 2 with generator 1 is as under:*

i. The prime mover of generator 2 is brought up to the rated speed. Now switch S2 in the field circuit of the generator 2 is closed.

ii. Next circuit breaker CB-2 is closed and the excitation of generator 2 is adjusted till it generates voltage equal to the bus-bars voltage. This is indicated by voltmeter V2.

iii. Now the generator 2 is ready to be paralleled with generator 1. The main switch DPST2 is closed, thus putting generator 2 in parallel with generator 1. Note that generator 2 is not supplying any load because its generated emf is equal to bus-bars voltage. The generator is said to be “floating” (i.e. not supplying any load) on the bus-bars.

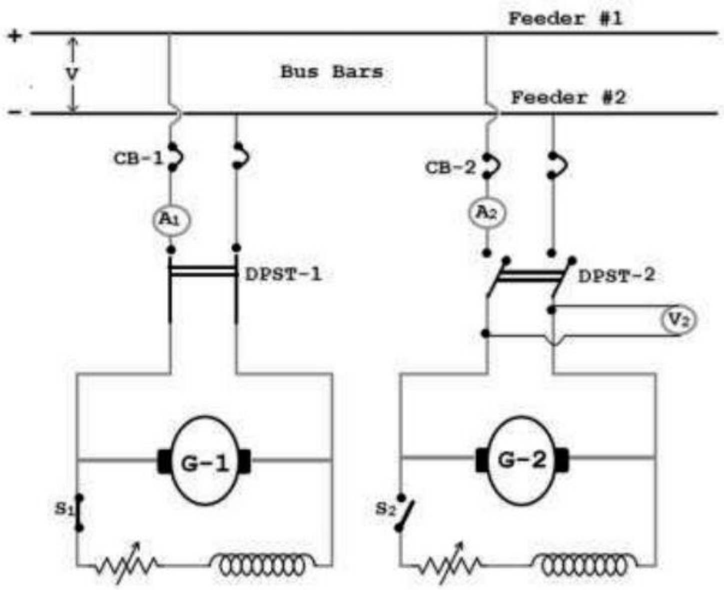


**Fig. 1.35 Schematic diagram of DC generator connected in parallel**

iv. If generator 2 is to deliver any current, then its generated voltage $E_g$ should be greater than the bus-bars voltage $V_T$. In that case, current supplied by it is $I = (E_g - V_T)/R_a$ where $R_a$ is the resistance of the armature circuit. By increasing the field current (and hence induced emf $E_g$), the generator 2 can be made to supply proper amount of load.

v. The load may be shifted from one shunt generator to another merely by adjusting the field excitation. Thus if generator 1 is to be shut down, the whole load can be shifted onto generator 2 provided it has the capacity to supply that load. In that case, reduce the current supplied by generator 1 to zero (This will be indicated by ammeter A1) open C.B.-1 and then open the main switch DPST1.

## 1.29 Equalizer Bar:

**Compound Generators in Parallel:** Under-compounded generators also operate satisfactorily in parallel but over compounded generators will not operate satisfactorily unless their series fields are paralleled. This is achieved by connecting two negative brushes together as shown in Fig. 1.36. The conductor used to connect these brushes is generally called equalizer bar. Suppose that an attempt is made to operate the two generators in parallel without an equalizer bar. If, for any reason, the current supplied by generator 1 increases slightly, the current in its series field will increase and raise the generated voltage. This will cause generator 1 to take more load. Since total load supplied to the system is constant, the current in generator 2 must decrease and as a result its series field is weakened. Since this effect is cumulative, the generator 1 will take the entire load and drive generator 2 as a motor. After machine 2 changes from a generator to a motor, the current in the shunt field will remain in the same direction, but the current in the armature and series field will reverse. Thus the magnetizing action, of the series field opposes that of the shunt field. As the current taken by the machine 2 increases, the demagnetizing action of series field becomes greater and the resultant field becomes weaker. The resultant field will finally become zero and at that time machine 2 will be short circuited machine 1, opening the breaker of either or both machines.

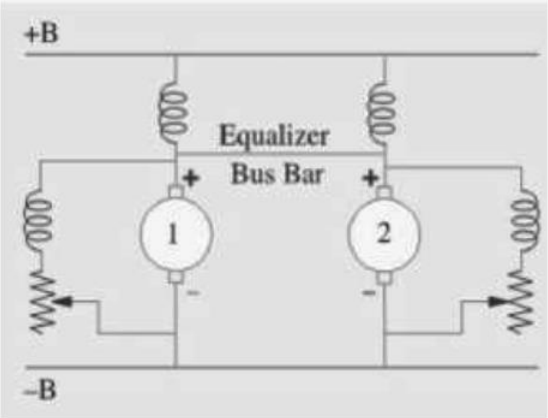


**Fig. 1.36 Connection of equalizer bar in parallel connection of DC compound generator**

When the equalizer bar is used, a stabilizing action exists and neither machine tends to take all the load. To consider this, suppose that current delivered by generator 1 increases. The increased current will not only pass through the series field of generator 1 but also through the equalizer bar and series field of generator 2. Therefore, the voltage of both the machines increases and the generator 2 will take a part of the load.

## 1.30 Load Sharing:

The load sharing between shunt generators in parallel can be easily regulated because of their drooping characteristics. The load may be shifted from one generator to another merely by adjusting the field excitation. Let us discuss the load sharing of two generators which have unequal no-load voltages. Let $E_1$, $E_2$ = no-load voltages of the two generators $R_1$, $R_2$ = their armature resistances

$V_T$ = common terminal voltage (Bus-bars voltage). Then

$$I_1=\frac{E_1\text{-}V_T}{R_1} \text{ and } I_2=\frac{E_2\text{-}V_T}{R_2}$$

Thus current output of the generators depends upon the values of $E_1$ and $E_2$. These values may be changed by field rheostats. The common terminal voltage (or bus-bars voltage) will depend upon (i) the emfs of individual generators and (ii) the total load current supplied. It is generally desired to keep the busbars voltage constant. This can be achieved by adjusting the field excitations of the generators operating in parallel.

# Module II

## [TRANSFORMER]

TOPICS

**Transformers:** Single phase transformer, Constructional details, Core, windings, Insulation, principle of operation, emf equation, magnetising current and core losses, no load and on load operation, Phasor diagram, equivalent circuit, losses and efficiency, condition for maximum efficiency, voltage regulation, approximate expression for voltage regulation, open circuit and short circuit tests, Sumpner’s test, Inrush of switching currents, harmonics in single phase transformers, magnetizing current wave form, Parallel operation of transformers.

[Topics are arranged as per above sequence]

# Transformers

## 2.1 Introduction

The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems. It can change the magnitude of alternating voltage or current from one value to another. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little attention. They also have a very high efficiency as high as 99%.

## 2.2. Single Phase Transformer

A transformer is a static device of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig 1. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage $V_1$ whose magnitude is to be changed is applied to the primary.

Depending upon the number of turns of the primary ($N_1$) and secondary ($N_2$), an alternating e.m.f. $E_2$ is induced in the secondary. This induced e.m.f. $E_2$ in the secondary causes a secondary current $I_2$. Consequently, terminal voltage $V_2$ will appear across the load.

If $V_2 > V_1$, it is called a step up-transformer.

If $V_2 < V_1$, it is called a step-down transformer.

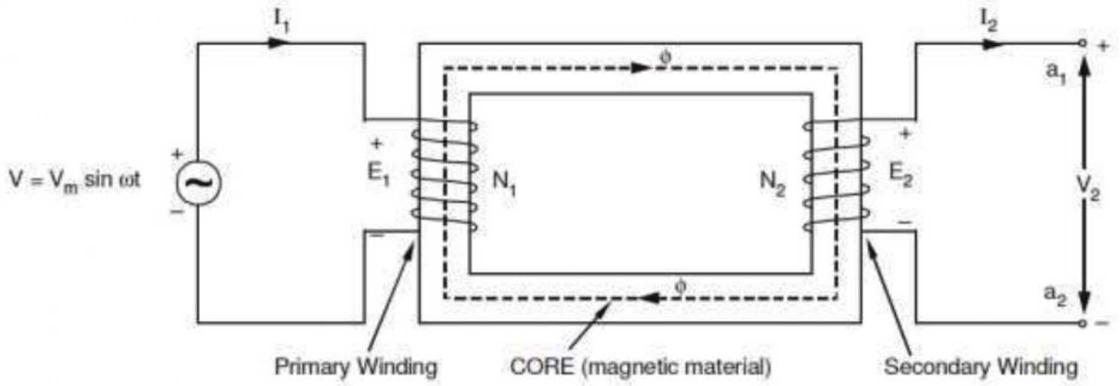


**Fig. 2.1 Schematic diagram of single phase transformer**

## 2.3 Constructional Details

Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called (a) core type, and (b) shell type.

### 2.3.1 Core-type and Shell-type Construction

In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high-voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure 2.1(a) shows the cross-section of the arrangement. In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in Figure 2.1(b). The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.

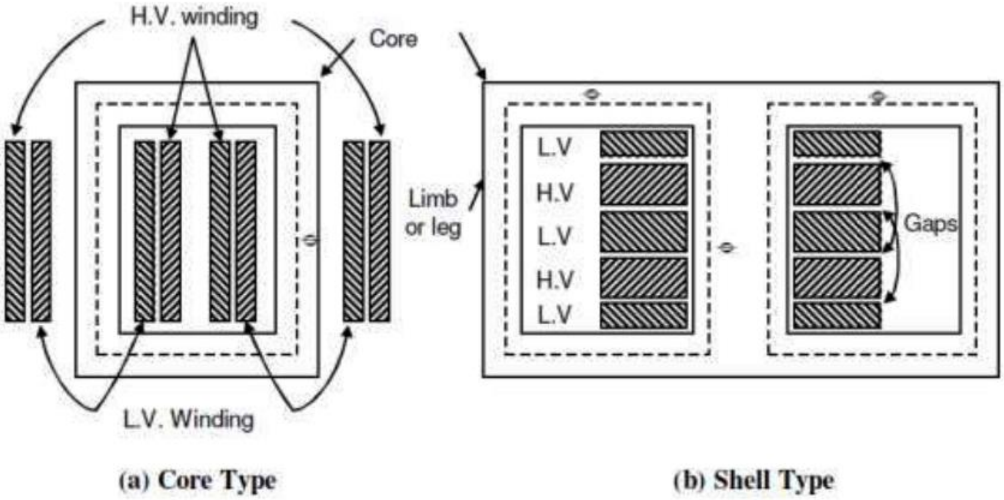


Fig: 2.1 Core type & shell type transformer

## Core

The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer. The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysteresis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel. Conductor material used for windings is mostly copper. However, for small distribution transformer aluminium is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

## Insulating Oil

In oil-immersed transformer, the iron core together with windings is immersed in insulating oil. The insulating oil provides better insulation, protects insulation from moisture and transfers the heat produced in core and windings to the atmosphere.

The transformer oil should possess the following qualities:

(a) High dielectric strength,

(b) Low viscosity and high purity,

(c) High flash point, and

(d) Free from sludge.

Transformer oil is generally a mineral oil obtained by fractional distillation of crude oil.

### Tank and Conservator

The transformer tank contains core wound with windings and the insulating oil. In large transformers small expansion tank is also connected with main tank is known as conservator. Conservator provides space when insulating oil expands due to heating. The transformer tank is provided with tubes on the outside, to permits circulation of oil, which aides in cooling. Some additional devices like breather and Buchholz relay are connected with main tank. Buchholz relay is placed between main tank and conservator. It protect the transformer under extreme heating of transformer winding. Breather protects the insulating oil from moisture when the cool transformer sucks air inside. The silica gel filled breather absorbs moisture when air enters the tank. Some other necessary parts are connected with main tank like, Bushings, Cable Boxes, Temperature gauge, Oil gauge, Tappings, etc.

## 2.4 Principle of Operation

When an alternating voltage V1 is applied to the primary, an alternating flux ϕ is set up in the core. This alternating flux links both the windings and induces e.m.f.s E1 and E2 in them according to Faraday's laws of electromagnetic induction. The e.m.f. E1 is termed as primary e.m.f. and e.m.f. E2 is termed as secondary e.m.f.

$$\text{Clearly,} \qquad E_1 = -N_1 \frac{d\phi}{dt}$$

$$\text{and} \qquad E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \qquad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E2 and E1 depend upon the number of turns on the secondary and primary respectively.

If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer. If $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer.

If load is connected across the secondary winding, the secondary e.m.f. $E_2$ will cause a current $I_2$ to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

**The following points may be noted carefully:**

(a) The transformer action is based on the laws of electromagnetic induction.

(b) There is no electrical connection between the primary and secondary.

(c) The a.c. power is transferred from primary to secondary through magnetic flux.

(d) There is no change in frequency i.e., output power has the same frequency as the input power.

(e) The losses that occur in a transformer are:

(a) ***core losses***—eddy current and hysteresis losses
(b) ***copper losses***—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

## 2.4.1 E.M.F. Equation of a Transformer

Consider that an alternating voltage $V_1$ of frequency f is applied to the primary as shown in Fig.2.3. The sinusoidal flux $\phi$ produced by the primary can be represented as:

$\phi=\phi_m \sin\omega t$

When the primary winding is excited by an alternating voltage $V_1$, it is circulating alternating current, producing an alternating flux $\phi$.

$\phi$ - Flux

$\phi_m$ - maximum value of flux

$N_1$ - Number of primary turns

$N_2$ - Number of secondary turns

$f$ - Frequency of the supply voltage

$E_1$ - R.M.S. value of the primary induced e.m.f

$E_2$ - R.M.S. value of the secondary induced e.m.f

The instantaneous e.m.f. $e_1$ induced in the primary is -

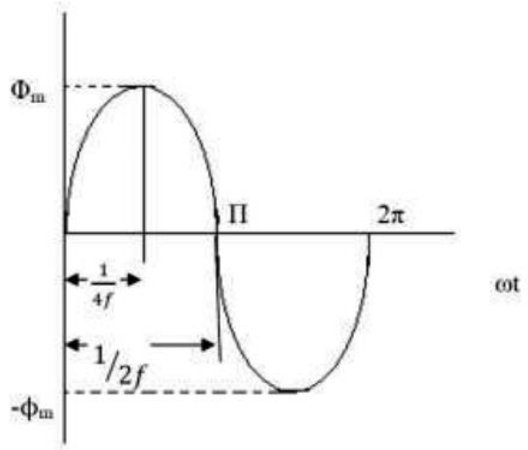


**Fig. 2.3**

From Faraday's law of electromagnetic induction -

$$\text{Average e.m.f per turns} = \frac{d\emptyset}{dt}$$

$d\phi$ = change in flux
$dt$ = time required for change in flux

The flux increases from zero value to maximum value $\phi m$ in 1/4f of the time period that is in 1/4f seconds.

The change of flux that takes place in 1/4f seconds = $\phi_m - 0 = \phi_m$ webers

$$\frac{d\phi}{dt} = \frac{dt}{1/4f} = 4f\phi_m \ w_b/\text{sec.}$$

Since flux ϕ varies sinusoidally, the R.m.s value of the induced e.m.f is obtained by multiplying the average value with the form factor

$$\text{Form factor of a sinwave} = \frac{\text{R.m.s value}}{\text{Average value}} = 1.11$$

R.M.S Value of e.m.f induced in one turns = $4\phi_m f \times 1.11$ Volts.

$= 4.44\phi_m f$ Volts.

R.M.S Value of e.m.f induced in primary winding = $4.44\phi_m f N1$Volts.

R.M.S Value of e.m.f induced in secondary winding = $4.44\phi_m f N2$Volts.

The expression of $E_1$ and $E_2$ are called e.m.f equation of a transformer

$$\boxed{\begin{array}{l} V_1 = E_1 = 4.44\phi_m f N_1 \text{Volts.} \\ V_2 = E_2 = 4.44\phi_m f N_2 \text{Volts.} \end{array}}$$

## 2.4.2 Voltage Ratio

Voltage transformation ratio is the ratio of e.m.f induced in the secondary winding to the e.m.f induced in the primary winding.

$$\frac{E2}{E1} = \frac{4.44\phi mf\ N2}{4.44\phi mf\ N1}$$

$$\boxed{\frac{E2}{E1} = \frac{N2}{N1} = K}$$

This ratio of secondary induced e.m.f to primary induced e.m.f is known as voltage transformation ratio

$$E_2 = KE_1 \qquad \text{where } K = \frac{N2}{N1}$$

1. If $N_2>N_1$ i.e. $K>1$ we get $E_2>E_1$ then the transformer is called step up transformer.

2. If $N_2< N_1$ i.e. $K<1$ we get $E_2< E_2$ then the transformer is called step down transformer.

3. If $N_2= N_1$ i.e. $K=1$ we get $E_2= E_2$ then the transformer is called isolation transformer or 1:1

transformer.

### 2.4.3 Current Ratio

Current ratio is the ratio of current flow through the primary winding ($I_1$) to the current flowing through the secondary winding ($I_2$). In an ideal transformer -

Apparent input power = Apparent output power.

$$V_1I_1 = V_2I_2$$

$$\frac{I1}{I2} = \frac{V2}{V1} = \frac{N2}{N1} = K$$

### Volt-Ampere Rating

i) The transformer rating is specified as the products of voltage and current (VA rating).

ii) On both sides, primary and secondary VA rating remains same. This rating is generally expressed in KVA (Kilo Volts Amperes rating).

$$\frac{V1}{V2} = \frac{I2}{I1} = K$$

$$V_1I_1 = V_2I_2$$

$$\text{KVA Rating of a transformer} = \frac{V1I1}{1000} = \frac{V2I2}{1000} \text{ (1000 is to convert KVA to VA)}$$

$V_1$ and $V_2$ are the $V_t$ of primary and secondary by using KVA rating we can calculate $I_1$ and $I_2$ Full load current and it is safe maximum current.

$$I_1 \text{ Full load current} = \frac{KVA\ Rating\ X\ 1000}{V1}$$

$$I_1 \text{ Full load current} = \frac{KVA\ Rating\ X\ 1000}{V2}$$

### 2.4.4 Transformer on No-load

a) Ideal transformer

b) Practical transformer

***a) Ideal Transformer***

An ideal transformer is one that has

(i) No winding resistance

(ii) No leakage flux i.e., the same flux links both the windings

(iii) No iron losses (i.e., eddy current and hysteresis losses) in the core

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.

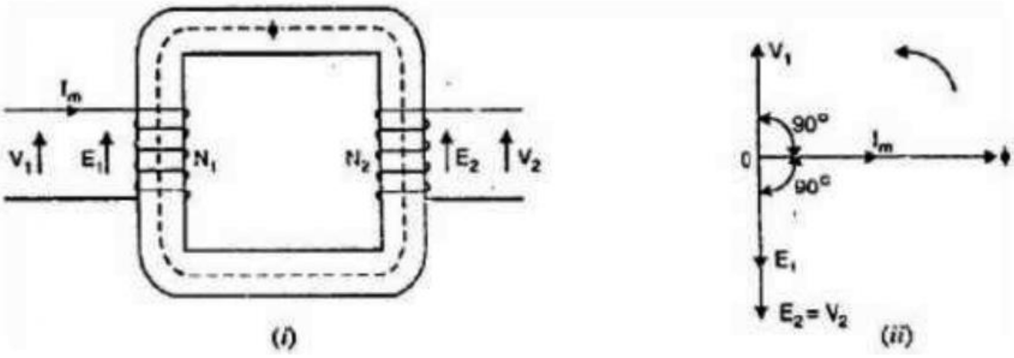


**Fig: 2.4**

Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in *Fig.2.4 (i)*. under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage $V_1$ is applied to the primary, it draws a small magnetizing current $I_m$ which lags behind the applied voltage by 90°. This alternating current $I_m$ produces an alternating flux ϕ which is proportional to and in phase with it. The alternating flux ϕ links both the windings and induces e.m.f. $E_1$ in the primary and e.m.f. $E_2$ in the secondary. The primary e.m.f. $E_1$ is, at every instant, equal to and in opposition to $V_1$ (Lenz's law). Both e.m.f.s $E_1$ and $E_2$ lag behind flux ϕ by 90°.However, their magnitudes depend upon the number of primary and secondary turns. *Fig. 2.4 (ii)* shows the phasor diagram of an ideal transformer on no load. Since flux ϕ is common to both the windings, it has been taken as the reference phasor. The primary e.m.f. E1 and secondary e.m.f. $E_2$ lag behind the flux ϕ by 90°. Note that $E_1$ and $E_2$ are in phase. But $E_1$ is equal to $V_1$ and 180° out of phase with it.

$$\boxed{\frac{E2}{E1} = \frac{V2}{V1} = K}$$

## 2.4.5 Phasor Diagram

i) Φ (flux) is reference

ii) $I_m$ produce ϕ and it is in phase with ϕ, $V_1$ Leads $I_m$ by 90˚

iii) $E_1$ and $E_2$ are in phase and both opposing supply voltage $V_1$, winding is purely inductive

So current has to lag voltage by 90˚.

iv) The power input to the transformer

$$P = V_1.I_1.\cos(90^\circ) \qquad \ldots\ldots\ldots\ldots (\cos 90^\circ = 0)$$

$$P = 0 \text{ (ideal transformer)}$$

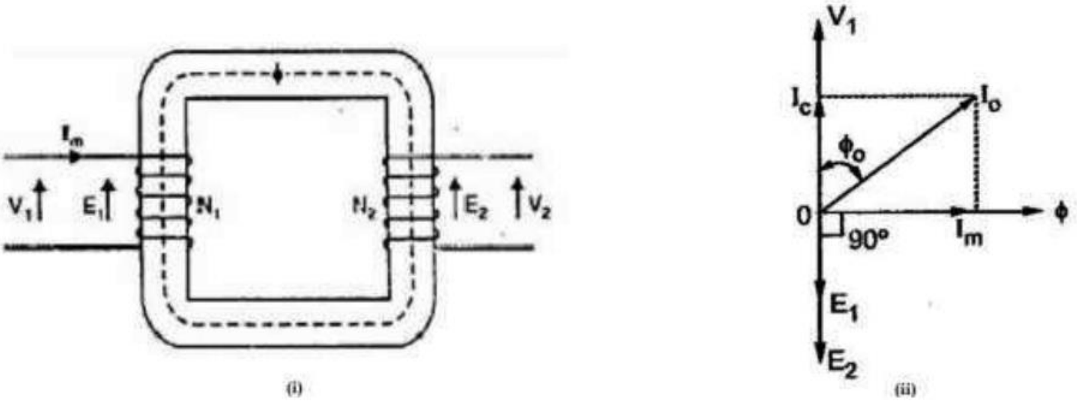
***b)i) Practical Transformer* on no load**

A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) iron losses (ii) winding resistances and (iii) Magnetic leakage

**(i) Iron losses**. Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.

**(ii) Winding resistances.** Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance $R_1$ and secondary resistance $R_2$ act in series with the respective windings as shown in Fig. When current flows through the windings, there will be power loss as well as a loss in voltage due to IR drop. This will affect the power factor and $E_1$ will be less than $V_1$ while $V_2$ will be less than $E_2$.

Consider a practical transformer on no load i.e., secondary on open-circuit as Shown in Fig 2.5.



(i) (ii)

**Fig: 2.5 Phasor diagram of transformer at noload**

Here the primary will draw a small current $I_0$ to supply -

(i) the iron losses and

(ii) a very small amount of copper loss in the primary.

Hence the primary no load current I0 is not 90° behind the applied voltage $V_1$ but lags it by an angle $\phi_0 < 90°$ as shown in the phasor diagram.

No load input power, $W_0 = V_1 I_0 \cos \phi 0$

As seen from the phasor diagram in Fig.2.5 (ii), the no-load primary current $I_0$

(i) The component $I_c$ in phase with the applied voltage $V_1$. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$I_c = I_0 \cos \phi_0$

The component Im lagging behind $V_1$ by 90° and is known as magnetizing component. It is this component which produces the mutual flux $\phi$ in the core.

$I_m = I_0 \sin \phi_0$

Clearly, Io is phasor sum of Im and $I_c$,

$$I_o = \sqrt{Im^2 + Ic^2}$$

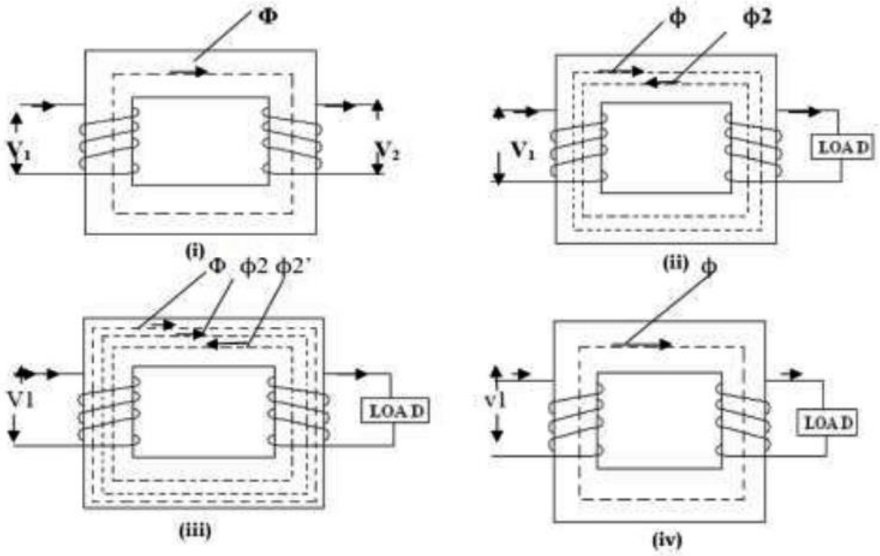
$$\text{No load P.F.,} \quad \cos\phi o = \frac{Ic}{Io}$$

The no load primary copper loss (i.e. $Io^2R1$) is very small and may be neglected.

Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

No load input power, $W_0 = V_1 I_o \cos\phi_o = Pi$ = Iron loss

***b) ii)Practical Transformer* on Load**



(i) (ii) (iii) (iv)

Fig: 2.6

At no load, there is no current in the secondary so that $V_2 = E_2$. On the primary side, the drops in $R_1$ and $X_1$, due to $I_0$ are also very small because of the smallness of $I_0$. Hence, we can say that at no load, $V_1 = E_1$.

i) When transformer is loaded, the secondary current $I_2$ is flows through the secondary winding.

ii) Already $I_m$ magnetizing current flow in the primary winding fig. 2.6(i).

iii) The magnitude and phase of $I_2$ with respect to $V_2$ is determined by the characteristics of the load.

a) $I_2$ in phase with $V_2$ (resistive load)

b) $I_2$ lags with $V_2$ (Inductive load)

c) $I_2$ leads with $V_2$ (capacitive load)

iv) Flow of secondary current $I_2$ produce new Flux $\phi_2$ fig.2.6 (ii)

v) $\Phi$is main flux which is produced by the primary to maintain the transformer as constant magnetising component.

vi) $\Phi_2$ opposes the main flux $\phi$, the total flux in the core reduced. It is called demagnetising Ampere-turns due to this $E_1$ reduced.

vii) To maintain the $\phi$ constant primary winding draws more current ($I_2$') from the supply (load component of primary) and produce $\phi_2$' flux which is oppose $\phi_2$ (but in same direction as $\phi$), to maintain flux constant flux constant in the core fig.2.6 (iii).

viii) The load component current $I_2$' always neutralizes the changes in the load.

ix) Whatever the load conditions, the net flux passing through the core is approximately the same as at no-load. An important deduction is that due to the constancy of core flux at all loads, the core loss is also practically the same under all load conditions fig.2.6 (iv).

$$\boxed{\Phi_2 = \phi_2' \quad N_2I_2 = N_1I_2' \quad I_2' = \frac{N2}{N1} X I_2 = KI_2}$$

## 2.4.6 Phasor Diagram

i) Take ($\phi$) flux as reference for all load

ii) The no load $I_o$ which lags by an angle $\phi o$. $Io = \sqrt{Ic^2 + Im^2}$.

iii) The load component $I_2$', which is in anti-phase with $I_2$ and phase of $I_2$ is decided by the load.

iv) Primary current $I_1$ is vector sum of $I_o$ and $I_2$'

$$\vec{I_1} - \vec{I_o} + \vec{I_2'}$$

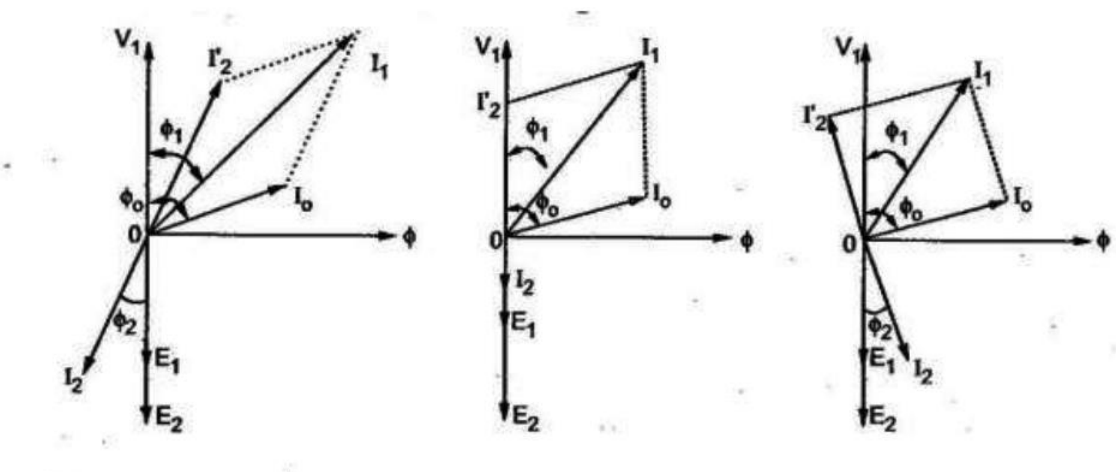
$$I_1 = \sqrt{Io^2 + I2'^2}$$

a) If load is Inductive, $I_2$ lags $E_2$ by $\phi_2$, shown in phasor diagram fig 2.7 (a).

b) If load is resistive, $I_2$ in phase with $E_2$ shown in phasor diagram fig. 2.7 (b).

c) If load is capacitive load, $I_2$ leads $E_2$ by $\phi_2$ shown in phasor diagram fig. 2.7 (c).

For easy understanding at this stage here we assumed $E_2$ is equal to $V_2$ neglecting various drops.



(a) Inductive load (b) Resistive load (c) Capacitive load

Fig: 2.7.a

$$I_1 \cong I_2'$$

Balancing the ampere – turns

$$N_1I_2' = N_1I_1 + N_2I_2$$

$$\frac{I1}{I2} = \frac{N2}{N1} = K$$

Now we going to construct complete phasor diagram of a transformer (shown in Fig: 2.7.b)

**Effect of Winding Resistance**

In practical transformer it process its own winding resistance causes power loss and also the voltage drop.

$R_1$ – primary winding resistance in ohms.
$R_2$ – secondary winding resistance in ohms.

The current flow in primary winding make voltage drop across it is denoted as $I_1R_1$ here supply voltage $V_1$ has to supply this drop primary induced e.m.f $E_1$ is the vector difference between $V_1$ and $I_1R_1$.

$$\overrightarrow{E_1} = \overrightarrow{V_1} - \overrightarrow{I_1R_1}$$

Similarly the induced e.m.f in secondary $E_2$, The flow of current in secondary winding makes voltage drop across it and it is denoted as $I_2R_2$ here $E_2$ has to supply this drop.

## Equivalent Resistance

1) It would now be shown that the resistances of the two windings can be transferred to any one of the two winding.

2) The advantage of concentrating both the resistances in one winding is that it makes calculations very simple and easy because one has then to work in one winding only.

3) Transfer to any one side either primary or secondary without affecting the performance of the transformer.

The total copper loss due to both the resistances.

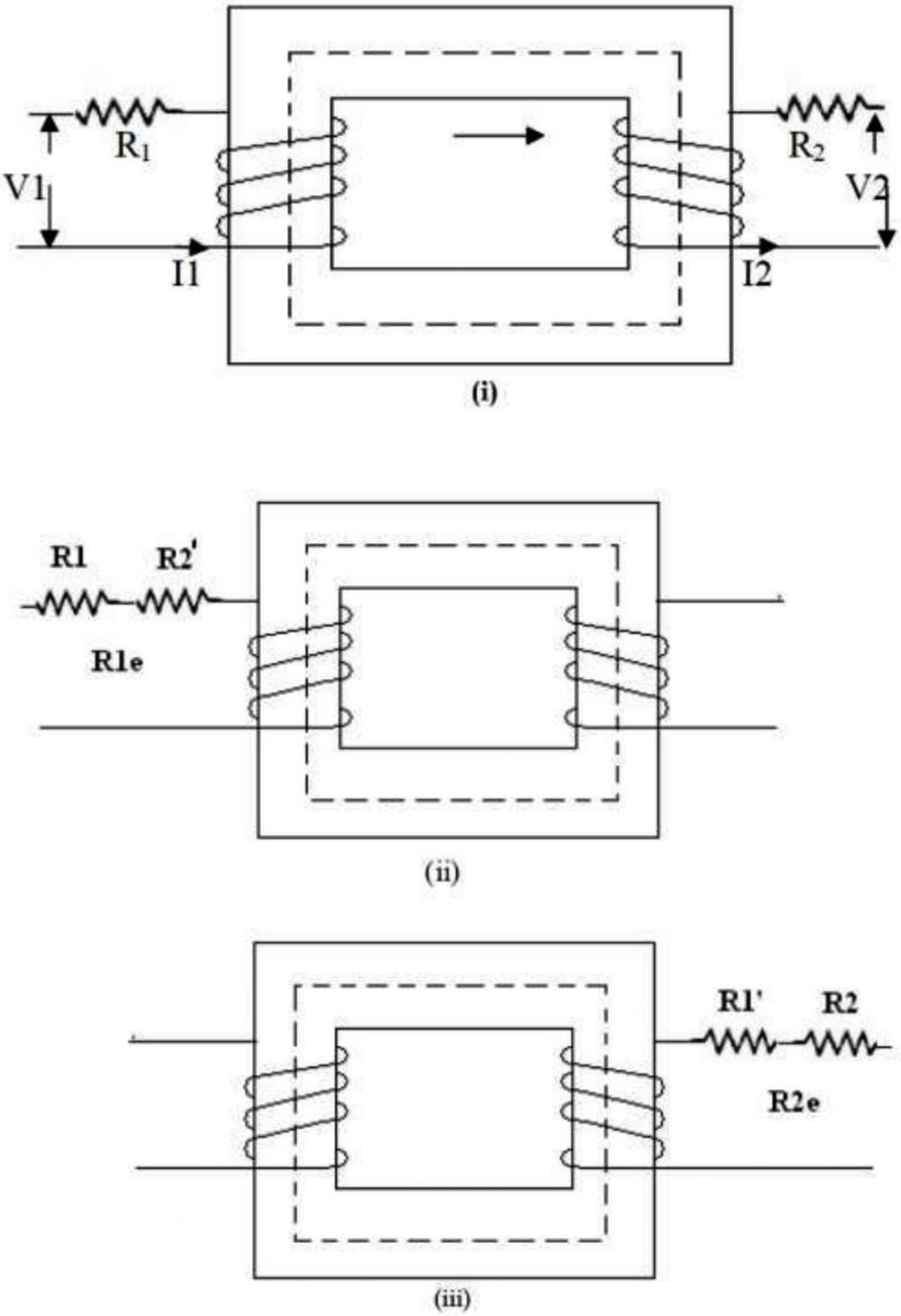
$$\begin{aligned}\text{Total copper loss} &= I_1^2R_1 + I_2^2R_2 \\ &= I_1^2[R_1 + \frac{I2^2}{I1^2}] \\ &= I_1^2[R_1 + \frac{1}{K}R_2]\end{aligned}$$

$\frac{R2}{K^2}$ is the resistance value of $R_2$ shifted to primary side and denoted as $R_2'$.
$R_2'$ is the equivalent resistance of secondary referred to primary

$$\boxed{R_2' = \frac{R2}{K^2}}$$

Equivalent resistance of transformer referred to primary fig (ii)

$$\boxed{R_{1e} = R_1 + R_2' = R_1 + \frac{R2}{K^2}}$$



(i)

(ii)

(iii)

**Fig:2.8**

Similarly it is possible to refer the equivalent resistance to secondary winding.

Total copper loss = $I_1^2R_1 + I_2^2R_2 = I_2^2\left[\frac{I1^2}{I2^2} R_1 + R_2\right]$

$$= I_2^2[K^2R_1 + R_2]$$

$K^2R1$ is primary resistance referred to secondary denoted as R1'.

$R_1' = K^2R_1$

Equivalent resistance of transformer referred to secondary, denoted as R2e

$$\boxed{R_{2e} = R_2 + R_1' = R_2 + K^2R_1}$$

Total copper loss = $I_2^2 R_{2e}$

*Note:*

*Note:*

i) When a resistance is to be transferred from the primary to secondary, it must be multiplied by $K^2$, it must be divided by $K^2$ while transferred from the secondary to primary.

High voltage side → low current side → high resistance side

Low voltage side → high current side → low resistance side

## Effect of Leakage Reactance

i) It has been assumed that all the flux linked with primary winding also links the secondary winding. But, in practice, it is impossible to realize this condition.

ii) However, primary current would produce flux $\phi$ which would not link the secondary winding. Similarly, current would produce some flux $\phi$ that would not link the primary winding.

iii) The flux $\phi_{L1}$ complete its magnetic circuit by passing through air rather than around the core, as shown in fig.2.9. This flux is known as primary leakage flux and is proportional to the primary ampere – turns alone because the secondary turns do not links the magnetic circuit of $\phi_{L1}$. It induces an e.m.f $e_{L1}$ in primary but not in secondary.

iv) The flux $\phi_{L2}$ complete its magnetic circuit by passing through air rather than around the core, as shown in fig. This flux is known as secondary leakage flux and is proportional to the secondary ampere – turns alone because the primary turns do not links the magnetic circuit of $\phi_{L2}$. It induces an e.m.f $e_{L2}$ in secondary but not in primary.

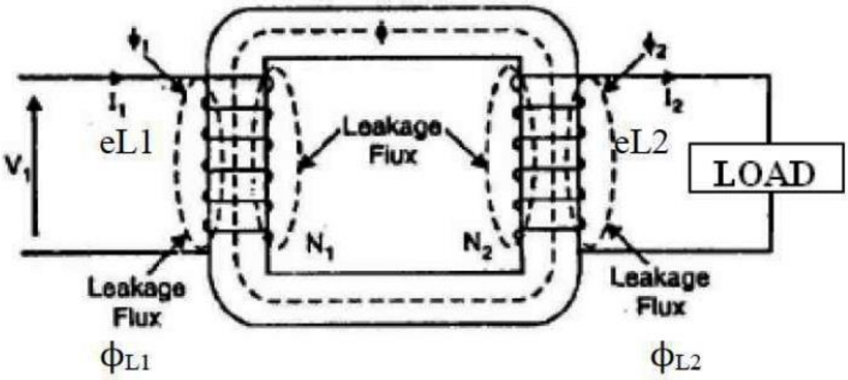


Fig: 2.9

$\phi_{L1}$ – primary leakage flux
$\phi_{L2}$ – secondary leakage flux
$e_{L1}$ – self induced e.m.f (primary)
$e_{L2}$ –self induced e.m.f (secondary)

**Equivalent Leakage Reactance**

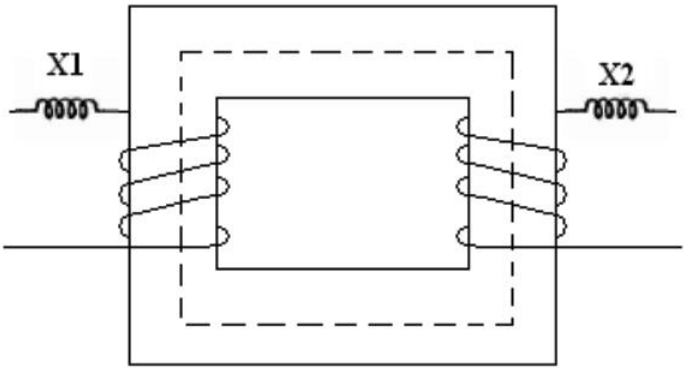


Fig: 2.10

Similarly to the resistance, the leakage reactance also can be transferred from primary to secondary. The relation through $K^2$ remains same for the transfer of reactance as it is studied earlier for the resistance

$X_1$ – leakage reactance of primary.

$X_2$ - leakage reactance of secondary.

Then the total leakage reactance referred to primary is $X_{1e}$ given by

$$X_{1e} = X_1 + X_2'$$

$$X_2' = \frac{X2}{K^2}$$

The total leakage reactance referred to secondary is $X_{2e}$ given by

$$X_{2e} = X_2 + X_1'$$
$$X_1' = K^2X_1$$

$$\boxed{\begin{array}{l} X_{1e} = X_1 + X_2' \\ X_{2e} = X_2 + X_1' \end{array}}$$

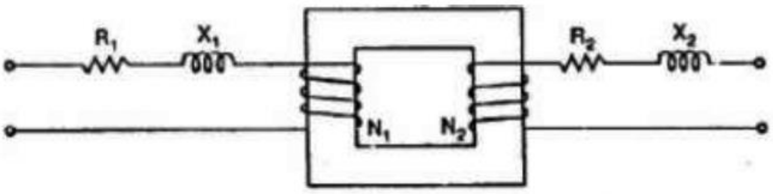
## Equivalent Impedance

The transformer winding has both resistance and reactance ($R_1$, $R_2$, $X_1$,$X_2$).Thus we can say that the total impedance of primary winding is$Z_1$ which is,

$Z_1 = R_1 + jX_1$ ohms

On secondary winding,

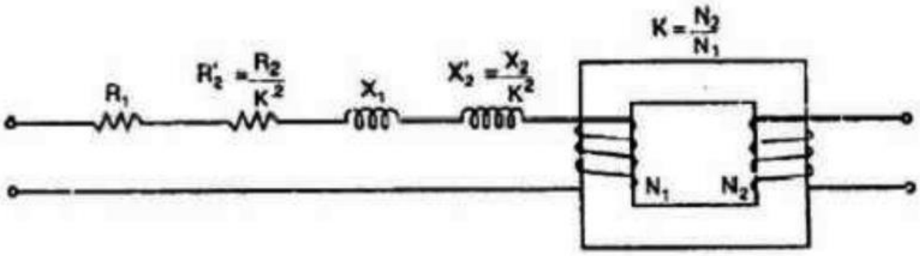
$Z_2 = R_2 + jX_2$ ohms



Individual magnitude of Z1 and Z2 are

$$Z_1 = \sqrt{R1^2 + X1^2}$$
$$Z_2 = \sqrt{R2^2 + X2^2}$$

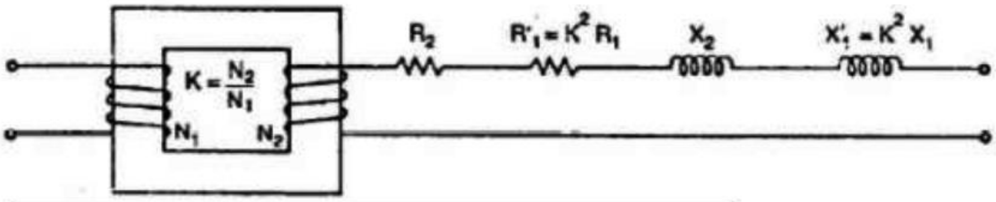
Similar to resistance and reactance, the impedance also can be referred to any one side,



$Z_{1e}$ = total equivalent impedance referred to primary

$$Z_{1e} = R_{1e} + jX_{1e} = Z_1 + Z_2' = Z_1 + \frac{Z2}{K^2}$$

$Z_{2e}$ = total equivalent impedance referred to secondary.



$$Z_{2e} = R_{2e} + jX_{2e} = Z_2 + Z_1' = Z_2 + K^2Z_1$$

The magnitudes of Z1e and Z2e

$$Z_1 = \sqrt{R1e^2 + X1e^2}$$
$$Z_2 = \sqrt{R2e^2 + X2e^2}$$

It can be noted that

$Z_{2e} = K^2Z_{1e}$ and $Z_{1e} = \frac{Z2e}{K^2}$

### 2.4.7 Complete Phasor Diagram of a Transformer *(for Inductive Load or Lagging pf)*

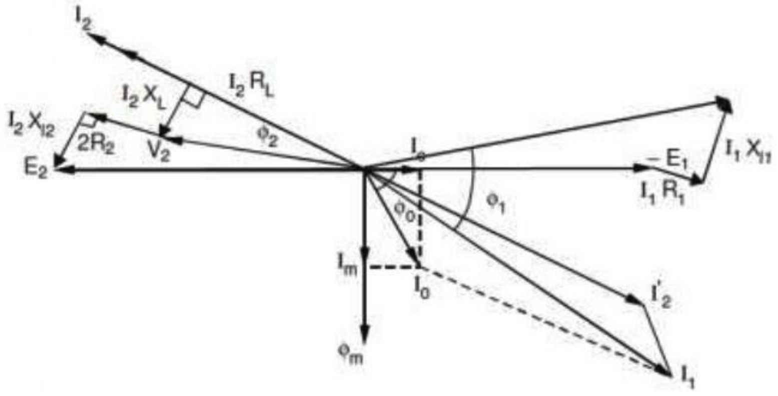
We now restrict ourselves to the more commonly occurring load i.e. inductive along with resistance, which has a lagging power factor.

For drawing this diagram, we must remember that

$$\bar{V}_2 = \bar{E}_2 - \bar{I}_2 (R_2 + j X_{l2})$$

and

$$\bar{V}_1 = -\bar{E}_1 + \bar{I}_2 (R_1 + j X_{l1})$$



## 2.5 Equivalent Circuit of Transformer

*No load equivalent circuit*

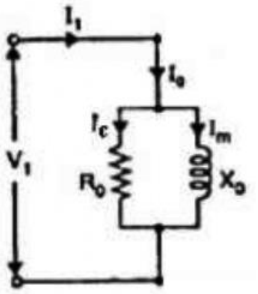


Fig:11

$I_m = I_o \sin \phi_o$ = magnetizing component

$I_c = I_o \cos \phi_o$ = Active component

$R_o = \frac{V1}{Ic}$, $X_o = \frac{V1}{Im}$

i) $I_m$ produces the flux and is assumed to flow through reactance $X_o$ called no load reactance while $I_c$ is active component representing core losses hence is assumed to flow through the resistance $R_0$

ii) Equivalent resistance is shown in fig.2.12.

iii) When the load is connected to the transformer then secondary current $I_2$ flows causes voltage drop across $R_2$ and $X_2$. Due to $I_2$, primary draws an additional current.

$$I_2' = \frac{I2}{K}$$

$I_1$ is the phasor addition of $I_o$ and $I_2$'. This $I_1$ causes the voltage drop across primary resistance $R_1$ and reactance $X_1$.

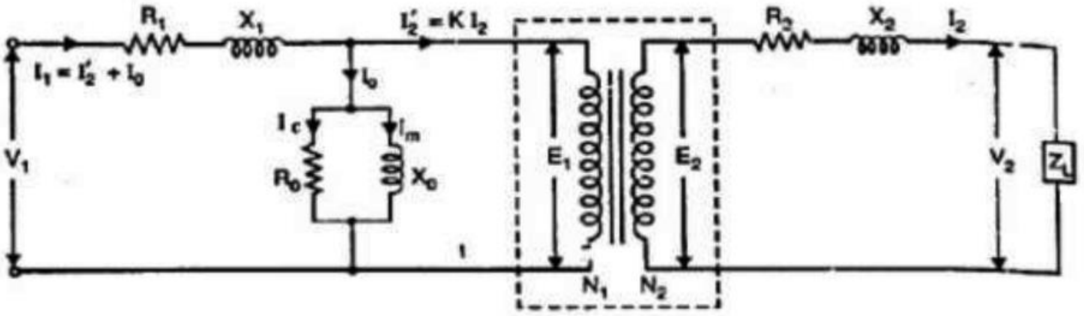


Fig: 2.12

To simplified the circuit the winding is not taken in equivalent circuit while transfer to one side.

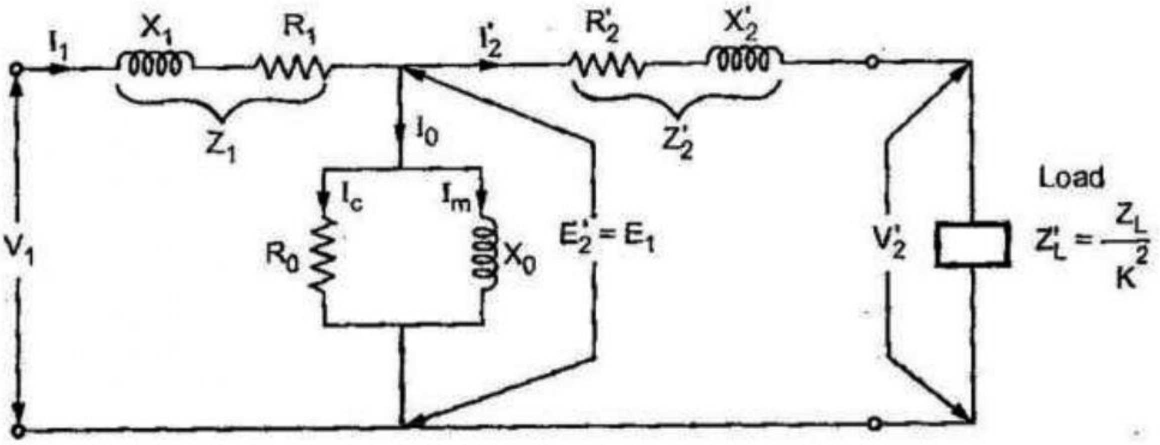


Fig: 2.13

## 2.5.1 Exact equivalent circuit referred to primary

Transferring secondary parameter to primary -

$$\boxed{R_2' = \frac{R2}{K^2},\ X_2' = \frac{X2}{K^2},\ Z_2' = \frac{Z2}{K^2},\ E_2' = \frac{E2}{K'},\ I_2' = KI_2,\ K = \frac{N2}{N1}}$$

High voltage winding ➡ low current ➡ high impedance

Low voltage winding ➡ high current ➡ low impedance

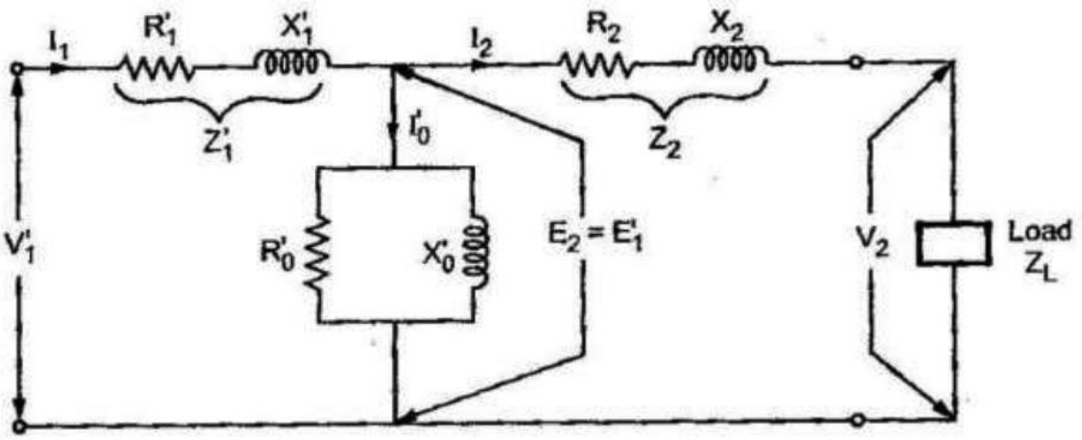


Fig: 2.14

## 2.5.2 Exact equivalent circuit referred to secondary

$$\boxed{\begin{array}{l} R_1' = R_1K^2,\ X_1' = K^2X_1\ ,\ E_1' = KE_1 \\[1em] Z_1' = K^2Z1\ ,\ I_1' = \frac{I1}{K'},\ I_o = \frac{Io}{K} \end{array}}$$

Now as long as no load branch i.e. exciting branch is in between $Z_1$ and $Z_2$', the impedances cannot be combined. So further simplification of the circuit can be done. Such circuit is called approximate equivalent circuit.

## 2.5.3 Approximate Equivalent Circuit

i) To get approximate equivalent circuit, shift the no load branch containing $R_o$ and $X_o$ to the left of $R_1$ and $X_1$.

ii) By doing this we are creating an error that the drop across $R_1$ and $X_1$ to $I_o$ is neglected due to this circuit because simpler.

iii) This equivalent circuit is called approximate equivalent circuit Fig: 2.15 & Fig: 2.16.

In this circuit new $R_1$ and $R_2$' can be combined to get equivalent circuit referred to primary $R_{1e}$,similarly

$X_1$ and $X_2$' can be combined to get $X_{1e}$.

$$R_{1e} = R_1 + R_2' = R_1 + \frac{R2}{K^2}$$
$$X_{1e} = X_1 + X_2' = X_1 + \frac{X2}{K^2}$$
$$Z_{1e} = R_{1e} + jX_1E, \qquad R_o = \frac{V1}{Ic}, \quad \text{and } X_o = \frac{V1}{Im}$$
$$I_c = I_o \cos\phi_o, \quad \text{and } I_m = I_o \sin\phi_o$$

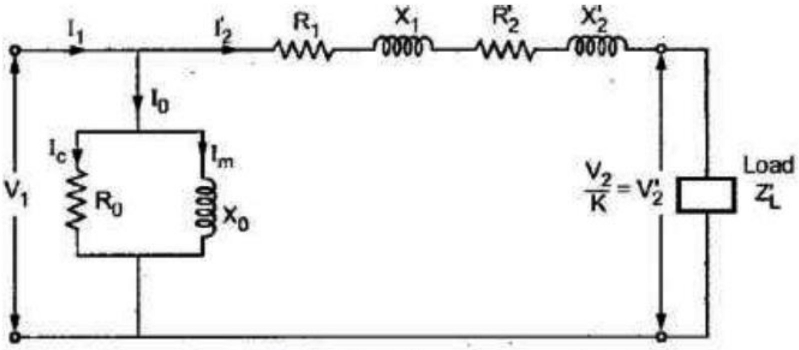


Fig:2.15Approximate equivalent circuit referred to primary

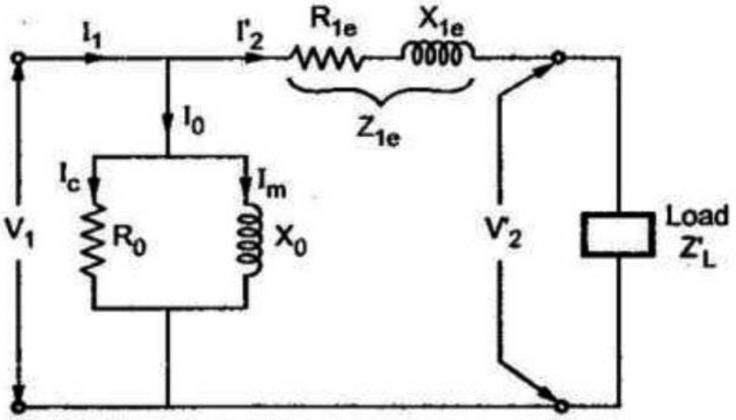
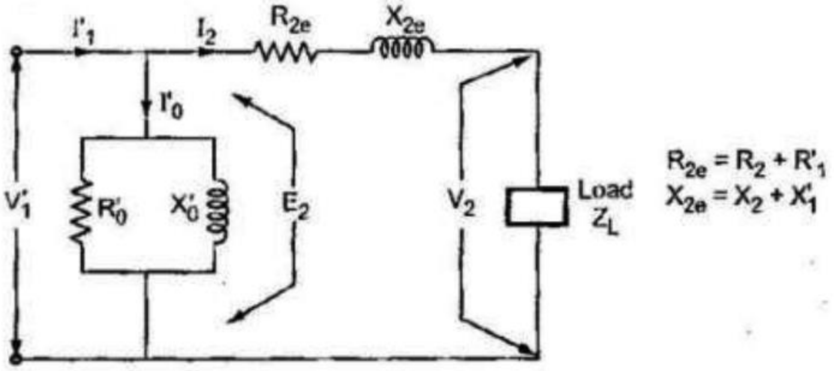


Fig:2.16Simplified equivalent circuit

## 2.6 Approximate Voltage Drop in a Transformer



$$\vec{E_2} = \overrightarrow{I_2R_{2e} + I_2X_{2e}} + \vec{V_2} = \vec{V_2} + \vec{I_2}\,(R_{2e} + jX_{2e}), \quad \vec{E_2} = \vec{V_2} + \overrightarrow{I_2Z_{2e}}$$

Fig. 2.17

Primary parameter is referred to secondary there are no voltage drop in primary. When there is no load, $I_2 = 0$ and we get no load terminal voltage drop in

$V_{2o} = E_2$ = no load terminal voltage
$V_2$ = terminal voltage on load

### 2.6.1 For Lagging P.F.

i) The current $I_2$ lags $V_2$ by angle $\phi_2$

ii) Take $V_2$ as reference

iii) $I_2R_2e$ is in phase with $I_2$ while $I_2 X_{2e}$ leads $I_2$ by 90˚

iv) Draw the circle with O as centre and OC as radius cutting extended OA at M. as OA

= $V_2$ and now OM = $E_2$.

v) The total voltage drop is AM = $I_2Z_{2e}$.

vi) The angle α is practically very small and in practice M&N are very close to each other. Due to this the approximate voltage drop is equal to AN instead of AM

AN – approximate voltage drop

To find AN by adding AD& DN

AD = AB cosϕ = $I_2R_{2e}$ cosϕ

DN = BL sinϕ = $I_2X_{2e}$ sinϕ

AN = AD + DN = $I_2R_{2e}$ cos$\phi_2$ + $I_2X_{2e}$ sin$\phi_2$

Assuming: $\phi_2 = \phi_1 = \phi$

Approximate voltage drop = $I_2R_{2e}$ cosϕ+$I_2X_{2e}$ sinϕ (referred to secondary)

Similarly: Approximate voltage drop = $I_1R_{1e}$ cosϕ+$I_1X_{1e}$ sinϕ (referred to primary)

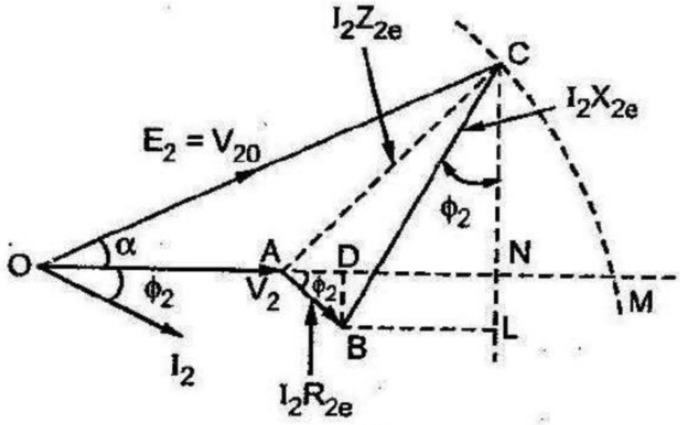


Fig:2.18

## 2.6.2 For Leading P.F Loading

$I_2$ leads $V_2$ by angle $\phi_2$

Approximate voltage drop = $I_2R_{2e}$ cosϕ - $I_2X_{2e}$ sinϕ (referred to secondary)

Similarly: Approximate voltage drop = $I_1R_{1e}$ cosϕ - $I_1X_{1e}$ sinϕ (referred to primary)

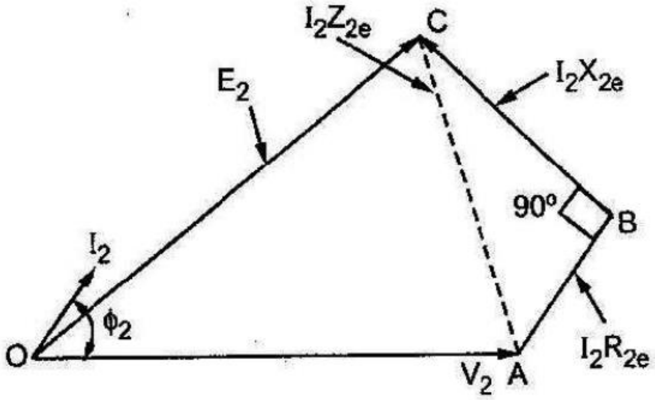
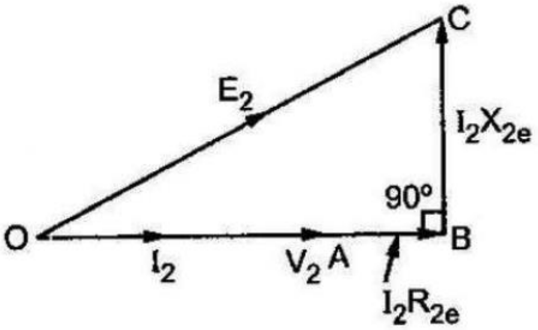


Fig: 2.19

### 2.6.3 For Unity P.F. Loading

Approximate voltage drop = $I_2R_{2e}$ (referred to secondary)

Similarly: Approximate voltage drop = $I_1R_{1e}$ (referred to primary)



Cosϕ = 1
Sinϕ = 0

Fig: 2.20

Approximate voltage drop = $E_2 - V_2$

$$= I_2R_{2e}\cos\phi \pm I_2X_{2e}\sin\phi \text{ (referred to secondary)}$$

$$= I_1R_{1e}\cos\phi \pm I_1X_{1e}\sin\phi \text{ (referred to primary)}$$

## 2.7 Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core or Iron losses

2. Copper losses

These losses appear in the form of heat and produce (i) an increase in Temperature and (ii) a drop in efficiency.

### *2.7.1 Core or Iron losses (Pi)*

These consist of hysteresis and eddy current losses and occur in the transformer core due to the alternating flux. These can be determined by open-circuit test.

Hysteresis loss = $k_h f B_m^{1.6}$ watts /m$^3$

$K_h$ – hysteresis constant depend on material

f - Frequency

$B_m$ – maximum flux density

Eddy current loss = $K_e f^2 B_m^2 t^2$ watts /m$^3$

Ke – eddy current constant

t - Thickness of the core

Both hysteresis and eddy current losses depend upon

(i) Maximum flux density $B_m$ in the core

(ii) Supply frequency f. Since transformers are connected to constant-frequency, constant voltage supply, both f and Bm are constant. Hence, core or iron losses are practically the same at all loads.

Iron or Core losses, Pi = Hysteresis loss + Eddy current loss = Constant losses (Pi)

The hysteresis loss can be minimized by using steel of high silicon content .Whereas eddy current loss can be reduced by using core of thin laminations.

### *Copper losses ($P_{cu}$)*

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test. The copper loss depends on the magnitude of the current flowing through the windings.

$$\text{Total copper loss} = I_1^2R_1 + I_2^2R_2 = I_1^2(R_1 + R_2') = I_2^2\,(R_2 + R_1')$$

$$\boxed{\text{Total loss} = \text{iron loss} + \text{copper loss} = Pi + Pcu}$$

## 2.8 Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.

Power output = power input – Total losses

Power input = power output + Total losses

= power output + Pi + $P_{cu}$

$$\text{Efficiency} = \frac{power\ output}{power\ input}$$

$$\text{Efficiency} = \frac{power\ output}{power\ input + Pi + Pcu}$$

Power output = $V_2I_2\cos\phi$, $\cos\phi$ = load power factor

Transformer supplies full load of current $I_2$ and with terminal voltage $V_2$

Pcu = copper losses on full load = $I_2^2 R_{2e}$

$$\text{Efficiency} = \frac{\text{V2I2}\cos\phi}{\text{V2I2}\cos\phi + Pi + I2^2R2e}$$

$V_2I_2$ = VA rating of a transformer

$$\text{Efficiency} = \frac{(\text{VA rating})\text{X}\cos\phi}{(\text{VA rating})\text{X}\cos\phi + Pi + I2^2R2e}$$

$$\boxed{\%\ \text{Efficiency} = \frac{(\text{VA rating})\text{X}\cos\phi}{(\text{VA rating})\text{X}\cos\phi + Pi + I2^2R2e}\ \text{X } 100}$$

This is full load efficiency and $I_2$ = full load current.

We can now find the full-load efficiency of the transformer at any p.f. without actually loading the transformer.

$$\text{Full load Efficiency} = \frac{(\text{Full load VA rating})\text{X} \cos\phi}{(\text{Full load VA rating})\text{X} \cos\phi + Pi + I2^2R2e}$$

Also for any load equal to n x full-load,

Corresponding total losses = $P_i + n^2P_{Cu}$

$$n = \text{fractional by which load is less than full load} = \frac{actual\ load}{full\ load}$$

$$n = \frac{half\ load}{fullload} = \frac{(\frac{1}{2})}{1} = 0.5$$

$$\boxed{\text{Corresponding (n) \% Efficiency} = \frac{n(\text{VA rating})\text{X} \cos\phi}{n(\text{VA rating})\text{X} \cos\phi + Pi + n^2Pcu} \text{ X } 100}$$

## 2.8.1 Condition for Maximum Efficiency

Voltage and frequency supply to the transformer is constant the efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it loaded further the efficiency start decreases as shown in fig. 2.21.

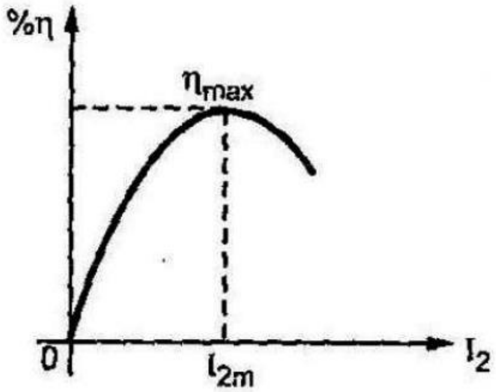


Fig: 2.21

The load current at which the efficiency attains maximum value is denoted as $I_{2m}$ and maximum efficiency is denoted as ηmax, now we find -

(a) condition for maximum efficiency

(b) load current at which $\eta$max occurs

(c) KVA supplied at maximum efficiency

Considering primary side,

Load output = $V_1I_1 \cos\phi 1$

Copper loss = $I_1{}^2 R_{1e}$ or $I2^2 R2e$

Iron loss = hysteresis + eddy current loss = $P_i$

$$\text{Efficiency} = \frac{V1I1\cos\phi 1 - \text{losses}}{V1I1\cos\phi 1} = \frac{V1I1\cos\phi 1 - I1^2R1e + Pi}{V1I1\cos\phi 1}$$

$$= 1 - \frac{I1R1e}{V1I21\cos\phi 1} = \frac{Pi}{V1I1\cos\phi 1}$$

Differentiating both sides with respect to I2, we get

$$\frac{d\eta}{dI2} = 0 - \frac{R1e}{V1\cos\phi 1} = \frac{Pi}{V1I1^2\cos\phi 1}$$

For **η** to be maximum, $\frac{d\eta}{dI2} = 0$. Hence, the above equation becomes

$$\frac{R1e}{V1\cos\phi 1} = \frac{Pi}{V1I1^2\cos\phi 1} \text{ or } Pi = I_1{}^2R_{1e}$$

$P_{cu}$ loss = $P_i$ iron loss

The output current which will make $P_{cu}$ loss equal to the iron loss. By proper design, it is possible to make the maximum efficiency occur at any desired load.

### 2.8.2 Load current I2m at maximum efficiency

For $\eta_{max}$ $I_2^2R_{2e} = P_i$ but $I_2 = I_{2m}$

$$I^2_{2m}\,R_{2e} = Pi \qquad I_{2m} = \sqrt{\frac{Pi}{R2e}}$$

This is the load current at $\eta_{max}$
$(I_2)F.L$ = full load current

$$\frac{I2m}{(I2)F.L} = \frac{1}{(I2)FL}\sqrt{\frac{Pi}{R2e}}$$

$$\frac{I2m}{(I2)F.L} = \sqrt{\frac{Pi}{[(I2)F.L]^2R2e}} = \sqrt{\frac{Pi}{[Pcu]F.L}}$$

$$\boxed{I_{2m} = (I_2)\text{ F.L.}\sqrt{\frac{Pi}{[Pcu]F.L}}}$$

This is the load current at $\eta_{max}$ in terms of full load current

### 2.8.3 KVA Supplied at Maximum Efficiency

For constant $V_2$ the KVA supplied is the function of load current.

$$\text{KVA at } \eta_{max} = I_{2m} V_2 = V_2(I_2)\text{F.L.} \times \sqrt{\frac{Pi}{[Pcu]\text{F.L}}}$$

$$\boxed{\text{KVA at } \eta_{max} = (\text{KVA rating}) \times \sqrt{\frac{Pi}{[Pcu]\text{F.L}}}}$$

Substituting condition for $\eta_{max}$ in the expression of efficiency, we can write expression for $\eta_{max}$ as ,

as Pcu = Pi

$$\% \, \eta_{max} = \frac{V2\ I2m\ \cos\phi}{V2\ I2m \cos\phi + 2Pi} \times 100$$

$$\% \, \eta_{max} = \frac{KVA\ for \eta \max \cos\varphi}{KVA\ for \eta \max \cos\varphi + 2Pi}$$

## 12.8.4 All Day Efficiency (Energy Efficiency)

In electrical power system, we are interested to find out the all-day efficiency of any transformer because the load at transformer is varying in the different time duration of the day. So all day efficiency is defined as the ratio of total energy output of transformer to the total energy input in 24 hours.

$$\text{All day efficiency} = \frac{\text{kWh output during a day}}{\text{kWh input during the day}}$$

Here, kWh is kilowatt hour.

## 12.9 Testing of Transformer

The testing of transformer means to determine efficiency and regulation of a transformer at any load and at any power factor condition.

There are two methods

i) Direct loading test

ii) Indirect loading test

a. ***Open circuit test***
b. ***Short circuit test***

## i) Load test on transformer

This method is also called as direct loading test on transformer because the load is directly connected to the transformer. We required various meters to measure the input and output reading while change the load from zero to full load. Fig. 2.22 shows the connection of transformer for direct load test. The primary is connected through the variac to change the input voltage as we required. Connect the meters as shown in the figure below.

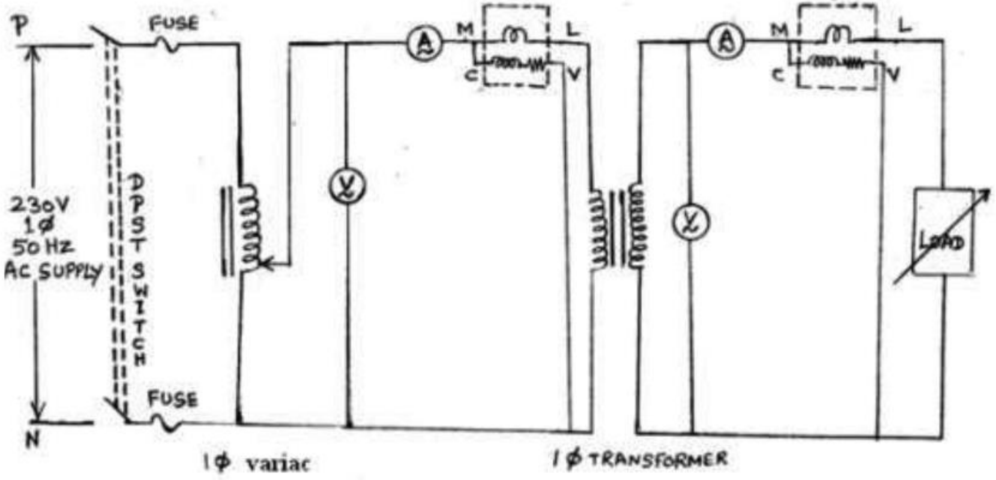


Fig: 2.22

The load is varied from no load to full load in desired steps. All the time, keep primary voltage $V_1$ constant at its rated value with help of variac and tabulated the reading. The first reading is to be noted on no load for which $I_2 = 0$ A and $W_2 = 0$W.

## Calculation

From the observed reading

$W_1$ = input power to the transformer

$W_2$ = output power delivered to the load

$$\boxed{\% \eta = \frac{W2}{W1} \times 100}$$

The first reading is no load so $V2 = E_2$

The regulation can be obtained as

$$\boxed{\% R = \frac{E2-V2}{V2} \times 100}$$

The graph of % $\eta$ and % R on each load against load current $I_L$ is plotted as shown in fig. 2.23.

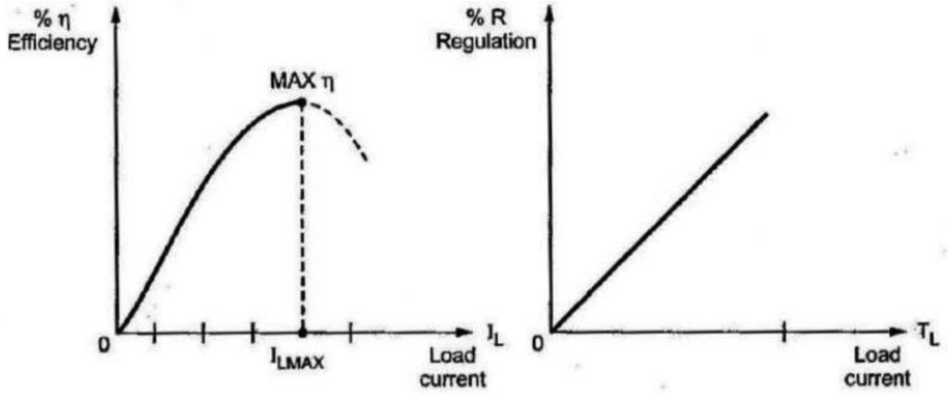


Fig: 2.23

***Advantages:***

1) This test enables us to determine the efficiency of the transformer accurately at any load.

2) The results are accurate as load is directly used.

***Disadvantages:***

1) There are large power losses during the test.

2) Load not avail in lab while test conduct for large transformer.

## ii) a. Open-Circuit or No-Load Test

This test is conducted to determine the iron losses (or core losses) and parameters $R_0$ and $X_0$ of the transformer. In this test, the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open circuited. The applied primary voltage $V_1$ is measured by the voltmeter, the no load current $I_0$ by ammeter and no-load input power $W_0$ by wattmeter as shown in Fig.2.24.a. As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core. Hence wattmeter will record the iron losses and small copper loss in the primary. Since no-load current $I_0$ is very small (usually 2-10 % of rated current). Cu losses in the primary under no-load condition are negligible as compared with iron losses. Hence, wattmeter reading practically gives the iron losses in the transformer. It is reminded that iron losses are the same at all loads.

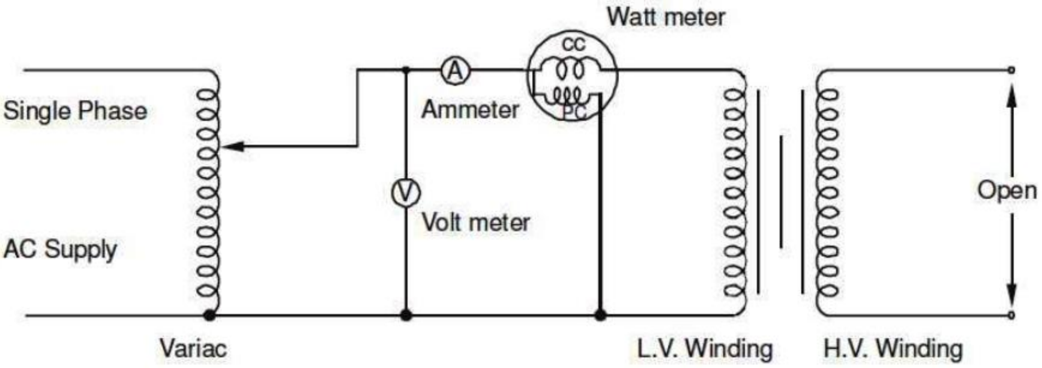


Fig: 2.24.a

Iron losses, $P_i$ = Wattmeter reading = $W_0$

No load current = Ammeter reading = $I_0$

Applied voltage = Voltmeter reading = $V_1$

Input power, $W_0 = V_1 I_0 \cos \phi_0$

No - load p.f., $\cos\phi = \frac{Wo}{Vo\,Io}$ = no load power factor

$I_m = I_o \sin \phi_o$ = magnetizing component

$I_c = I_o \cos \phi_o$ = Active component

$$R_o = \frac{Vo}{Ic}\ \Omega, \quad X_o = \frac{Vo}{Im}\ \Omega$$

Under no load conditions the PF is very low (near to 0) in lagging region. By using the above data we can draw the equivalent parameter shown in Figure 2.24.b.

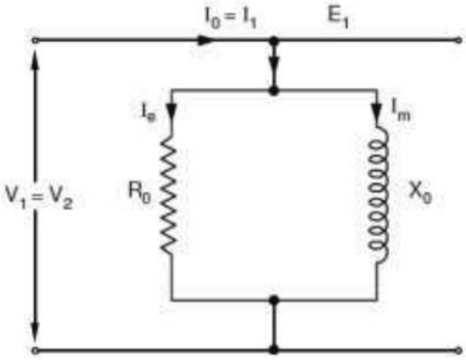


Fig: 2.24.b

Thus open-circuit test enables us to determine iron losses and parameters $R_0$ and $X_0$ of the transformer.

### ii) b. Short-Circuit or Impedance Test

This test is conducted to determine $R_{1e}$ (or $R_{2e}$), $X_{1e}$ (or $X_{2e}$) and full-load copper losses of the transformer. In this test, the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary as shown in Fig.2.25. The low input voltage is gradually raised till at voltage $V_{SC}$, full-load current $I_1$ flows in the primary. Then $I_2$ in the secondary also has full-load value since $I_1/I_2 = N_2/N_1$. Under such conditions, the copper loss in the windings is the same as that on full load. There is no output from the transformer under short-circuit conditions. Therefore, input power is all loss and this loss is almost entirely copper loss. It is because iron loss in the core is negligibly small since the voltage $V_{SC}$ is very small. Hence, the wattmeter will practically register the full load copper losses in the transformer windings.

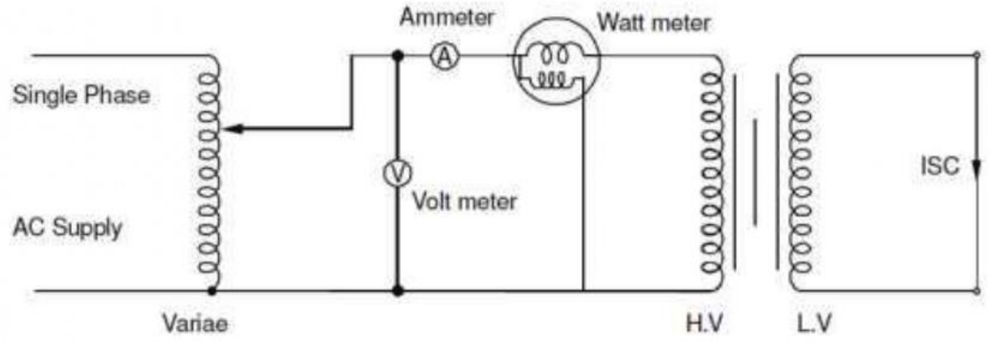


Fig: 2.25.a

Full load Cu loss, PC = Wattmeter reading = $W_{sc}$
Applied voltage = Voltmeter reading = $V_{SC}$
F.L. primary current = Ammeter reading = $I_1$

$P_{cu} = I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_{1e}$, $R_{1e} = \frac{Pcu}{I1^2}$

Where $R_{1e}$ is the total resistance of transformer referred to primary.

Total impedance referred to primary, $Z1e = \sqrt{Z1e^2 - R1e^2}$ ,

short – circuit P.F, $\cos\Phi = \frac{Pcu}{Vsc\ I1}$ Thus short-circuit lest gives full-load Cu loss, R1e and X1e.

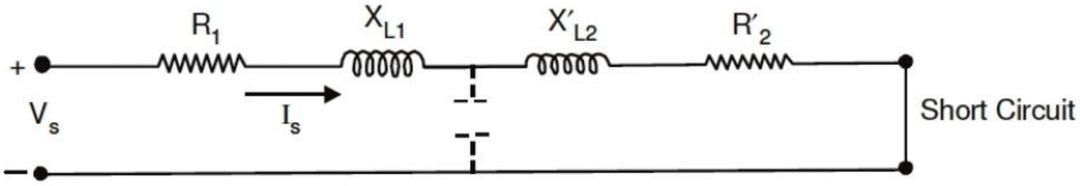


Fig: 2.25.b

From fig: 2.25.b we can calculate,

$$\text{equivalent resistance } R_{eq} = \frac{W_s}{I_s^2} = R_1 + R_2'$$

$$\text{and equivalent impedance } Z_{eq} = \frac{V_s}{I_s}$$

So we calculate equivalent reactance

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = X_{L1} + X_{L2}'$$

These $R_{eq}$ and $X_{eq}$ are equivalent resistance and reactance of both windings referred in HV side. These are know as equivalent circuit resistance and reactance.

## 2.8 Voltage Regulation of Transformer

Under no load conditions, the voltage at the secondary terminals is $E_2$ and

$$E_2 \approx V_1 \cdot \frac{N_2}{N_1}$$

(This approximation neglects the drop $R_1$ and $X_{l1}$ due to small no load current). As load is applied to the transformer, the load current or the secondary current increases. Correspondingly, the primary current

$I_1$ also increases. Due to these currents, there is a voltage drop in the primary and secondary leakage reactances, and as a consequence the voltage across the output terminals or the load terminals changes. In quantitative terms this change in terminal voltage is called Voltage Regulation.

Voltage regulation of a transformer is defined as the drop in the magnitude of load voltage (or secondary terminal voltage) when load current changes from zero to full load value. This is expressed as a fraction of secondary rated voltage.

$$\text{Regulation} = \frac{\text{Secondary terminal voltage at no load} - \text{Secondary terminal voltage at any load}}{\text{Secondary rated voltage}}$$

The secondary rated voltage of a transformer is equal to the secondary terminal voltage at no load (i.e. $E_2$), this is as per IS.

Voltage regulation is generally expressed as a percentage.

$$\text{Percent voltage regulation (\% VR)} = \frac{E_2 - V_2}{E_2} \times 100.$$

Note that $E_2$, $V_2$ are magnitudes, and not phasor or complex quantities. Also note that voltage regulation depends not only on load current, but also on its power factor. Using approximate equivalent circuit referred to primary or secondary, we can obtain the voltage regulation. From approximate equivalent circuit referred to the secondary side and phasor diagram for the circuit.

$$E_2 = V_2 + I_2\, r_{eq} \cos \phi_2 \pm I_2\, x_{eq} \sin \phi_2$$

where $r_{eq} = r_2 + r_1^1$ (referred to secondary) $x_e = x_2 + x_1^1$ (+ sign applies lagging power factor load and – sign applies to leading pf load).

So $$\frac{E_2 - V_2}{E_2} = \frac{I_2\, r_{eq} \cos \phi_2 \pm I_2\, x_{eq} \sin \phi_2}{E_2}$$

$$\frac{E_2 - V_2}{E_2} = \frac{I_2\, r_{eq}}{E_2} \cos \phi_2 \pm \frac{I_2\, x_{eq}}{E_2} \sin \phi_2$$

% Voltage regulation = (% resistive drop) cos $\phi_2$ ± (% reactive drop) sin $\phi_2$.

Ideally voltage regulation should be zero.

## 2.9 Auto-transformers

The transformers we have considered so far are two-winding transformers in which the electrical circuit connected to the primary is electrically isolated from that connected to the secondary. An auto-transformer does not provide such isolation, but has economy of cost combined with increased efficiency. Fig.2.26 illustrates the auto-transformer which consists of a coil of $N_A$ turns between terminals 1 and 2, with a third terminal 3 provided after $N_B$ turns. If we neglect coil resistances and leakage fluxes, the flux linkages of the coil between 1 and 2 equals $N_A\, \phi m$ while the portion of coil between 3 and 2 has a flux linkage $N_B\, \phi m$. If the induced voltages are designated as $E_A$ and $E_B$, just as in a two winding transformer,

$$\frac{\bar{E}_A}{\bar{E}_B} = \frac{N_A}{N_B}$$

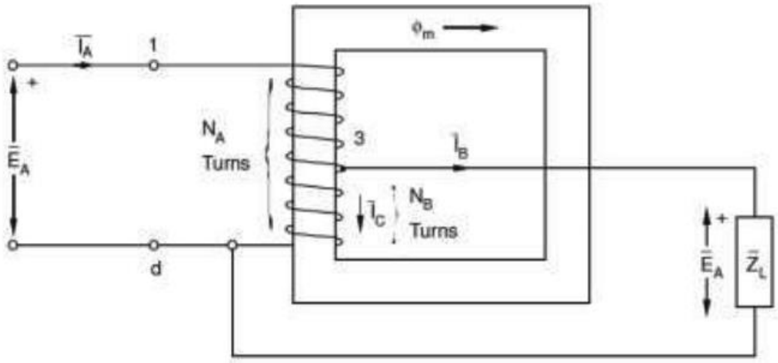


Fig: 2.26

Neglecting the magnetizing ampere-turns needed by the core for producing flux, as in an ideal transformer, the current $I_A$ flows through only $(N_A - N_B)$ turns. If the load current is $I_B$, as shown by

Kirchhoff's current law, the current $I_C$ flowing from terminal 3 to terminal 2 is $(I_A - I_B)$. This current flows through $N_B$ turns. So, the requirement of a net value of zero ampere-turns across the core demands that

$$(N_A - N_B)\,\bar{I}_A + (\bar{I}_A - \bar{I}_B)\,N_B = 0$$

or

$$N_A\,\bar{I}_A - N_B\,\bar{I}_B = 0$$

Hence, just as in a two-winding transformer,

$$\frac{\bar{I}_A}{\bar{I}_B} = \frac{N_B}{N_A}$$

Consequently, as far as voltage, current converting properties are concerned, the autotransformer of Figure: 26 behaves just like a two-winding transformer. However, in the autotransformer we don't need two separate coils, each designed to carry full load values of current.

## 2.10 Parallel Operation of Transformers

It is economical to install numbers of smaller rated transformers in parallel than installing a bigger rated electrical power transformers. This has mainly the following advantages,

To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfil the total demand. In this way we can run the system with maximum efficiency.

To maximize electrical power system availability: If numbers of transformers run in parallel, we can shut down any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.

To maximize power system reliability: if any one of the transformers run in parallel, is tripped due to fault of other parallel transformers is the system will share the load, hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.

To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfil the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

### 2.10.1 Conditions for Parallel Operation of Transformers

When two or more transformers run in parallel, they must satisfy the following conditions for satisfactory performance. These are the conditions for parallel operation of transformers.

- ***Same voltage ratio of transformer.***
- ***Same percentage impedance.***
- ***Same polarity.***
- ***Same phase sequence.***
- ***Same Voltage Ratio***

#### Same voltage ratio of transformer.

If two transformers of different voltage ratio are connected in parallel with same primary supply voltage, there will be a difference in secondary voltages. Now say the secondary of these transformers are connected to same bus, there will be a circulating current between secondaries and therefore between primaries also. As the internal impedance of transformer is small, a small voltage difference may cause sufficiently high circulating current causing unnecessary extra I2R loss.

#### Same Percentage Impedance

The current shared by two transformers running in parallel should be proportional to their MVA ratings. Again, current carried by these transformers are inversely proportional to their internal impedance. From these two statements it can be said that, impedance of transformers running in parallel are inversely proportional to their MVA ratings. In other words, percentage impedance or per unit values of impedance should be identical for all the transformers that run in parallel.

### Same Polarity

Polarity of all transformers that run in parallel, should be the same otherwise huge circulating current that flows in the transformer but no load will be fed from these transformers. Polarity of transformer means the instantaneous direction of induced emf in secondary. If the instantaneous directions of induced secondary emf in two transformers are opposite to each other when same input power is fed to both of the transformers, the transformers are said to be in opposite polarity. If the instantaneous directions of induced secondary e.m.f in two transformers are same when same input power is fed to the both of the transformers, the transformers are said to be in same polarity.

### Same Phase Sequence

The phase sequence or the order in which the phases reach their maximum positive voltage, must be identical for two parallel transformers. Otherwise, during the cycle, each pair of phases will be short circuited.

The above said conditions must be strictly followed for parallel operation of transformers but totally identical percentage impedance of two different transformers is difficult to achieve practically, that is why the transformers run in parallel may not have exactly same percentage impedance but the values would be as nearer as possible.

## 2.11 Why Transformer Rating in kVA?

An important factor in the design and operation of electrical machines is the relation between the life of the insulation and operating temperature of the machine. Therefore, temperature rise resulting from the losses is a determining factor in the rating of a machine. We know that copper loss in a transformer depends on current and iron loss depends on voltage. Therefore, the total loss in a transformer depends on the volt-ampere product only and not on the phase angle between voltage and current i.e., it is independent of load power factor. For this reason, the rating of a transformer is in kVA and not kW.

## Acknowledgement

The committee members gratefully acknowledge google, scribd, NPTEL, openoffice, sumantra pdf, scilab for myriad suggestions and help for preparing this lecture note. The committee members also wants to express their gratitude to the persons out there who thinks knowledge should be free and be accessible and sharable without any restrictions so that every single person on this planet has the same opportunity to explore, expand and become enlightened by the collective gifts of mankind.

However apart from this lecture note students/readers are strongly recommended to follow the below mentioned books in the references and above all confer with the faculty for thorough knowledge of this authoritative subject of electrical engineering.

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**Best of Luck to All the Students**

# LECTURE NOTES
# On
# Electrical Machine 1

## ***Name of the Department- Electrical Engineering***

**NAME OF THE SUBJECT- ELECTRICAL MACHINE1 (PART 2)**

**SEMESTER- 3RD**

**BRANCH- EE&EEE**

**PART2- MODULE3+ MODULE4**

# MODULE-III

DC MOTOR

## TOPICS

DC Motors: Principle of operation, Back E.M.F., Torque equation, characteristics and application of shunt, series and compound motors, Armature reaction and commutation, Starting of DC motor, Principle of operation of 3 point and 4 point starters, drum controller, Constant & Variable losses, calculation of efficiency, condition for maximum efficiency.

Speed control of DC Motors: Armature voltage and field flux control methods, Ward Leonard method.

Methods of Testing: direct, indirect and regenerative testing, brake test, Swinburne's test, Load test, Hopkinson's test, Field's test, Retardation test, separation of stray losses in a DC motor test.

[Topics are arranged as per above sequence]

## 3.1 Principle of Operation

DC motor operates on the principle that when a current carrying conductor is placed in a magnetic field, it experiences a mechanical force given by F = BIL newton. Where 'B' = flux density in wb, 'I' is the current and 'L' is the length of the conductor. The direction of force can be found by Fleming's left hand rule. From the point of construction, there is no difference between a DC generator and DC motor. Figure 3.1 shows a multipolar DC motor. Armature conductors are carrying current downwards under North Pole and upwards under South Pole. When the field coils are excited, with current carrying armature conductors, a force is experienced by each armature conductor whose direction can be found by Fleming's left hand rule. This is shown by arrows on top of the conductors. The collective force produces a driving torque which sets the armature into rotation. The function of a commutator in DC motor is to provide a continuous and unidirectional torque.

In DC generator the work done in overcoming the magnetic drag is converted into electrical energy. Conversion of energy from electrical form to mechanical form by a DC motor takes place by the work done in overcoming the opposition which is called the 'back emf'.

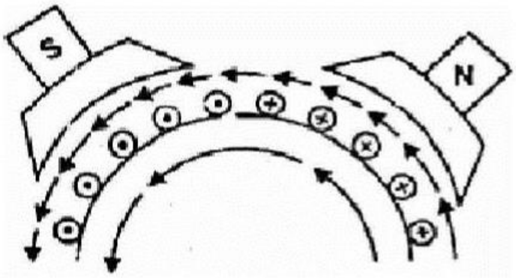


**Fig. 3.1 Generation of force in DC motor**

## 3.2 Back EMF

It is the dynamically induced emf in the armature conductors when the armature rotates following principle of DC motor. The direction of this induced emf can be determined using Fleming's right hand rule. This emf act in opposition to the supply voltage of the armature. It opposes the supply voltage that is why it is called back emf. The value of this induced emf is same as the value of the emf induced in dc generator. The work done in overcoming this opposition is converted into mechanical energy.

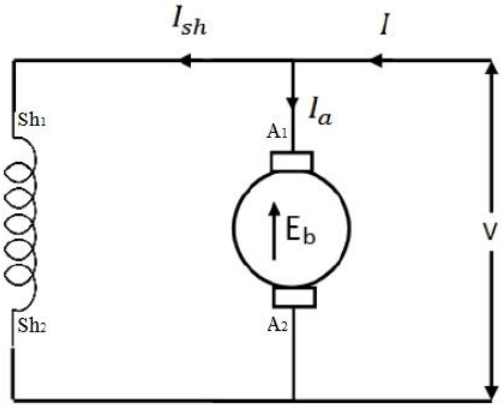


**Fig. 3.2 Schematic diagram of DC shunt motor**

Fig shown 3.2 a DC shunt motor the rotating armature generating the back emf $E_b$. The armature current can be written as

$$I_a = \frac{V_T - E_b}{r_a}$$

Where $r_a$ is armature resistance,

$$\boxed{E_b = \frac{P\phi ZN}{60A}}$$

Armature current is proportional to back emf. So back emf is a controlling factor of armature current.

## 3.3Torque Equation

Let $T_a$ = armature torque in N –m developed by the armature of a motor running at N.rps.

Therefore $P = T_a X \frac{2\pi N}{60}$ Watts

Electrical equivalent of mechanical power developed $P_m = E_b I_a$

$$P_m = E_b I_a = P = T_a \times \frac{2\pi N}{60}$$

$T_a = \frac{E_b I_a}{2\pi N} \times 60$ Also, on substituting for Eb i.e., $E_b = \frac{P\phi ZN}{60A}$

Therefore,

$$T_a = \frac{I_a \times 60}{2\pi N} \times \frac{P\phi ZN}{60A}$$

$$T_a = \frac{I_a}{2\pi} \times \frac{P\phi Z}{A}$$

$$\boxed{T_a = \frac{P\phi ZI_a}{2\pi A}} \text{N-m}$$

From the above equation for torque, it is seen that

(i) $T_a = k\phi I_a$

(ii) $T_a \propto I_a^{\,2}$ - For series motor (because $\phi \propto I_a$) before saturation. After saturation $T_a \propto I_a$

*(iii)* $T_a \propto I_a$ - For shunt motor. (because $\phi$ is constant in a shunt motor)

## 3.4 Characteristics of DC Motors

**There are three important characteristics-**

1. Armature torque vs armature current $T_a$ vs $l_a$ *(Electrical characteristics)*
2. Speed vs armature current characteristic N vs $I_a$
3. Speed vs torque N vs $T_a$ (Mechanical characteristics)

### 3.4.1 Characteristics of DC shunt motor

### *3.4.1.1 Armature torque vs armature current $T_a$ vs $I_a$ characteristics*

For a shunt motor flux can be assumed practically constant (at heavy loads, decreases, due to increased armature reaction)

$T_a = k\phi I_a$

$\phi$ is constant, $T_a \propto I_a$

Therefore electrical characteristic of a shunt motor is a straight line through origin shown by dotted line in figure 3.3. Armature reaction weakens the flux hence $T_a$ vs $I_a$ characteristic bends as shown by dark line in figure 3.3, Shunt motors should never be started on heavy loads, since it draws heavy current under such condition.

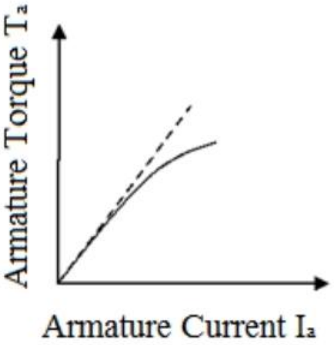


**Fig. 3.3 Torque Current Characteristic of DC shunt motor**

### 3.4.1.2 Speed vs armature current $N_a$ vs $I_a$ characteristics

$$N \propto \frac{E_b}{\phi}$$

$$N=\frac{V-I_a r_a}{k\phi}$$

$$N=\frac{V}{k\phi}-\frac{I_a r_a}{k\phi}$$

V is constant and in dc shunt motor $\phi$ is also constant. Thus with armature current speed drops and the speed current characteristics is drooping in nature is shown in figure 3.4.

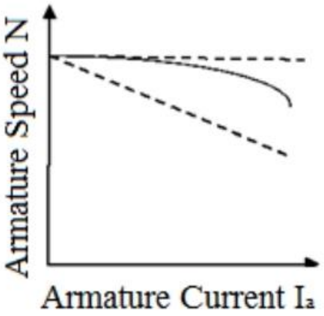


**Fig. 3.4 Speed vs armature current characteristics of DC shunt motor**

### 3.4.1.3 Speed vs armature torque $N_a$ vs $T_a$ characteristics

$$N=\frac{V}{k\phi}-\frac{I_a r_a}{k\phi}$$

$$I_a=\frac{T_a}{k\phi}$$

$$N=\frac{V}{k\phi}-\frac{r_a}{k\phi^2}T_a$$

Thus with increase with torque the speed of DC shunt motor decreases. The nature of the characteristics is drooping in nature shown in figure 3.5.

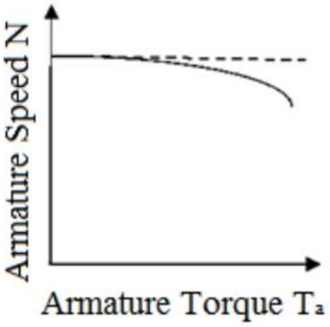


**Fig. 3.5 Speed vs armature torque characteristics of DC shunt motor**

### 3.4.2 Characteristics of DC series motor

#### *3.4.2.1 Armature torque vs armature current $T_a$ vs $I_a$ characteristics*

$T_a = k\phi I_a$

$T_a \propto I_a^2$ - For series motor (because $\phi \propto I_a$) before saturation

After saturation $\phi$ becomes constant thus $T_a \propto I_a$

At light loads, $I_a$ and hence $\phi$ is small. But as $I_a$ increases $T_a$ increases as the square of the current up-to saturation. After saturation $\phi$ becomes constant, the characteristic becomes a straight line as shown in Figure 3.6. Therefore a series motor develops a torque proportional to the square of the armature current. This characteristic is suited where huge starting torque is required for accelerating heavy masses.

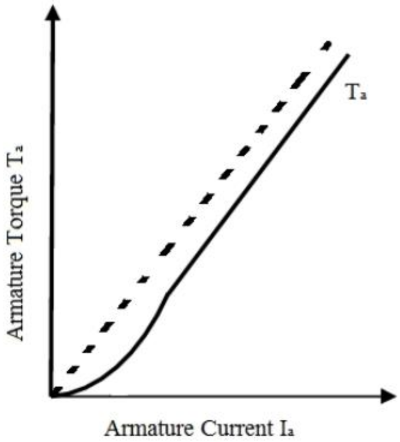


**Fig. 3.6 Torque Current Characteristic of DC series motor**

### *3.4.2.2 Speed vs armature current $N_a$vs $I_a$ characteristics*

$$N \propto \frac{E_b}{\phi}$$

$$N = \frac{V - I_a r_a}{k\phi}$$

$$N = \frac{V}{k\phi} - \frac{I_a r_a}{k\phi}$$

$$I_a \propto \phi$$

$$N = \frac{V}{kk_1 I_a} - \frac{k_1 r_a}{k}$$

$$N \propto \frac{1}{I_a}$$

If $I_a$ increases, speed decreases. This characteristic is shown in figure 3.7. Therefore the speed is inversely proportional to armature current $I_a$. When load is heavy $I_a$ ia heavy thus speed is low. When load is low $I_a$ is low thus speed becomes dangerously high. Hence series motor should never started without load on it.

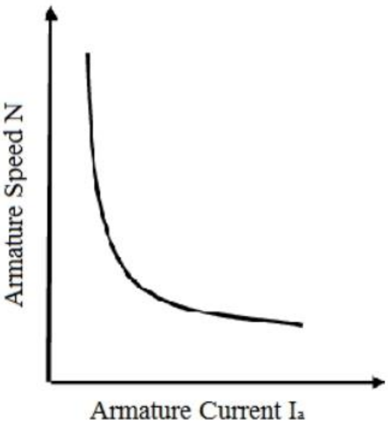


**Fig. 3.7 Speed vs armature current characteristics of DC series motor**

### *3.4.2.2 Speed vs armature torque $N_a$vs $T_a$ characteristics*

$N=\frac{V}{k\phi}-\frac{I_a r_a}{k\phi}$ and $\phi = k_1 I_a$

$\therefore N=\frac{V}{kk_1 I_a}-\frac{I_a r_a}{kk_1 I_a}$

$\Rightarrow N=\frac{V}{kk_1 I_a}-\frac{r_a}{kk_1}$

Now, $T_a = kI_a^{\,2} \;\therefore I_a = \sqrt{\frac{T_a}{k}}$

Substituting $I_a$

$N=\frac{V\sqrt{k}}{kk_1\sqrt{T_a}}-\frac{r_a}{kk_1}$

$\Rightarrow N=\frac{\text{Const.}}{\sqrt{T_a}}-\text{Const.}$

Thus Speed is inversely proportional to torque. The characteristics is shown in figure 3.8.

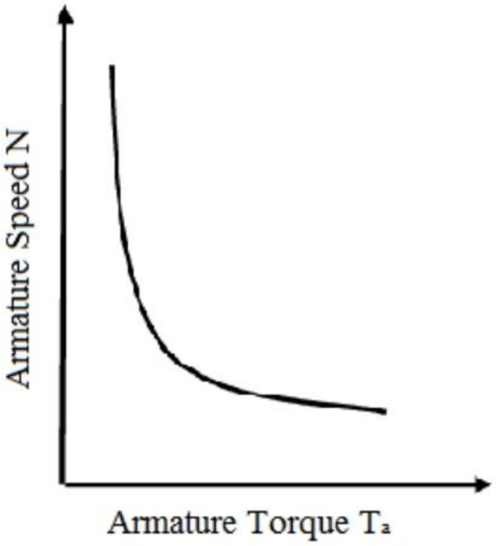


**Fig. 3.8 Speed vs armature torque characteristics of DC series motor**

### 3.4.3 Characteristics of DC compound motor

There are two different types of compound motors in common use, they are the cumulative compound motor and the differential compound motor. In the cumulative compound motor, the field produced by the series winding aids the field produced by the shunt winding. The speed of this motor falls more rapidly with increasing current than does that of the shunt motor because the field increases. In the differential compound motor, the flux from the series winding opposes the flux from the shunt winding. The field flux, therefore, decreases with increasing load current. Because the flux decreases, the speed may increases with increasing load. Depending on the ratio of the series-to-shunt field ampere-turns, the motor speed may increases very rapidly.

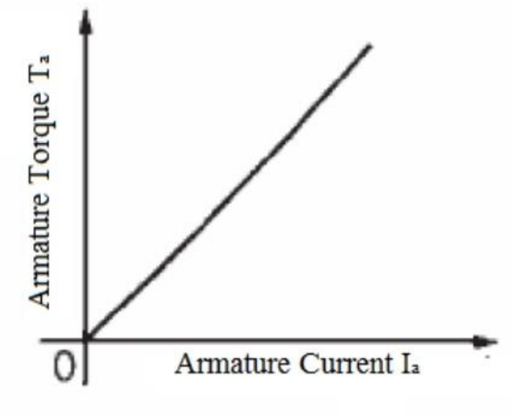


**Fig. 3.9 Armature torque vs armature current characteristics of DC compound motor**

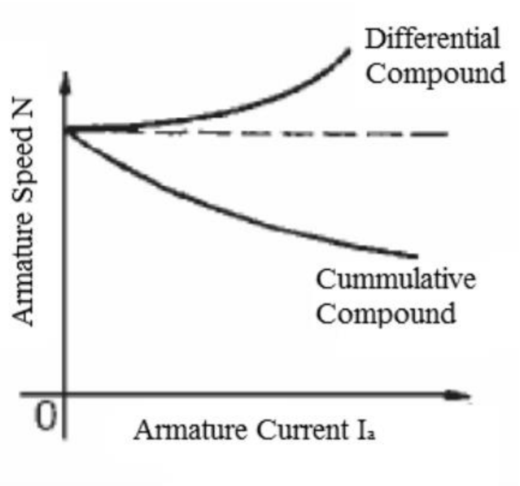


**Fig. 3.10 Speed vs armature current characteristics of DC compound motors**

### *3.4.3.1The torque-speed (c/s) of a cumulatively compound D.C motor*

In the cumulative compounded D.C. motor, there is a component of flux which is constant and another component which is proportional to its armature current (and thus to its load). Therefore, the cumulatively compounded motor has a higher starting torque than a shunt motor (whose flux is constant) but a lower starting torque than a series motor (whose entire flux is proportional to armature current). At light loads, the series field has a very small effect, so the motor behaves approximately as a shunt D.C. motor. As the load gets very large, the series flux becomes quite important and the torque-speed curve begins to look like a series motor's (c/s). A comparison of the torque-speed (c/s) of each of these type of machines is shown in figure 3.11.

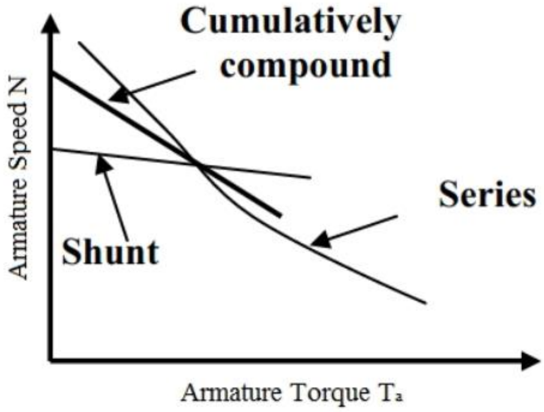


Fig. 3.11 Speed vs armature torque characteristics of DC motors

### *3.4.3.2 The torque-speed (c/s) of a differentially compound D.C motor*

In a differentially compound D.C. motor, the shunt magneto motive force and series magneto motive force subtract from each other. This means that as the load on the motor increases, $I_a$ increases and the flux in the motor decreases. But as the flux decreases, the speed of the motor increases. This speed increases causes anther increases in load, which further increases $I_a$, further decreasing the flux, and increasing the speed again. The result is that a differentially compounded motor is unstable and tends to run away. It is so bad that a differentially compounded motor is unsuitable for any application. The torque speed characteristics is shown in figure 3.12.

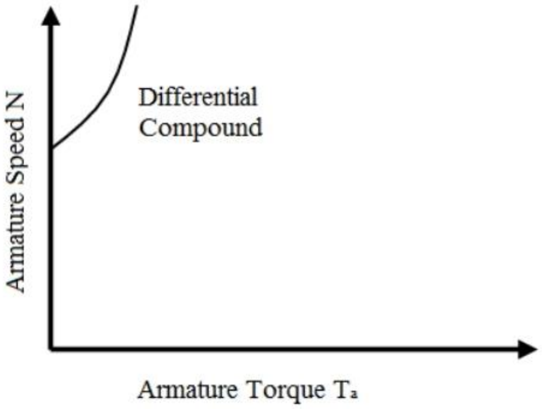


Fig. 3.12 Speed vs armature torque characteristics of DC differential compound motor

## 3.5 Application of DC motors

### 3.5.1 Application of DC shunt motor

The characteristics of a DC shunt motor give it a very good speed regulation, and it is classified as a constant speed motor, even though the speed does slightly decrease as load is increased. Shunt wound motors are used in industrial and automotive applications where precise control of speed and torque are required.

### 3.5.2 Application of DC series motor

For a given input current, the starting torque developed by a DC series motor is greater than that developed by a shunt motor. Hence series motors are used where huge starting torques are necessary. Ex. Cranes, hoists, electric traction etc. The DC series motor responds by decreasing its speed for the increased in load. The current drawn by the DC series motor for the given increase in load is lesser than DC shunt motor. The drop in speed with increased load is much more prominent in series motor than that in a shunt motor. Hence series motor is not suitable for applications requiring a constant speed.

### 3.5.3 Application of DC compound motor

Cumulative compound wound motors are virtually suitable for almost all applications like business machines, machine tools, agitators and mixers etc. Compound motors are used to drive loads such as shears, presses and reciprocating machines.

Differential compound motors are seldom used in practice (because of rising speed characteristics).

## 3.6 Armature Reaction

The action of magnetic field set up by armature current on the distribution of flux under main poles of a DC machine is called armature reaction.

When the armature of a DC machines carries current, the distributed armature winding produces its own mmf. The machine air gap is now acted upon by the resultant mmf distribution caused by the interaction of field ampere turns ($AT_f$) and armature ampere turns ($AT_a$). As a result the air gap flux density gets distorted.

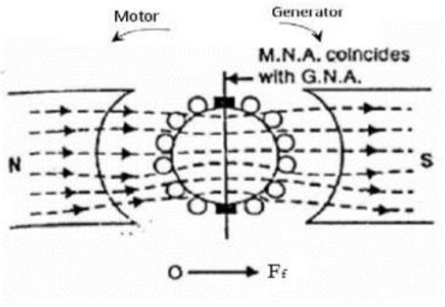


Fig. a

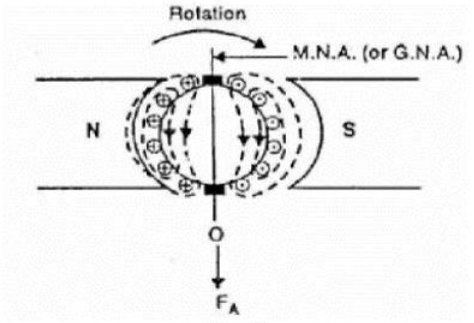


Fig. b

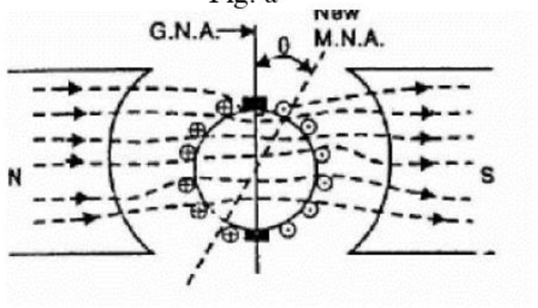


Fig. c

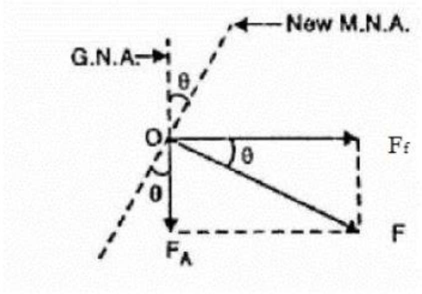


Fig. d

Figure a shows a two pole machine with single equivalent conductor in each slot and the main field mmf ($F_f$) acting alone. The axis of the main poles is called the direct axis (d-axis) and the interpolar axis is called quadrature axis (q-axis). It can be seen from the Figure b that armature mmf ($F_a$) is along the interpolar axis. $F_a$ which is at $90^0$ to the main field axis is known as cross magnetizing mmf.

Figure c shows the practical condition in which a DC machine operates when both the Field flux and armature flux are existing. Because of both fluxes are acting simultaneously, there is a shift in brush axis and crowding of flux lines at the trailing pole tip and flux lines are weakened or thinned at the leading pole tip. (The pole tip which is first met in the direction of rotation by the armature conductor is leading pole tip and the other is trailing pole tip).

If the iron in the magnetic circuit is assumed unsaturated, the net flux/pole remains unaffected by the armature reaction though the air gap flux density distribution gets distorted. If the main pole excitation is such that the iron is in the saturated region of magnetization (practical case) the increase in flux density at one end of the poles caused by armature reaction is less than the decrease at the other end, so that there is a net reduction in the flux/pole. This is called the demagnetizing effect.

Thus it can be summarized that the nature of armature reaction in a DC machine is

1. Cross magnetizing with its axis along the q-axis.
2. It causes no change in flux/pole if the iron is unsaturated but causes reduction in flux/pole in the presence of iron saturation. This is termed as demagnetizing effect. The resultant mmf ‘F’ is shown in figure d.

## 3.7 Commutation

The process of reversal of current in the short circuited armature coil is called ‘Commutation’. This process of reversal takes place when coil is passing through the interpolar axis (q-axis), the coil is short circuited through commutator segments and brush.

The process of commutation of coil ‘CD’ is shown Fig. 3.13. In sub figure ‘c’ coil ‘CD’ carries 20A current from left to right and is about to be short circuited in figure ‘d’ brush has moved by a small width and the brush current supplied by the coil are as shown. In figure ‘e’ coil ‘CD’ carries no current as the brush is at the middle of the short circuit period and the brush current in supplied by coil ‘AB’ and coil ‘EF’. In sub figure ‘f’ the coil ‘CD’ which was carrying current from left to right carries current from right to left. In sub fig ‘g’ spark is shown which is due to the reactance voltage. As the coil is embedded in the armature slots, which has high permeability, the coil possess appreciable amount of self inductance. The current is changed from +20 to –20. So due to self inductance and variation in the current from +20 to –20, a voltage is induced in the coil which is given by L dI/dt. This emf opposes the change in current in coil ‘CD’ thus sparking occurs.

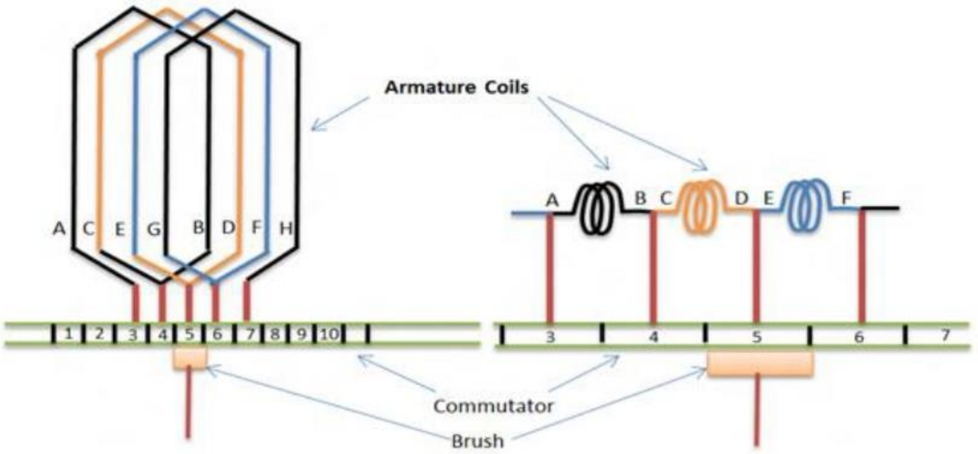


Fig a

Fig b

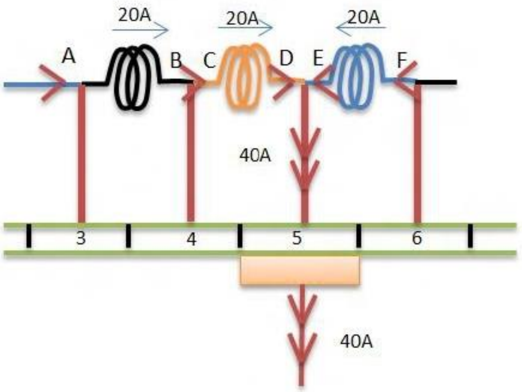


Fig.c

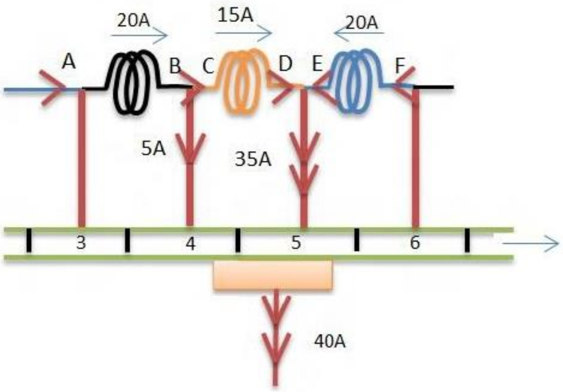


Fig.d

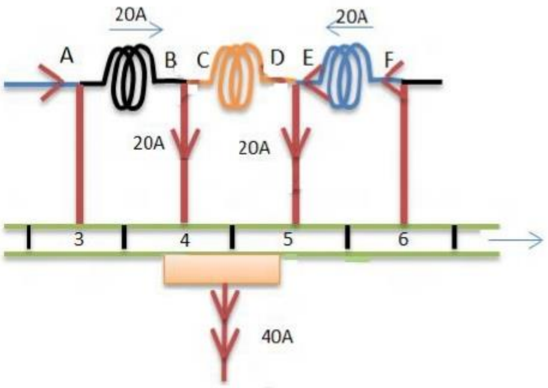


Fig.e

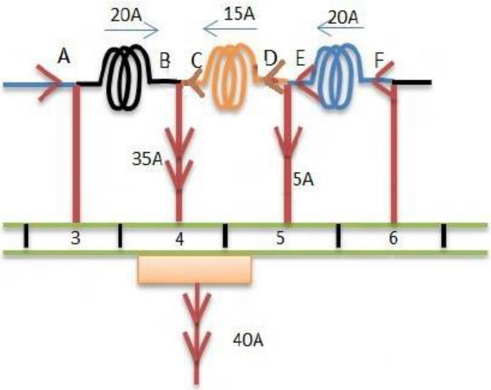


Fig.f

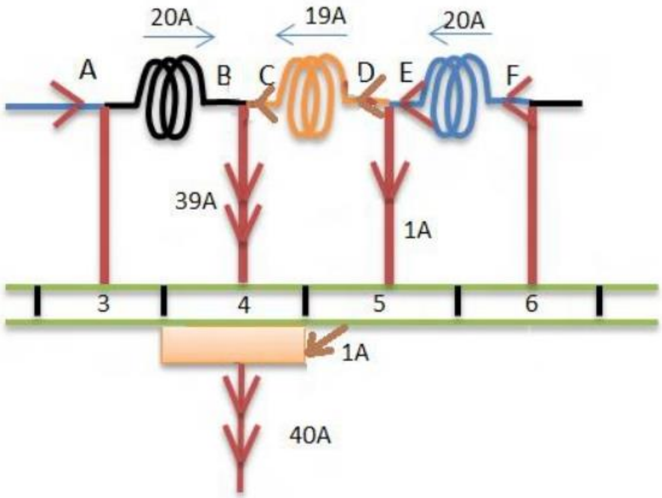


Fig.g

**Fig. 3.13 a-g shows the process of commutation**

## 3.8 Starting of DC Motor

**Necessity of starter:**

The current drawn by the armature is given by

$$I_a = \frac{V_T - E_b}{r_a}$$

At starting, as N=0 so Eb = 0 thus

$$I_a = \frac{V_T}{r_a}$$

Armature resistance will be very low. Therefore, the current drawn by the motor will be very high. In order to limit this high current, a starting resistance is connected in series with the armature. The starting resistance will be excluded from the circuit after the motor attains its rated speed. From there on back emf limits the current drawn by the motor.

### 3.8.1 Three Point Starter

The arrangement is shown in the figure 3.14 shows a three point starter for shunt motor.

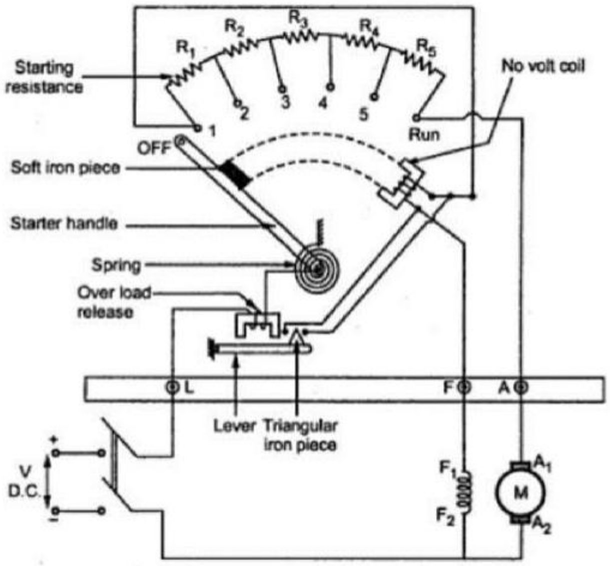


**Fig. 3.14 Internal view of three point starter**

It consists of resistances arranged in steps, $R_1$ to $R_5$ connected in series with the armature of the shunt motor. Field winding is connected across the supply through a protective device called 'NO – Volt Coil'. Another protection given to the motor in this starter is 'over load release coil'. To start the motor the starter handle is moved from OFF position to Run position gradually against the tension of a hinged spring. An iron piece is attached to the starter handle which is kept hold by the No-volt coil at Run position. The function of No volt coil is to get de-energized and release the handle when there is failure or disconnection or a break in the field circuit so that on restoration of supply, armature of the motor will not be connected across the lines without starter resistance. If the motor is over loaded beyond a certain predetermined value, then the electromagnet of overload release will exert a force enough to attract the lever which short circuits the electromagnet of No volt coil. Short circuiting of No volt coil results in de-energisation of it and hence the starter handle will be released and return to its off position due to the tension of the spring.

### 3.8.2 Four Point Starter

One important change is the No Volt Coil has been taken out of the shunt field and has been connected directly across the line through a Protecting resistance ‘R’. When the arm touches stud one. The current divides into three paths, 1. Through the starter resistance and the armature, 2. Through shunt field and the field rheostat and 3.Through No-volt Coil and the protecting resistance ‘R’. With this arrangement, any change of current in shunt field circuit does not affect the current passing though the NO-volt coil because, the two circuits are independent of each other. Thus the starter handle will not be released to its off position due to changes in the field current which may happen when the field resistance is varied. Fig 3.15 shows internal view of 4-point starter.

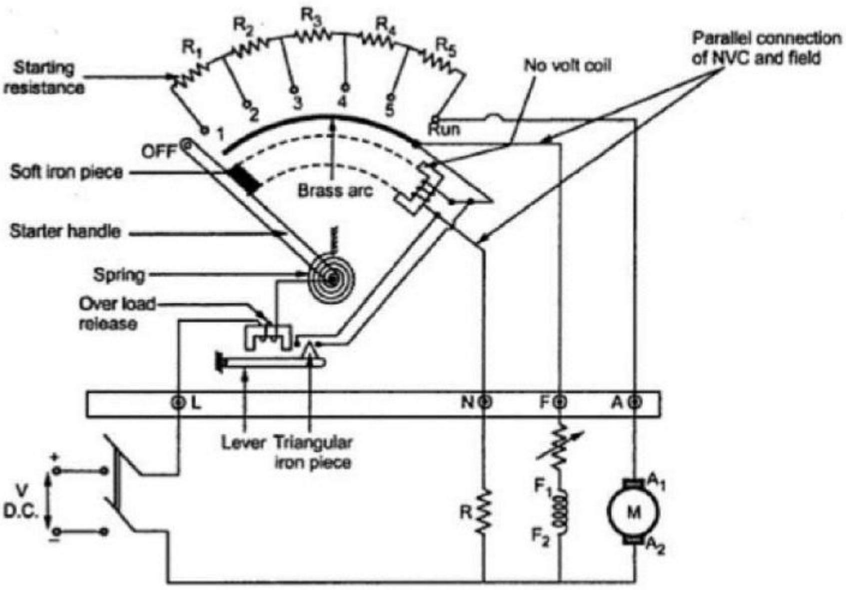


**Fig. 3.15 Internal view of three point starter**

## 3.9 DRUM CONTROLLERS

Drum controllers are used when an operator is controlling the motor directly. The drum controller is used to start, stop, reverse, and vary the speed of a motor. This type of controller is used on crane motors, elevators, machine tools, and other applications in heavy industry. As a result, the drum controller must be more rugged than the starting rheostat.

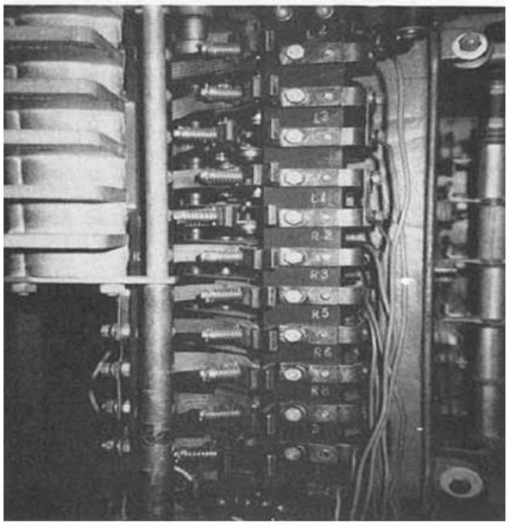


Fig. 3.16 Drum type controller shows contact fingers.

A drum controller with its cover removed is illustrated in 3.16. The switch consists of a series of contacts mounted on a movable cylinder. The contacts, which are insulated from the cylinder and from one another, are called movable contacts. There is another set of contacts, called stationary contacts, located inside the controller. These stationary contacts are arranged to touch the movable contacts as the cylinder is rotated. A handle, keyed to the shaft for the movable cylinder and contacts, is located on top of the drum controller. This handle can be moved either clockwise or counterclockwise to give a range of speed control in either direction of rotation. The handle can remain stationary in either the forward or reverse direction due to a roller and a notched wheel. A spring forces the roller into one of the notches at each successive position of the controller handle to keep the cylinder and movable contacts stationary until the handle is moved by the operator.

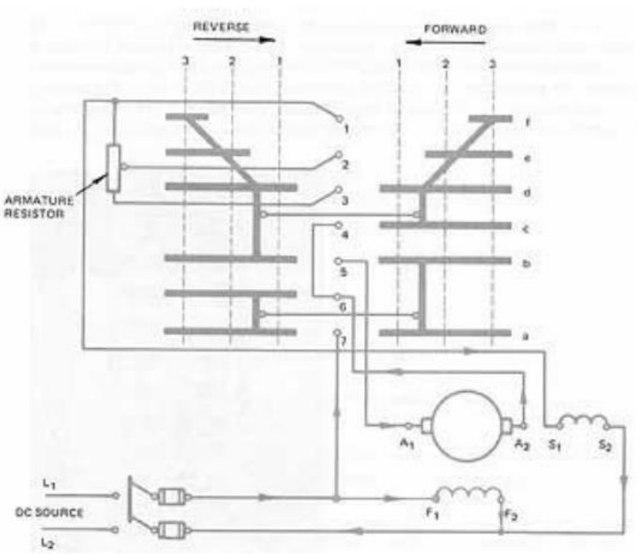


Fig. 3.17 Schematic diagram of a drum controller connected to a compound-wound motor

A drum controller with two steps of resistance is illustrated in 3.17. The contacts are represented in a flat position in this schematic diagram to make it easier to trace the circuit connections. To operate the motor in the forward direction, the set of contacts on the right must make contact with the center stationary contacts. Operation in the reverse direction requires that the set of movable contacts on the left makes contact with the center stationary contacts.

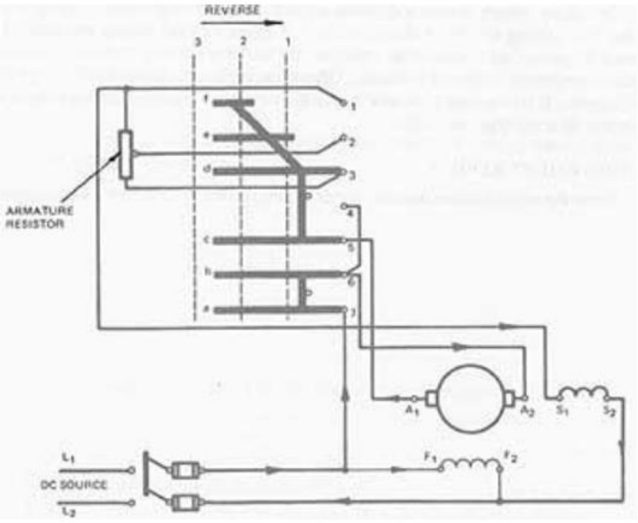


Fig. 3.18 First position of controller for reverse direction

Note in figure 3.17 that there are three forward positions and three reverse positions to which the controller handle can be set. In the first forward position, all of the resistance is in series with the armature. The circuit path for the first forward position is as follows:

1. Movable fingers a, b, c, and d contact the stationary contacts 7, 5, 4, and 3.
2. The current path is from the positive side of the line to contact 7, from 7 to a, from a to b, from b to 5, and then to armature terminal A1.
3. After passing through the armature winding to terminal A the current path is to stationary contact 6, and then to stationary contact 4.
4. From contact 4 the current path is to contact c, to d, and then to contact 3.
5. The current path then goes through the armature resistor, to the series field, and then back to the negative side of the line.

The shunt field of the compound motor is connected across the source voltage. On the second forward position of the controller handle, part of the resistance is cut out. The third forward position cuts out all of the resistance and puts the armature circuit directly across the source voltage.

In the first reverse position, all of the resistance is inserted in series with the armature. Fig. 3.18 shows the first position of the controller in the reverse direction. The current in the armature circuit's reversed. However, the current direction in the shunt and series fields is the same as the direction for the forward positions. A change in current direction in the armature only resulted in a change in the direction of rotation.

The second reverse position cuts out part of the resistance circuit. The third reverse position cuts out all of the resistance and puts the armature circuit directly across the source. Drum controllers with more positions for a greater control of speed can be obtained. However, these controllers all use the same type of circuit arrangement shown in this unit.

DC series motors require a different starting controller than shunt or compound motor. The holding circuit for the controller is in series with the starting resistance. If there is a low-voltage or no-voltage condition, the starter is returned to the off position. Drum controllers are still used frequently. Often drum controllers are used with ac as well as dc motors. It is important to be able to read the connection diagrams and the sequence diagrams on drum-type controllers.

## 3.10 Losses and efficiency of DC Machines

It is convenient to determine the efficiency of a rotating machine by determining the losses than by direct loading. Further it is not possible to arrange actual load for large and medium sized machines. By knowing the losses, the machine efficiency can be found by

$$\eta = \frac{\text{Output}}{\text{Output+Losses}} \text{(for Generator)}$$

$$\eta = \frac{\text{Input-Losses}}{\text{Input}} \text{(for Motor)}$$

In the process of energy conversion in rotating machines-current, flux and rotation are involved which cause losses in conductors, ferromagnetic materials and mechanical losses respectively.

Various losses occurring in a DC machine are listed below-

Total losses can be broadly divided into two types.

1) Constant losses

2) Variable losses

These losses can be further divided as

1) Constant losses –

i) Core loss or iron loss

  a) Hysteresis loss

  b) Eddy current loss

ii) Mechanical loss

  a) Windage loss

  b) Friction loss – brush friction loss and Bearing friction loss.

2) Variable losses –

i) copper loss ($I^2 r$)

a) Armature copper loss
b) Field copper loss
c) Brush contact loss

ii) Stray load loss

a) Copper stray load loss

b) Core stray load loss

Core loss or iron loss occurs in the armature core is due to the rotation of armature core in the magnetic flux produced by the field system. Iron loss consists of a) Hysteresis loss and b) Eddy current loss.

**Hysteresis loss**: This loss is due to the reversal of magnetization of armature core as the core passes under north and south poles alternatively. This loss depends on the volume and grade of iron, maximum value of flux density and frequency. Hysteresis loss is given by Steinmetz formula.

$W_h = K_h B_m^{1.6} f \ V$ Joule/sec or watt

Where $K_h$ = Constant of proportionality- depends on core material.

$B_m$ = Maximum flux density in $Wb/m^2$

f = Frequency in Hz

V = Volume of the armature core in $m^3$

**Eddy Current Loss:** Eddy currents are the currents set up by the induced emf in the armature core when the core cuts the magnetic flux. The loss occurring due to the flow of eddy current is known as eddy current loss. To reduce this loss the core is laminated, stacked and riveted. These laminations are insulated from each other by a thin coating of varnish. The effect of lamination is to reduce the current path because of increased resistance due to reduced cross section area of laminated core. Thus the magnitude of eddy current is reduced resulting in the reduction of eddy current loss.

Eddy Current loss is given by

$W_e = K_e B_m^2 f^2 t^2 \mathrm{V}$ Watt

Where $K_e$ = Constant of proportionality

$B_m$ = Maximum flux density in $Wb/m^2$

$f$ = Frequency in Hz

V = Volume of the armature core in $m^3$

t = Thickness of the lamination in meters

**ii) Mechanical loss:** these losses include losses due to windage, brush friction and bearing friction losses.

2) **Variable losses**: Variable losses consist of

(i) Copper loss:

a) Armature copper loss: This loss occurs in the armature windings because of the resistance of armature windings, when the current flows through them. The loss occurring is termed as copper loss or r loss. This loss varies with the varying load.

b) Field copper loss: This is the loss due to current flowing in the field windings of the machine. c) Brush contact drop: This is due the contact resistance between the brush and the commutator. This loss remains constant with load.

(ii) Stray load loss: The additional losses which vary with the load but cannot be related to current in a simple manner are called stray load loss. Stray load losses are.

Copper stray load loss: the loss occurring in the conductor due to skin effect and loss due to the eddy currents in the conductor set up by the flux passing through them are called copper stray load loss.

**Core stray load loss**: When the load current flows through the armature conductors, the flux density distribution gets distorted in the teeth and core. The flux density decreases at one end of the flux density wave and increases at the other. Since the core loss is proportional to the square of the flux density, the decrease in flux density will be less than the increase due to the increase in flux density, resulting in a net increase in the core loss predominantly in the teeth, is known as stray load loss in the core.

Further under highly saturated conditions of teeth, flux leaks through the frame and end shields causing eddy current loss in them. This loss is a component of stray load loss. Stray load loss is difficult to calculate accurately and therefore it is taken as 1 % of the output of a DC machine.

## 3.11EFFICIENCY OF A DC GENERATOR:

Power flow in a DC generator is shown in figure 3.19.

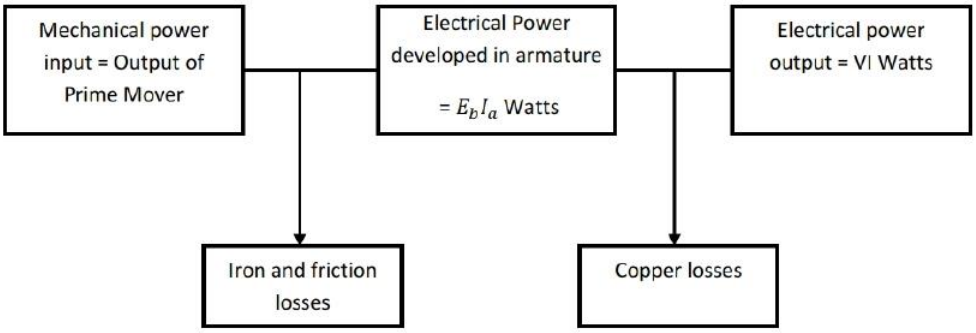


**Fig. 3.19 Power flow in a DC generator**

## 3.12 CONDITION FOR MAXIMUM EFFICIENCY

Generator output = VI;

Generator input = VI + losses.

Input $= VI + I_a^2 r_a + w_c$

If the shunt field current is negligible, then $I_a = I$

For maximum efficiency $\frac{d}{dI}(\eta) = 0$

$I_a^2 r_a = w_c$

Hence efficiency is maximum when variable loss = constant loss.

The load current corresponding to maximum efficiency is $I = \sqrt{\frac{w_c}{r_a}}$

## EFFICIENCY OF DC MOTOR:

The power flow in a DC motor is shown in figure 3.20.

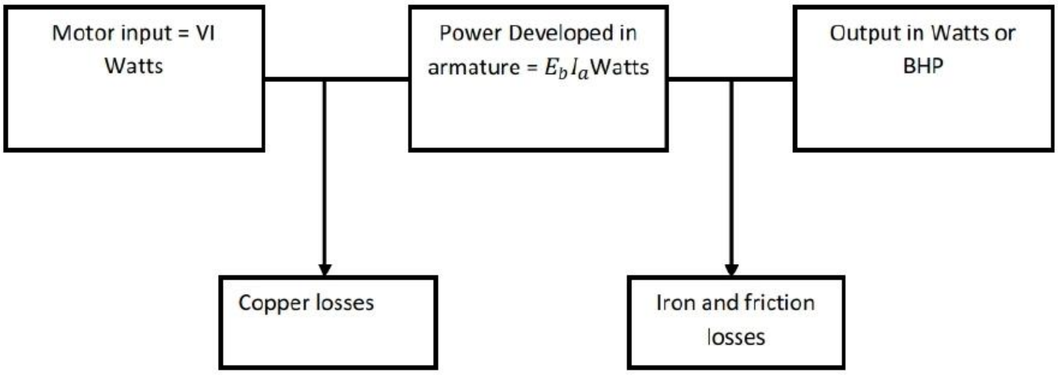


**Fig. 3.20 Power flow in a DC generator**

Efficiency $\eta = \frac{\text{Input-Losses}}{\text{Input}}$ (for Motor)

$$\eta = \frac{VI - I_a^2 r_a - w_c}{VI}$$

Efficiency is maximum when variable loss = constant loss.

## 3.12 Speed Control of DC Motor

DC motors are in general much more adaptable speed drives than AC motors. Speed of a DC motor can be controlled in a wide range.

$$N = \frac{E_b}{K\phi} = \frac{V - I_a r_a}{K\phi}$$

The speed equation shows that speed can be controlled by-

1. Variation of field current which varies the flux/pole ($\phi$) and is known as field control.
2. Variation of armature resistance known as armature voltage control.
3. Variation of terminal voltage 'V' known as Ward Leonard method.

### 3.12.1 Field Control

For a fixed terminal voltage, $\frac{N_2}{N_1} = \frac{\phi_2}{\phi_1} = \frac{If_2}{If_1}$

Limitations of speed control by field control:

1. ‘N’ below rated speed is not possible. Because $\phi$ can be decreased and cannot increase.
2. $N \propto \frac{1}{\phi}$ & $T \propto \phi$ for a given armature current, this method suits for constant kW drives only where ‘T’ decreases if speed decreases.
3. Not suited for speed reversal.

### 3.12.1.1 DC Shunt Motor

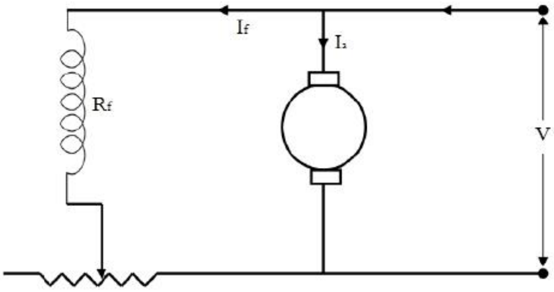


**Fig. 3.21 Circuit diagram for speed control using field control method**

Speed control is achieved by means of a rheostat in the field circuit as shown in the figure 3.21. The working range of the speed torque characteristics reduces with increasing speed in order for the armature current not to exceed the full load value with a weakening field.

### 3.12.1.2 DC Series Motor

Speed control is achieved by adjusting the field ampere turns. There are three ways for varying the field ampere turns.

*A. Diverter field control*

Diverter resistance $R_d$ is connected across the field winding as shown in figure 3.22. By varying $R_d$ the field current and hence the field ampere turns can be reduced.

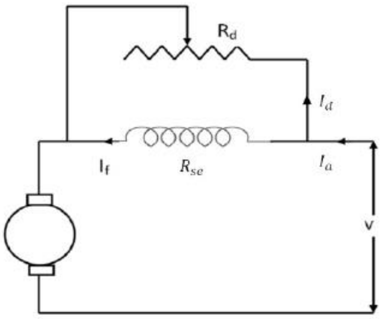


Fig. 3.22 Field diverter circuit

### B. Tapped field control:

The field ampere turns are adjusted in steps by varying the number of turns included in the circuit as shown in figure 3.23. By changing number of field winding turns effective ampere turns of the field is changed thus field flux can be controlled.

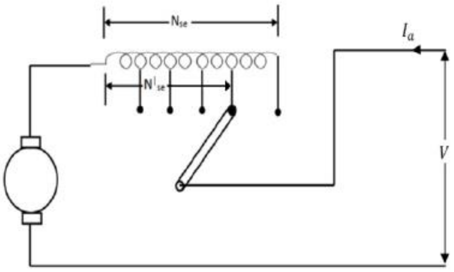


Fig. 3.23 Tapped field circuit

### C. Series parallel control

In this method, the field windings are divided into two equal halves and then connected in series or parallel to control the field ampere turns as shown in figure 3.24 and 3.25respectively.

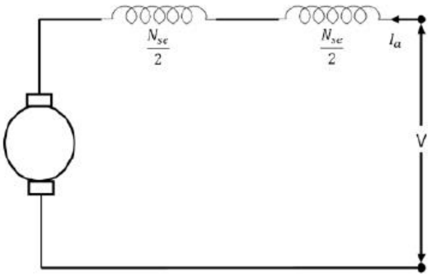


Fig. 3.24 Field circuit connected in series

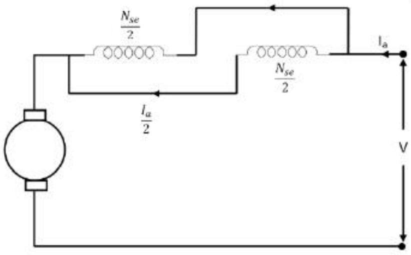


Fig. 3.25 Field circuit connected in parallel

### 3.12.2Armature Voltage Control

In this method, applied voltage across the armature of the DC motor is varied. This method is superior to field control in the following aspects:

1. This method provides a constant torque drive.(if the $\phi$ and $I_a$ are maximum, maximum torque can be obtained as $T \propto \phi I_a$ )
2. Since main field ampere turns are maintained at large value, flux density distortion caused by armature reaction is limited.
3. Unlike field control scheme, speed reversal can be easily implemented.
4. This method requires a variable voltage supply which makes this method costlier.

#### *3.12.2.1 DC Shunt Motor*

Following are the armature control schemes for DC shunt motor.

**1. Rheostatic control:**

Here the applied armature voltage is varied by placing an adjustable resistance 'R' in series with the armature as shown in the figure 3.26. N vs T for varying 'R' is shown in figure 3.27.

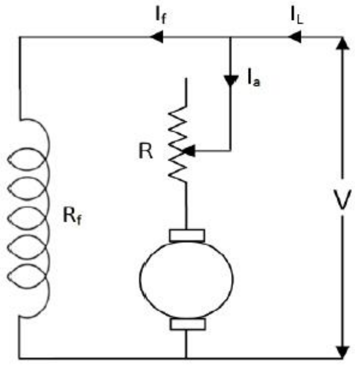


**Fig. 3.27 Circuit diagram for armature resistance control**

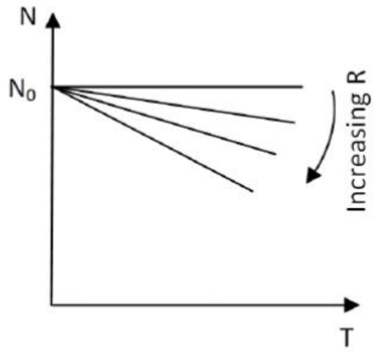


**Fig. 3.28 Speed variation with torque for different resistance**

Some of the limitations of the rheostatic method are:

Speeds only below rated speed

1. Range of speeds is limited because efficiency reduces drastically for large speed reductions
2. Speed regulation is poor. Because for a given resistance $r_e$, N varies directly with load.
3. Therefore this method is suitable for very small (fractional kW) or for short-time, intermittent slowdowns for medium sized motors.

### 2. Shunted armature control

In the armature rheostatic control method, the change in armature current due to change in load will affect the speed. Hence in this method the armature is shunted by an adjustable resistance as shown in figure 3.28.

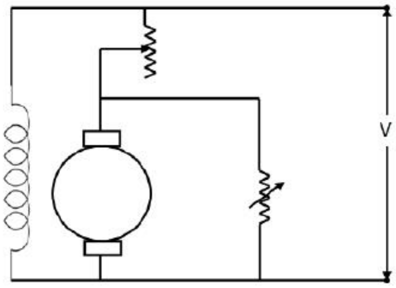


**Fig. 3.28 Shunted armature control circuit**

Advantages of this method are

1. Speed regulation will be better.

2. The changes in the armature current due to load will not be as effective as the armature is connected across a resistance.

### 3.12.3 Ward-Leonard Method

It is a combined armature and field control and is therefore operationally the most efficient method of speed control with a wide range. '$M_1$' is the main motor whose speed control is required. The field of this motor is permanently connected across the DC supply lines. Its armature is supplied by a variable voltage derived by a Motor-Generator set. The motor $M_2$ act as prime mover for the generator can be AC motor or DC motor. The field of the DC generator is separately excited. The entire arrangement is shown in the figure 3.29. The reversible switch provided for the generator field makes it possible to easily reverse the generator excitation thereby reversing the voltage polarity for reversing the direction of rotation of motor. Though expensive, this arrangement can be easily adapted to feedback schemes for automatic control of speed. This method provides both constant torque and constant HP (or kW) drive.

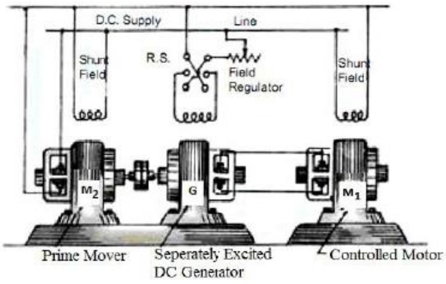


**Fig. 3.29 Ward-Leonard speed control scheme**

The armature and field winding of the motor are fed at maximum values at the base speed $N_{base}$. When armature voltage is reduced a constant torque speed control is obtained where the speed can be reduced below the base value, while the motor has full torque capability. When speed above $N_{base}$ is required then the field is gradually weakened. The motor torque therefore reduces as its speed increases which corresponds to constant kW (or HP) drive. This is shown in the figure 3.30.

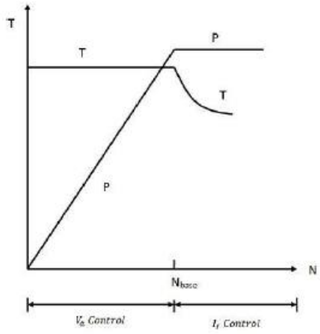


**Fig. 3.30 Speed torque relation during Ward –Leonard speed control**

Some of the features of the Ward Leonard system are given below:

1. As this method does not required any external resistance thus the efficiency is improved at all speeds and also when the generator emf becomes less than the back emf of the motor, the electrical power flows back from motor to generator, is converted to mechanical form and is returned to the mains via the driving AC motor.
2. Motor starts up smoothly therefore starting device is not required.
3. Reversal of speed is smoothly carried out.
4. Fine speed control from zero to rated value in both the directions are possible.

This method of speed control is used in

a. High speed elevators

b. Colliery winders

## 3.13 Testing of DC machines

Testing of DC machines can be broadly classified as

a) Direct method of Testing

b) Indirect method of testing

### 3.13.1 Direct method of testing

In this method, the DC machine is loaded directly by means of a brake applied to a water cooled pulley coupled to the shaft of the machine. The input and output are measured and efficiency is determined by $\eta = \frac{\text{Output}}{\text{Input}}$

It is not practically possible to arrange loads for machines of large capacity.

**BRAKE TEST**:

This is a direct method of testing. In this method of testing motor shaft is coupled to a Water cooled pulley which is loaded by means of weight as shown in figure 3.31.

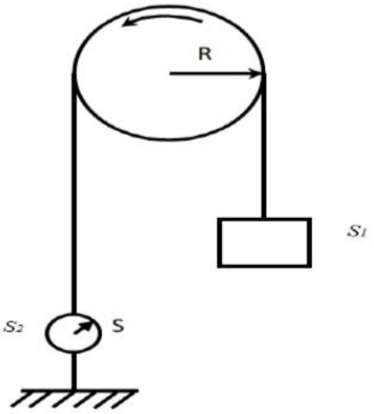


**Fig. 3.31 Brake pulley arrangement for direct load test**

$S_1$= suspended weight in kg

$S_2$= Reading in spring balance in kg

R = radius of pulley

n = speed in rps

V = Supply voltage

I = Full Load Current

Net pull due to friction = $(S_1 - S_2)$ kg

= 9.81 $(S_1 - S_2)$ Newton

Shaft torque = $(S_1 - S_2)$ R kg-mt

= 9.81 $(S_1 — S_2)$ R N-mt

Motor output power = $T_{sh}$ X $2\Pi n$ Watt

= $(S_1 — S_2)$ R X $2\Pi n$ watts

Or 9.81 $(S_1 — S_2)R \times 2\Pi n$ watt.

Input power = VI watts

Therefore efficiency $= \eta = \dfrac{\text{Output}}{\text{Input}} = \dfrac{9.81\ (S_1 — S2)R \times 2\Pi n}{VI}$

This method of testing can be used for small motors only because for a large motor it is difficult to arrange for dissipation of heat generated at the brake.

### 3.13.2 Indirect method of testing:

In this method, the machine is not actually loaded. The losses are determined. If the losses are known, then efficiency can be determined. Swinburne's test and Hopkinson's test are commonly used on shunt motors. But, as series motor cannot be started on No-load, these tests cannot be conducted on DC series motor.

#### *3.13.2.1Swinburne's Test*

For a d.c shunt motor change of speed from no load to full load is quite small. Therefore, mechanical loss can be assumed to remain same from no load to full load. Also if field current is held constant during loading, the core loss too can be assumed to remain same.

In this test, the motor is run at rated speed under *no load* condition at rated voltage. The current drawn from the supply $I_{L0}$ and the field current $I_f$ are recorded. Now we note that:

Input power to the motor, $P_{in} = VI_{L0}$
Cu loss in the field circuit $P_{fl} = VI_f$
Power input to the armature, $= VI_{L0} - VI_f = V(I_{L0} - I_f) = VI_{a0}$
Cu loss in the armature circuit $= I^2_{a0}\, r_a$
Gross power developed by armature $= VI_{a0} - I^2_{a0}\, r_a = (V - I_{a0} r_a) I_{a0} = E_{b0} I_{a0}$

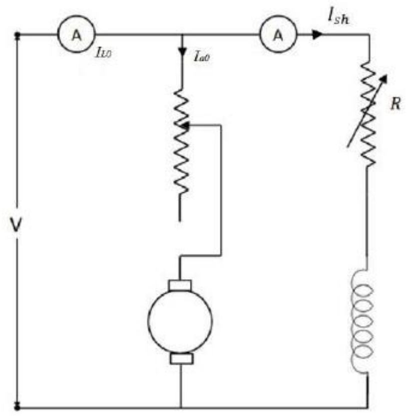


**Fig. 3.32 Circuit diagram for Swinburne'*S* Test**

Since the motor is operating under no load condition, net mechanical output power is zero. Hence the gross power developed by the armature must supply the core loss and friction & windage losses of the motor. Therefore,

$P_{core} + P_{friction} = (V - I_{a0} r_a) I_{a0} = E_{b0} I_{a0}$

Since, both $P_{core}$ and $P_{friction}$ for a shunt motor remains practically constant from no load to full load, the sum of these losses is called constant rotational loss i.e.,

Constant rotational loss, $P_{rot} = P_{core} + P_{friction}$

In the Swinburne's test, the constant rotational loss comprising of core and friction loss is estimated from the above equation.

After knowing the value of $P_{rot}$ from the Swinburne's test, we can fairly estimate the efficiency of the motor at any loading condition. Let the motor be loaded such that new current drawn from the supply is $I_L$ and the new armature current is $I_a$. To estimate the efficiency of the loaded motor we proceed as follows:

Input power to the motor, $P_{in} = VI_L$

Cu loss in the field circuit $P_{fl} = VI_f$

Power input to the armature, $= VI_L - VI_f = V(I_L - I_f) = VI_a$

Cu loss in the armature circuit = $I^2{}_a r_a$

Gross power developed by armature = $VI_a - I^2{}_a r_a = (V - I_a r_a) I_a = E_b I_a$

Net mechanical output power, $P_{net\ mech} = E_b I_a - P_{rot}$

Efficiency of the loaded motor,

$$\eta = \frac{E_b I_a - P_{rot}}{VI_L} = \frac{P_{net\ mech}}{P_{in}}$$

The estimated value of $P_{rot}$ obtained from Swinburne's test can also be used to estimate the efficiency of the shunt machine operating as a generator.

Output power of the generator, $P_{out} = VI_L$

Cu loss in the field circuit $P_{fl} = VI_f$

Output power of the armature, $= VI_L + VI_f = VI_a$

Mechanical input power, $P_{in\ mech} = VI_a + I^2{}_a r_a + P_{rot}$

Efficiency of the generator,

$$\eta = \frac{VI_L}{P_{in\ mech}} = \frac{VI_L}{VI_L + VI_f + I_a^2 r_a + P_{rot}}$$

As this test is done at no-load condition thus the power required is very less. From the test effect of armature reaction, temperature rise, commutation etc. cannot be predicted as the machine is not actually loaded.

### *3.13.2.2Load Test*

To assess the rating of a machine a load test has to be conducted. When the machine is loaded, certain fraction of the input is lost inside the machine and appears as heat, increasing the temperature of the machine. If the temperature rise is excessive then it affects the insulations, ultimately leading to the breakdown of the insulation and the machine. The load test gives the information about the efficiency of a given machine at any load condition. Also, it gives the temperature rise of the machine. If the temperature rise is below the permissible value for the insulation then the machine can be safely operated at that load, else the load has to be reduced. The maximum continuous load that can be delivered by the machine without exceeding the temperature rise for the insulation used, is termed as the continuous rating of the machine. Thus the load test alone can give us the proper information of the rating and also can help in the direct measurement of the efficiency.

### *3.13.2.3 Hopkinson's Test*

Here power drawn from the supply only corresponds to no load losses of the machines, the armature physically carries any amount of current (which can be controlled with ease). Two similar DC shunt motors are mechanically coupled. Electrically these two machines are eventually connected in parallel and controlled in such a way that one machine acts as a generator and the other as motor.

Two similar (same rating) machines are connected and coupled as shown in figure 3.33. With switch is open initially, the first machine is run as a shunt motor at rated speed. It may be noted that the second machine is operating as a separately excited generator because its field winding is excited and it is driven by the first machine. The value of the voltage across the switch is either close to twice supply voltage or small voltage. In fact the voltmeter practically reads the difference of the induced voltages in the armature of the machines. In case if the voltmeter reading is high, then the armature connection of the generator should be reversed and start afresh. Now if the voltmeter is found to read small voltage then any attempt to close the switch may result into large circulating current as the armature resistances are small. By adjusting the field current $I_{fg}$ of the generator the voltmeter reading may be adjusted to zero ($E_g \approx E_b$) and switch is now closed. Both the machines are now connected in parallel as shown in figure 3.33.

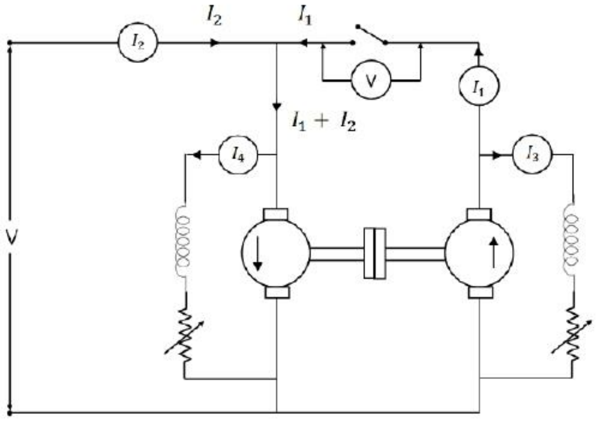


**Fig. 3.33 Connection of Hopkinson's Test**

After the machines are successfully connected in parallel, if the field current of generator is increased (by decreasing generator field resistance), then $E_g$ becomes greater than $E_b$ and both armature current of generator and motor increase, Thus by increasing field current of generator (alternatively decreasing field current of motor) one can make $E_g > E_b$ so as to make the second machine act as generator and first machine as motor. In practice, it is also required to control the field current of the motor to maintain speed constant at rated value. The interesting point to be noted here is that the armature current of generator and motor are not reflected in the supply side line. Thus current drawn from supply remains small (corresponding to losses of both the machines). The loading is sustained by the output power of the generator running the motor and vice versa. The machines can be loaded to full load current without the need of any loading arrangement.

## *Calculation*

V= supply voltage

Motor input = $V(I_1+I_2)$

Generator output = $VI_1$

If it is assumed both machines have the same efficiency ‘η’, then,

Output of motor = $\eta$ x input = $\eta$ x $V (I_1+I_2)$ = input to generator

Output of generator = $\eta$ x input = $\eta$ x $\eta V (I_1+I_2) = \eta^2 V(I_1+I_2)$

$VI_1 = \eta^2 V(I_1+I_2)$

Therefore, $\eta = \sqrt{\dfrac{I_1}{I_1+I_2}}$

Armature copper loss in motor = $(I_1 + I_2 - I_4)^2 r_a$

Shunt field copper loss in motor = $VI_4$

Armature copper loss in generator = $(I_1 + I_3)^2 r_a$

Shunt field copper loss in generator = $VI_3$

Power drawn from supply = $VI_2$

Therefore stray losses = $VI_2 - [(I_1 + I_2 - I_4)^2 r_a + VI_4 + (I_1 + I_3)^2 r_a + VI_3]$ = W (say)

Stray losses/motor = $\frac{W}{2}$

**Therefore for generator**

Total losses=$(I_1+I_2)^2 r_a + VI_3 + \frac{W}{2} = W_g$

Output = $VI_1$, therefore

$$\eta_{generator} = \frac{VI_1}{VI_1+W_g} = \frac{output}{output+losses}$$

**For motor,**

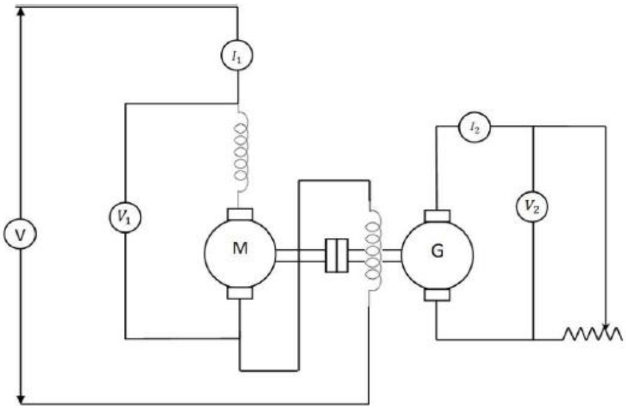
Total losses =$(I_1+I_2-I_4)^2 r_a + VI_4 + \frac{W}{2} = W_m$

Input to motor = $V(I_1+I_2)$

Therefore $\eta_{motor} = \frac{V(I_1+I_2)-W_m}{V(I_1+I_2)}$

### *3.13.2.4Field's Test*

Figure 3.34 shows the circuit for fields test. This test is applicable to two similar series motor. One of the machine runs as a motor and drives a generator whose output is wasted in a variable load 'R'. Both machine field coils are in series and both run at same speed so that iron and friction losses are made equal.



**3.34 Circuit diagram for Field's test on DC series motor**

Load resistance 'R' is varied till the motor current reaches its full load value.

V = Supply voltage

$I_1$ = Motor current

$V_2$ = Generator terminal voltage

$I_2$ = Load current

Input = $VI_1$ and output = $V_2I_2$

$R_a$ and $R_{se}$ = hot resistances.

Total losses in the set $W_t = VI_1 - V_2I_2$

Armature and Field copper losses $W_c = (R_a + 2\ r_{se})\ I_1^2 + I_a^2R_a$

Stray losses for the set = $W_t - W_c$

Stray losses per machine $W_s = \frac{W_t - W_c}{2}$

**Motor efficiency :**

Input=$V_1I_1$

Losses=$\left(R_a + R_{se}\right) I_1^2 + W_s = W_m$ (say)

$$\eta_{motor} = \frac{VI_1 + W_m}{VI_1}$$

**Generator efficiency**: η of generator is of little use, because its field winding is separately excited

Generator output = $VI_2$

Field copper loss = $I_1^2r_{se}$

Armature copper loss = $I_1^2r_a$

Total losses = $I_1^2r_{se} + I_1^2r_a + W_s = W_g$ (say)

$$\eta_{generator} = \frac{VI_2}{V_2 + W_g}$$

### *3.12.2.5Reatardation Test*

This method is applicable to shunt motors and generators and is used for finding the stray losses. If armature and shunt copper losses are known for a given load, efficiency can be calculated. The circuit is shown in figure 3.35.

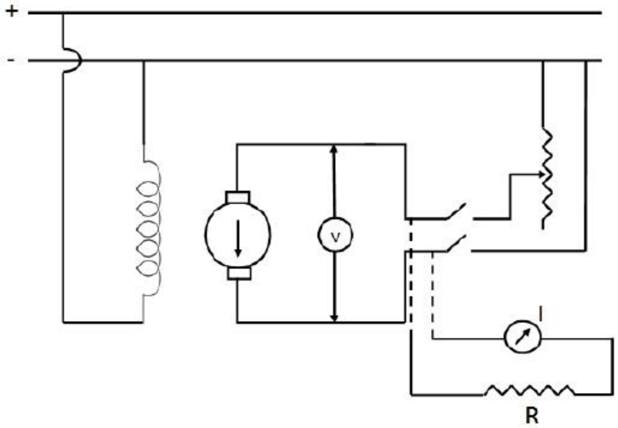


**Fig. 3.35 Circuit diagram for Retardation test on DC motor**

Machine is speeded up slightly beyond its rated speed and then supply is cut off from the armature while keeping the field excited. Armature will slow down and its kinetic energy is needed to meet rotational losses. i.e., friction and windage losses.

Kinetic energy of the armature = $\frac{1}{2}I\omega^2$

I = Moment of inertia of the armature

$\omega$ = Angular velocity.

Rotational losses;

N = Rate of loss of K.E.

Rate of loss of Kinetic energy $W = \frac{d}{dt}\left[\frac{1}{2}I\omega^2\right] = I\omega\frac{d\omega}{dt}$

Two quantities need to be known

(i) Moment of Inertia 'I

(ii) $\frac{d\omega}{dt}$ or $\frac{dN}{dt}$ (because $\omega \propto N$)

(i) Finding $\frac{d\omega}{dt}$:

The voltmeter "V" in the circuit shown in Fig. 3.35 is used as per speed indicator by suitably grading it because $E \propto N$. Then the supply is cut off, the armature speed and hence voltmeter reading falls. Voltage and time at different interval are noted and a curve is drawn between time and speed as shown in fig. 3.36.

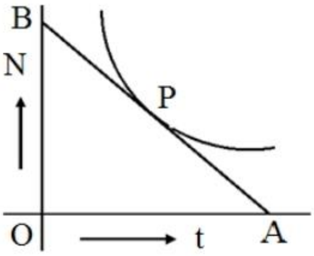


**Fig. 3.36 Change of speed with time**

In the fig. 3.36 AB- tangent drawn at P

Therefore $\frac{dN}{dt} = \frac{OB(rpm)}{OA(sec)}$

$$W = I \times \omega \times \frac{d\omega}{dt}$$

$$\omega = \frac{2\pi N}{60}$$

$$W = I\left(\frac{2\pi N}{60}\right)\frac{d}{dt}\left(\frac{2\pi N}{60}\right)$$

$$W = \left(\frac{2\pi}{60}\right)^2 .I.N.\frac{dN}{dt}$$

(ii) Finding Moment of Inertia "I":

There are two methods of finding the moment of inertia 'I'

**(a) I is calculated:**

(i) Slowing down curve with armature alone is calculated.

(ii) A fly wheel is keyed to the shaft and the curve is drawn again

For any given speed, $\frac{dN}{dt}$ and $\frac{dN}{dt_1}$ are determined as before.

Therefore $W=\left(\frac{2\pi}{60}\right)^2.I.N.\frac{dN}{dt_1}$ 1st case

$W=\left(\frac{2\pi}{60}\right)^2(I+I_1)N.\frac{dN}{dt_2}$ 2nd Case

The two cases are equal because losses in two cases will be almost same.

$I\frac{dN}{dt}=(I+I_1)\frac{dN}{dt}.\frac{I+I1}{I}\left(\frac{dN}{dt_2}\right)=\frac{dN}{dt_1}$

$\frac{I+I1}{I}=\frac{dN}{dt_2}$

$I=I_1\times\frac{t_2}{t_1-t_2}$

**(b) I is eliminated:**

In this method, time taken to slow down is noted with armature alone and then a retarding torque is applied electrically i.e., a non-inductive resistance is connected to the armature.

The additional loss is $I_a^2(R_a+R)\text{or}VI_a$

Let $W^1$ be the power then

$W=\left(\frac{2\pi}{60}\right)^2 IN.\frac{dN}{dt_1}$

$W+W^1=\left(\frac{2\pi}{60}\right)^2 IN.\frac{dN}{dt_2}$

$\frac{dN}{dt_1}=$ rate of change of speed without electrical load

$\frac{dN}{dt_2}=$ rate of change of speed with electrical load

$$\frac{W+W^1}{W}=\frac{\frac{dN}{dt_2}}{\frac{dN}{dt_1}}$$

or, $W=W^1\times\frac{dt_2}{dt_1-dt_2}$

or $W=W^1\times\frac{t_2}{t_1-t_2}$

# Module IV

## [THREE PHASE TRANSFORMER]

TOPICS

**Three Phase Transformers:** Constructional features of three phase transformers – three phase connection of transformers (Dd0, Dd6, Yy0, Yy6, Dy1, Dy11, Yd1, Yd11, zigzag), Scott connection, open delta connection, three phase to six phase connection, oscillating neutral, tertiary winding, three winding transformer, equal and unequal turns ratio, parallel operation, load sharing. Distribution transformers, all day efficiency, Autotransformers, saving of copper, applications, tap- changing transformers, cooling of transformers.

[Topics are arranged as per above sequence]

# Three Phase Transformers

## 4.1 Introduction

Electric power is generated in generating stations, using three phase alternators at 11 KV. This voltage is further stepped up to 66 KV, 110 KV, 230 KV or 400 KV using 3 phase power transformers and power is transmitted at this high voltage through transmission lines. At the receiving substations, these high voltages are stepped down by 3 phase transformers to 11 KV. This is further stepped down to 400 volts at load centers by means of distribution transformers. For generation, transmission and distribution, 3 phase system is economical. Therefore 3 phase transformers are very essential for the above purpose. The sectional view of a 3 phase power transformer is shown in Fig.4.1.

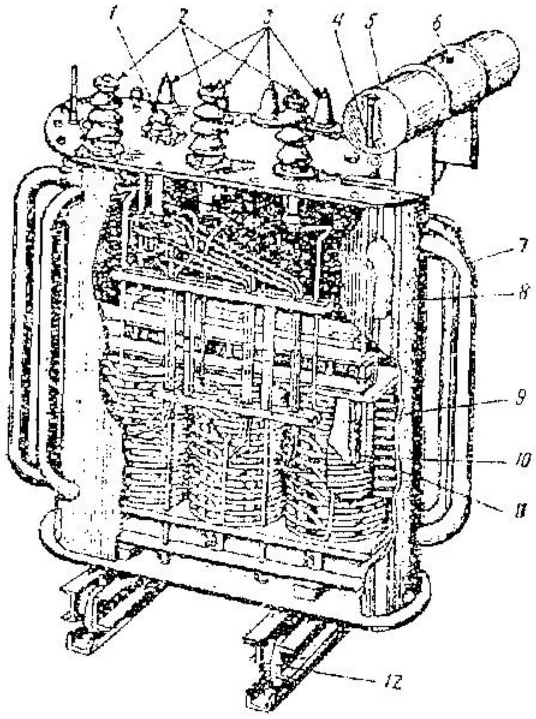


**Fig. 4.1 100 KVA oil immersed power transformer**

1. Tap-changer switch handle
2. Porcelain-bushing insulator (For high voltage)
3. Bushing insulators (For low voltages)
4. Oil gauge

5. Oil tank
6. Breather plug
7. Cooling pipes
8. Tank front wall
9. Core,
10. High voltage winding
11. Low voltage winding
12. Wheels or rollers.

## 4.2 Construction of Three phase Transformer

Three phase transformers comprise of three primary and three secondary windings. They are wound over the laminated core as we have seen in single phase transformers. Three phase transformers are also of core type or shell type as in single phase transformers. The basic principle of a three phase transformer is illustrated in fig 4.2 in which the primary windings and secondary windings of three phases are shown. The primary windings can be inter connected in star or delta and put across three phase supply.

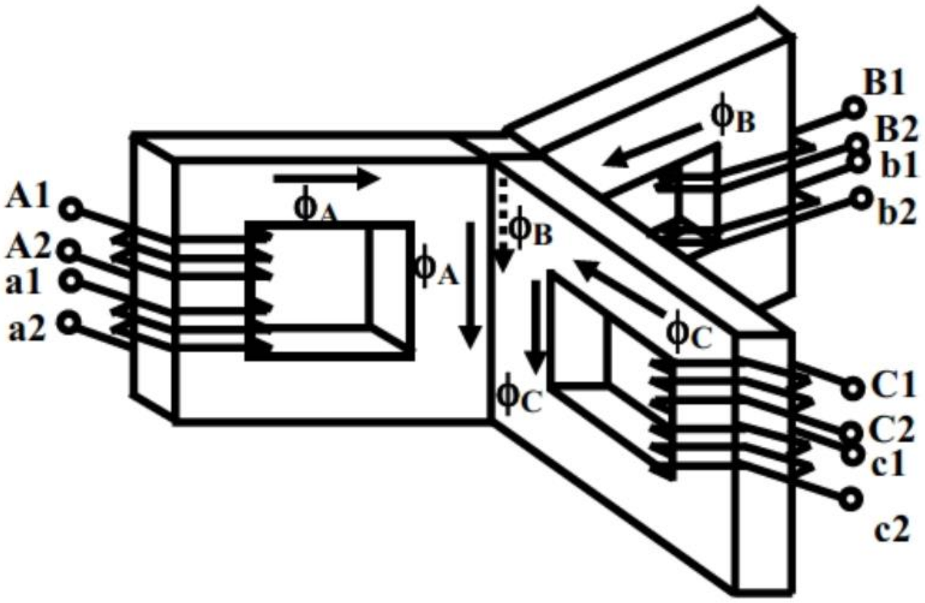


**Fig. 4.2 3-phase core-type Transformer**

The three cores are 120° apart and their unwound limbs are shown in contact with each other. The center core formed by these three limbs, carries the flux produced by the three phase currents $I_R$, $I_Y$ and $I_B$. As at any instant $I_R+I_y+I_B=0$, the sum of three fluxes (flux in the center limb) is also zero.

Therefore it will make no difference if the common limb is removed. All the three limbs are placed in one plane in case of a practical transformer as shown in fig 4.3.

The core type transformers are usually wound with circular cylindrical coils. The construction and assembly of laminations and yoke of a three phase core type transformer is shown in fig 4.4 one method of arrangement of windings in a three phase transformer is shown.

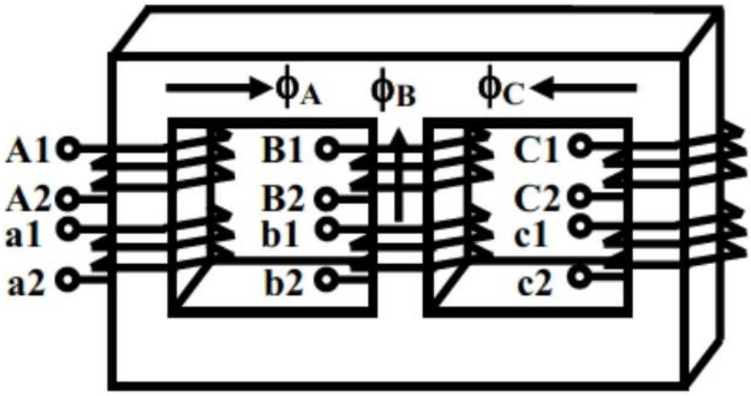


**Fig. 4.3 A practical core type three phase transformer**

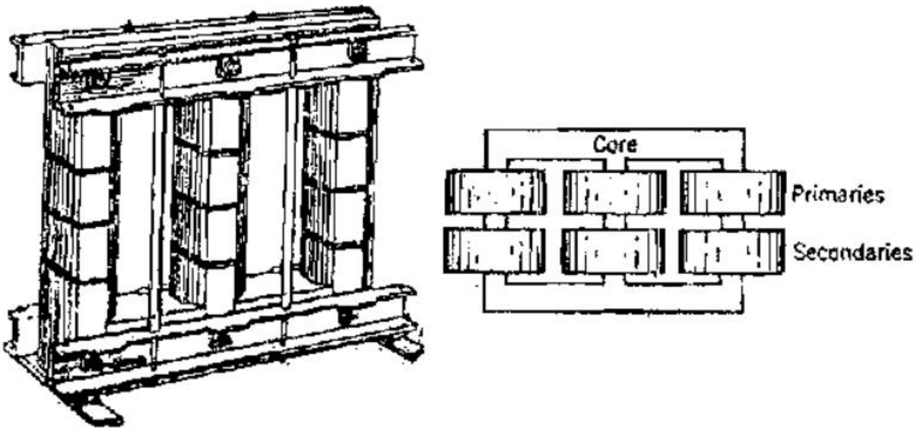


**Fig. 4.4 Core type transformer windings and construction**

In the other method the primary and secondary windings are wound one over the other in each limb. The low-tension windings are wound directly over the core but are, of course, insulated for it. The high tension windings are wound over the low— tension windings and adequate insulation is provided between the two windings.

The primary and secondary windings of the three phase transformer can also be interconnected as star or delta.

## 4.3 Three Phase Transformer connections:-

The identical single phase transformers can be suitably inter-connected and used instead of a single unit 3—phase transformer. The single unit 3 phase transformer is housed in a single tank. But the transformer bank is made up of three separate single phase transformers each with its own, tanks and bushings. This method is preferred in mines and high altitude power stations because transportation becomes easier. Bank method is adopted also when the voltage involved is high because it is easier to provide proper insulation in each single phase transformer.

As compared to a bank of single phase transformers, the main advantages of a single unit 3-phase transformer are that it occupies less floor space for equal rating, less weight costs about 20% less and further that only one unit is to be handled and connected.

There are various methods available for transforming 3 phase voltages to higher or lower 3 phase voltages. The most common connections are (i) star — star (ii) Delta—Delta (iii) Star —Delta (iv) Delta — Star.

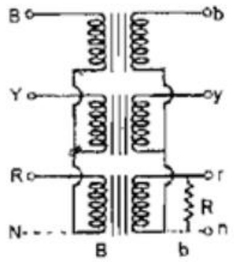


Fig 4.5 Star-star connection

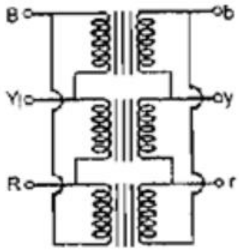


Fig. 4.6 Delta-delta connection

The star-star connection is most economical for small, high voltage transformers because the number of turns per phase and the amount of insulation required is minimum (as phase voltage is only 1/3 of line voltage. In fig. 4.5 a bank of three transformers connected in star on both the primary and the secondary sides is shown. The ratio of line voltages on the primary to the secondary sides is the same as a transformation ratio of single phase transformer.

The delta— delta connection is economical for large capacity, low voltage transformers in which insulation problem is not a serious one. The transformer connection are as shown in fig. 4.6.

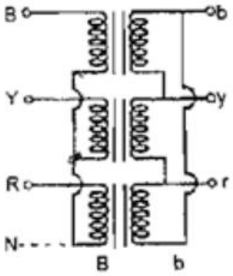


Fig. 4.7 Star-delta connection

The main use of star-delta connection is at the substation end of the transmission line where the voltage is to be stepped down. The primary winding is star connected with grounded neutral as shown in Fig. 4.7. The ratio between the secondary and primary line voltage is 1/3 times the transformation ratio of each single phase transformer. There is a 30° shift between the primary and secondary line voltages which means that a star-delta transformer bank cannot be paralleled with either a star-star or a delta-delta bank.

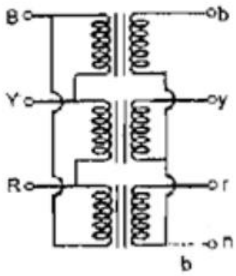


**Fig. 4.8 Delta-star connection**

Delta-Star connection is generally employed where it is necessary to step up the voltage. The connection is shown in fig. 4.8. The neutral of the secondary is grounded for providing 3-phase, 4-wire service. The connection is very popular because it can be used to serve both the 3-phase power equipment and single phase lighting circuits.

## 4.4Vector Group of 3-phase transformer

The secondary voltages of a 3-phase transformer may undergo a *phase shift* of either +30° leading or -30° lagging or 0° i.e, no phase shift or 180° reversal with respective line or phase to neutral voltages. On the name plate of a three phase transformer, the vector group is mentioned. Typical representation of the vector group could be Ydl or Dy 11 etc. The first capital latter Y indicates that the primary is connected in star and the second lower case latter d indicates delta connection of the secondary side. The third numerical figure conveys the angle of phase shift based on *clock convention*. The minute hand is used to represent the primary phase to neutral voltage and always shown to occupy the position 12. The hour hand represents the secondary phase to neutral voltage and may, depending upon phase shift, occupy position other than 12 as shown in the figure 4.9. The angle between two consecutive numbers on the clock is 30°.

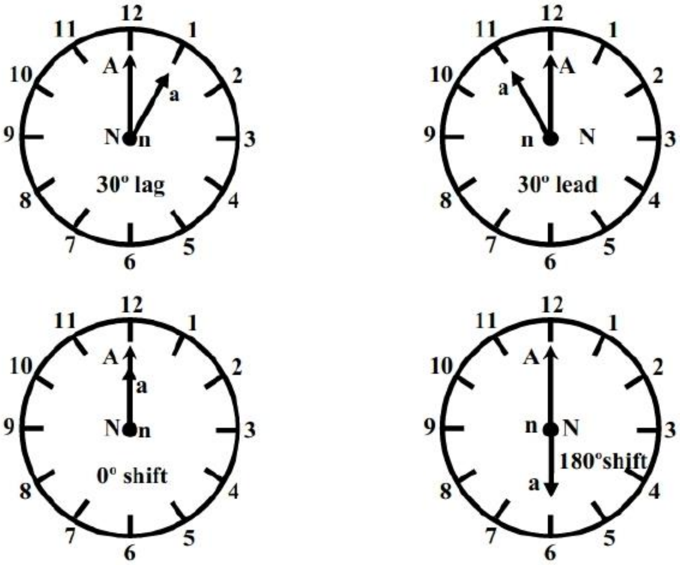


Fig. 4.9 Clock convention representing vector groups

### 4.4.1 Delta/delta (Dd0, Dd6) connection

The connection of Dd0 is shown in fig. 4.10 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is zero degree (0°).

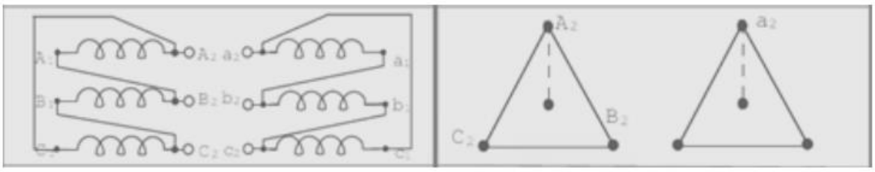


Fig 4.10 Dd0 connection and phasor diagram

The connection of Dd6 is shown in fig. 4.11 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 180°.

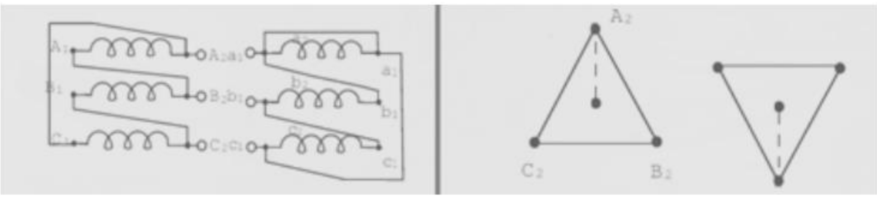


Fig 4.11 Dd6 connection and phasor diagram

This connection proves to be economical for large low voltage transformers as it increases number of turns per phase. Primary side line voltage is equal to secondary side line voltage. Primary side phase voltage is equal to secondary side phase voltage. There is no phase shift between primary and secondary voltages for Dd0 connection. There is 180° phase shift between primary and secondary voltages for Dd6 connection.

### *Advantages*

- **Sinusoidal Voltage at Secondary:** In order to get secondary voltage as sinusoidal, the magnetizing current of transformer must contain a third harmonic component. The delta connection provides a closed path for circulation of third harmonic component of current. The flux remains sinusoidal which results in sinusoidal voltages.

- **Suitable for Unbalanced Load:** Even if the load is unbalanced the three phase voltages remains constant. Thus it suitable for unbalanced loading also.

- **Carry 58% Load if One Transfer is Faulty in Transformer Bank**: If there is bank of single phase transformers connected in delta-delta fashion and if one of the transformers is disabled then the supply can be continued with remaining tow transformers of course with reduced efficiency.

- **No Distortion in Secondary Voltage**: there is no any phase displacement between primary and secondary voltages. There is no distortion of flux as the third harmonic component of magnetizing current can flow in the delta connected primary windings without flowing in the line wires .there is no distortion in the secondary voltages.

- **Economical for Low Voltage:** Due to delta connection, phase voltage is same as line voltage hence winding have more number of turns. But phase current is (1/√3) times the line current. Hence the cross-section of the windings is very less. This makes the connection economical for low voltages transformers.
- **Reduce Cross section of Conductor:** The conductor is required of smaller Cross section as the phase current is 1/√3 times of the line current. It increases number of turns per phase and reduces the necessary cross sectional area of conductors thus insulation problem is not present.
- **Absent of Third Harmonic Voltage:** Due to closed delta, third harmonic voltages are absent.
- The absence of star or neutral point proves to be advantageous in some cases.

### *Disadvantages*

- Due to the absence of neutral point it is not suitable for three phase four wire system.
- More insulation is required and the voltage appearing between windings and core will be equal to full line voltage in case of earth fault on one phase.

### *Application*

- Suitable for large, low voltage transformers.
- This Type of Connection is normally uncommon but used in some industrial facilities to reduce impact of SLG faults on the primary system
- It is generally used in systems where it need to be carry large currents on low voltages and especially when continuity of service is to be maintained even though one of the phases develops fault.

## 4.4.2 Star/star (YyO, Yy6) connection

This is the most economical one for small high voltage transformers. Insulation cost is highly reduced. Neutral wire can permit mixed loading. Triplen harmonics are absent in the lines. These triplen harmonic currents cannot flow, unless there is a neutral wire. This connection produces oscillating neutral. Three phase shell type units have large triplen harmonic phase voltage. However three phase core type transformers work satisfactorily. A tertiary mesh connected winding may be required to stabilize the oscillating neutral due to third harmonics in three phase banks.

The connection of Yy0 is shown in fig. 4.12 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is zero degree (0°).

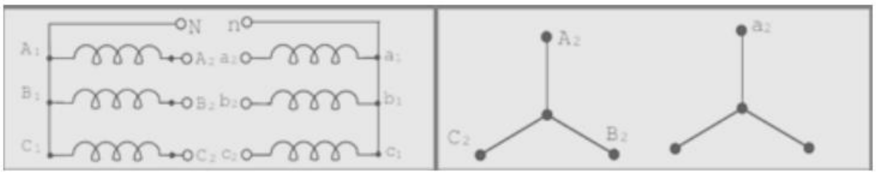


**Fig .4.12 Yy0 connection and phasor diagram**

The connection of Yy6 is shown in fig. 4.13 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 180°.

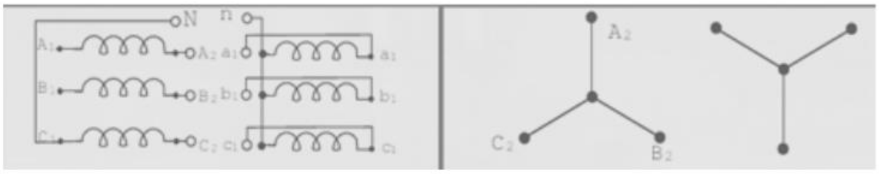


**Fig 4.13. Yy6 connection and phasor diagram**

- In Primary Winding Each Phase is120°electrical degrees out of phase with the other two phases.
- In Secondary Winding Each Phase is120°electrical degrees out of phase with the other two phases.

- Each primary winding is magnetically linked to one secondary winding through a common core leg. Sets of windings that are magnetically linked are drawn parallel to each other in the vector diagram. In the Y-Y connection, each primary and secondary winding is connected to a neutral point.
- The neutral point may or may not be brought out to an external physical connection and the neutral may or may not be grounded.

## *Advantages of Y-y connection*

- **No Phase Displacement:** The primary and secondary circuits are in phase; i.e., there are no phase angle displacements introduced by the Y-Y connection. This is an important advantage when transformers are used to interconnect systems of different voltages in a cascading manner. For example, suppose there are four systems operating at 800, 440, 220, and 66 kV that need to be interconnected. Substations can be constructed using Y-Y transformer connections to interconnect any two of these voltages. The 800 kV systems can be tied with the 66 kV systems through a single 800 to 66 kV transformation or through a series of cascading transformations at 440,220 and 66 kV.
- **Required Few Turns for winding:** Due to star connection, phase voltages is (1/√3) times the line voltage. Hence less number of turns is required. Also the stress on insulation is less. This makes the connection economical for small high voltage purposes.
- **Required Less Insulation Level**: If the neutral end of a Y-connected winding is grounded, then there is an opportunity to use reduced levels of insulation at the neutral end of the winding. A winding that is connected across the phases requires full insulation throughout the winding.
- **Handle Heavy Load:** Due to star connection, phase current is same as line current. Hence windings have to carry high currents. This makes cross section of the windings high. Thus the windings are mechanically strong and windings can bear heavy loads and short circuit current.
- **Use for Three phases Four Wires System:**As neutral is available, suitable for three phases four wire

system.

- **Eliminate Distortion in Secondary Phase Voltage:** The connection of primary neutral to the neutral of generator eliminates distortion in the secondary phase voltages by giving path to triple frequency currents toward to generator.
- **Sinusoidal voltage on secondary side:** Neutral give path to flow Triple frequency current to flow Generator side thus sinusoidal voltage on primary will give sinusoidal voltage on secondary side.
- **Used as Auto Transformer:** A Y-Y transformer may be constructed as an autotransformer, with the possibility of great cost savings compared to the two-winding transformer construction.
- **Better Protective Relaying:** The protective relay settings will be protecting better on the line to ground faults when the Y-Y transformer connections with solidly grounded neutrals are applied.

### *Disadvantages*

- **The Third harmonic issue:** The voltages in any phase of a Y-Y transformer are 1200 apart from the voltages in any other phase. However, the third-harmonic components of each phase will be in phase with each other. Nonlinearities in the transformer core always lead to generation of third harmonic. These components will add up resulting in large (can be even larger than the fundamental component) third harmonic component.
- **Overvoltage at Lighting Load:** The presence of third (and other zero-sequence) harmonics at an ungrounded neutral can cause overvoltage conditions at light load. When constructing a Y-Y transformer using single-phase transformers connected in a bank, the measured line-to-neutral voltages are not 57.7% of the system phase-to-phase voltage at no load but are about 68% and diminish very rapidly as the bank is loaded. The effective values of voltages at different frequencies combine by taking the square root of the sum of the voltages squared. With sinusoidal phase-to-phase voltage, the third-harmonic component of the phase-to-neutral

voltage is about 60%.

- **Voltage drop at Unbalance Load:** There can be a large voltage drop for unbalanced phase-to-neutral loads. This is caused by the fact that phase-to-phase loads cause a voltage drop through the leakage reactance of the transformer whereas phase-to-neutral loads cause a voltage drop through the magnetizing reactance, which is 100 to 1000 times larger than the leakage reactance.
- **Overheated Transformer Tank:** Under certain circumstances, a Y-Y connected three-phase trans- can produce severe tank overheating that can quickly destroy the transformer. This usually occurs with an open phase on the primary circuit and load on the secondary.
- **Over Excitation of Core in Fault Condition:** If a phase-to-ground fault occurs on the primary circuit with the primary neutral grounded, then the phase-to-neutral voltage on the un faulted phases increases to 173% of the normal voltage. This would almost certainly result in over excitation of the core, with greatly increased magnetizing currents and core losses
- If the neutrals of the primary and secondary are both brought out, then a phase-to-ground fault on the secondary circuit causes neutral fault current to flow in the primary circuit. Ground protection re- laying in the neutral of the primary circuit may then operate for faults on the secondary circuit
- **Neutral Shifting:** If the load on the secondary side unbalanced then the performance of this connection is not satisfactory then the shifting of neutral point is possible. To prevent this, star point of the primary is required to be connected to the star point of the generator.
- **Distortion of Secondary voltage:** Even though the star or neutral point of the primary is earthed, the third harmonic present in the alternator voltage may appear on the secondary side. This causes distortion in the secondary phase voltages.
- **Over Voltage at Light Load:** The presence of third (and other zero-sequence) harmonics at an ungrounded neutral can cause overvoltage conditions at light load.

- **Difficulty in coordination of Ground Protection:** In Y-Y Transformer, a low-side ground fault causes primary ground fault current, making coordination more difficult.
- **Increase Healthy Phase Voltage under Phase to ground Fault:** If a phase-to-ground fault occurs on the primary circuit with the primary neutral grounded, then the phase-to-neutral voltage on the UN faulted phase's increases to 173% of the normal voltage. If the neutrals of the primary and secondary are both brought out, then a phase-to-ground fault on the secondary circuit causes neutral fault current to flow in the primary circuit.
- **Trip the T/C in Line-Ground Fault:** All harmonics will propagate through the transformer, zero-sequence current path is continuous through the transformer, one line-to-ground fault will trip the transformer.
- **Suitable for Core Type Transformer:** The third harmonic voltage and current is absent in such type of connection with three phase wire system or shell type of three phase units, the third harmonic phase voltage may be high. This type of connection is more suitable for core type transformers.

***Application***

- This Type of Transformer is rarely used due to problems with unbalanced loads.
- It is economical for small high voltage transformers as the number of turns per phase and the amount of insulation required is less.

### 4.4.3 Star/Delta connection(Yd1/Yd11)

There is a +30 Degree or -30 Degree Phase Shift between Secondary Phase Voltage to Primary Phase Voltage. The connection of Yd1 is shown in fig. 4.14 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is -30°.

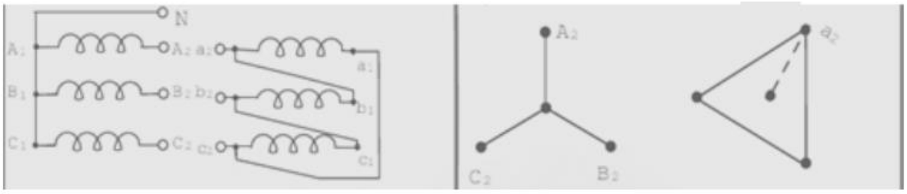


Fig 4.14. Yd1 connection and phasor diagram

The connection of Yd11 is shown in fig. 4.15 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 30°.

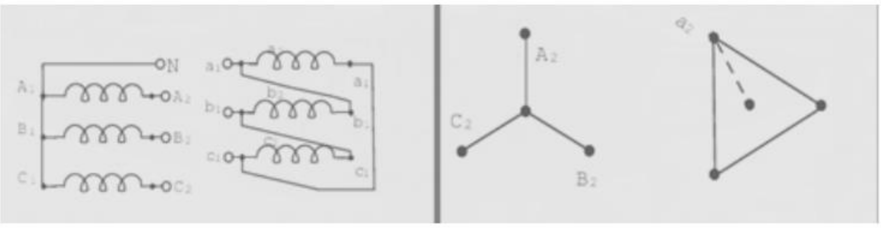


Fig 4.15. Yd11 connection and phasor diagram

### *Advantages*

- The primary side is star connected. Hence fewer numbers of turns are required. This makes the connection economical for large high voltage step down power transformers.
- The neutral available on the primary can be earthed to avoid distortion.
- The neutral point allows both types of loads (single phase or three phases) to be met.
- Large unbalanced loads can be handled satisfactory.
- The Y-D connection has no problem with third harmonic components due to circulating currents inD. It is also more stable to unbalanced loads since the D partially redistributes any imbalance that occurs.
- The delta connected winding carries third harmonic current due to which potential of neutral point is stabilized. Some saving in cost of insulation is achieved if HV side is star connected. But in practice the HV side is normally connected in delta so that the three phase loads like motors and single phase loads like lighting loads can be supplied by LV side using three phase

four wire system.

- **As Grounding Transformer:** In Power System Mostly grounded Y- Δ transformer is used for no other purpose than to provide a good ground source in ungrounded Delta system.

## *Disadvantages*

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.
- One problem associated with this connection is that the secondary voltage is shifted by $30^0$ with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts.
- If secondary of this transformer should be paralleled with secondary of another transformer without phase shift, there would be a problem

## *Application*

- It is commonly employed for power supply transformers.
- This type of connection is commonly employed at the substation end of the transmission line. The main use with this connection is to step down the voltage. The neutral available on the primary side is grounded. It can be seen that there is phase difference of 30° between primary and secondary line voltages.
- Commonly used in a step-down transformer, Y connection on the HV side reduces insulation costs the neutral point on the HV side can be grounded, stable with respect to unbalanced loads. As for example, at the end of a transmission line. The neutral of the primary winding is earthed. In this system, line voltage ratio is $1/\sqrt{3}$ Times of transformer turn-ratio and secondary voltage lags behind primary voltage by 30°. Also third harmonic currents flows in

to give a sinusoidal flux.

### 4.4.4 Delta-star connection (Dy1/Dy11)

In this type of connection, the primary connected in delta fashion while the secondary current is connected in star. There is s +30 Degree or -30 Degree Phase Shift between Secondary Phase Voltage to Primary Phase Voltage.

The connection of Dy1 is shown in fig. 4.16 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is -30°.

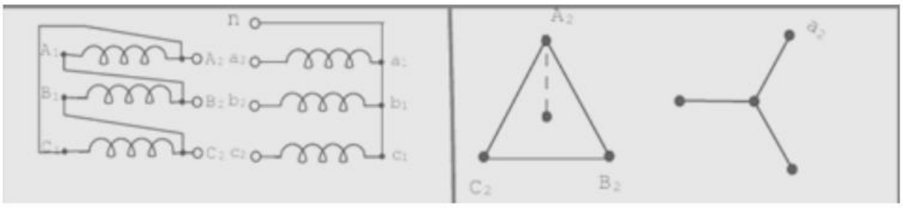


**Fig 4.16. Dy1 connection and phasor diagram**

The connection of Dy11 is shown in fig. 4.17 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 30°.

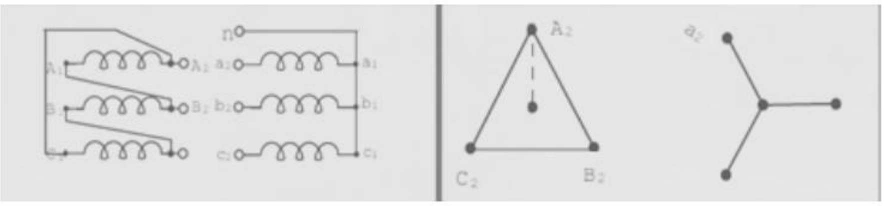


**Fig 4.17. Dy11 connection and phasor diagram**

***Advantages***

- **Cross section area of winding is less at Primary side**: On primary side due to delta connection winding cross-section required is less.

- **Used at Three phase four wire System:** On secondary side, neutral is available, due to which it can be used for 3-phase, 4 wire supply system.
- **No distortion of Secondary Voltage:** No distortion due to third harmonic components.
- **Handled large unbalanced Load:** Large unbalanced loads can be handled without any difficulty.
- **Grounding Isolation between Primary and Secondary:** Assuming that the neutral of the Y-connected secondary circuit is grounded, a load connected phase-to-neutral or a phase-to-ground fault produces two equal and opposite currents in two phases in the primary circuit without any neutral ground current in the primary circuit. Therefore, in contrast with the Y-Y connection, phase-to-ground faults or current unbalance in the secondary circuit will not affect ground protective relaying applied to the primary circuit. This feature enables proper coordination of protective devices and is a very important design consideration.
- The neutral of the Y grounded is sometimes referred to as a grounding bank, because it provides a local source of ground current at the secondary that is isolated from the primary circuit.
- **Harmonic Suppression:** The magnetizing current must contain odd harmonics for the induced voltages to be sinusoidal and the third harmonic is the dominant harmonic component. In a three-phase system the third harmonic currents of all three phases are in phase with each other because they are zero-sequence currents. In the Y-Y connection, the only path for third harmonic current is through the neutral. In the Δ -Y connection, however, the third harmonic currents, being equal in amplitude and in phase with each other, are able to circulate around the path formed by the Δ connected winding. The same thing is true for the other zero-sequence harmonics.
- **Grounding Bank:** It provides a local source of ground current at the secondary that is isolated from the primary circuit. For suppose an ungrounded generator supplies a simple radial system

through Δ-Y transformer with grounded Neutral at secondary as shown Figure. The generator can supply a single-phase-to-neutral load through the -grounded Y transformer.

### *Disadvantages*

- In this type of connection, the secondary voltage is not in phase with the primary. Hence it is not possible to operate this connection in parallel with star-star or delta-delta connected transformer.
- One problem associated with this connection is that the secondary voltage is shifted by $30^0$ with respect to the primary voltage. This can cause problems when paralleling 3-phase transformers since transformers secondary voltages must be in-phase to be paralleled. Therefore, we must pay attention to these shifts.
- If secondary of this transformer should be paralleled with secondary of another transformer without phase shift, there would be a problem.

### *Application*

- **Commonly used in a step-up transformer:** As for example, at the beginning of a HT transmission line. In this case neutral point is stable and will not float in case of unbalanced loading. There is no distortion of flux because existence of a Δ -connection allows a path for the third-harmonic components. The line voltage ratio is √3 times of transformer turn-ratio and the secondary voltage leads the primary one by 30°. In recent years, this arrangement has become very popular for distribution system as it provides 3- Ø, 4-wire system.
- **Commonly used in commercial, industrial, and high-density residential locations**: To supply three-phase distribution systems. An example would be a distribution transformer with a delta primary, running on three 11kV phases with no neutral or earth required, and a star (or wye) secondary providing a 3-phase supply at 400 V, with the domestic voltage of 230 available between each phase and an earthed neutral point.
- **Used as Generator Transformer:** The Δ-Y transformer connection is used universally for connecting generators to transmission systems.

## Delta-zigzag and Star zigzag connections (Dz0/Dz6 & Yz1/Yz6) –

The connection of Dz0 is shown in fig. 4.18 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 0°.

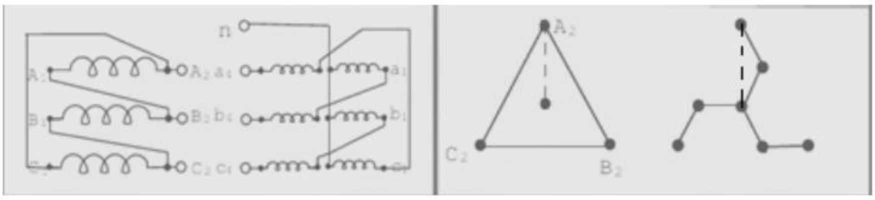


Fig 4.18. Dz0 connection and phasor diagram

The connection of Dz6 is shown in fig. 4.19 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 180°.

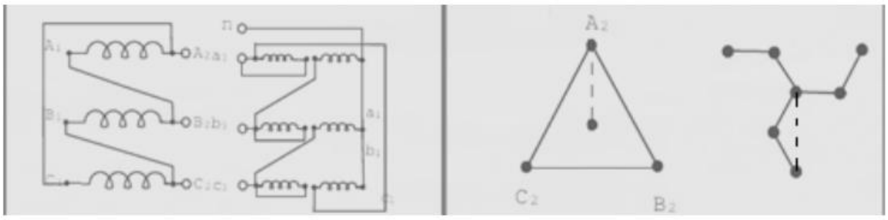


Fig 4.19. Dz6 connection and phasor diagram

The connection of Yz1 is shown in fig. 4.20 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is -30°.

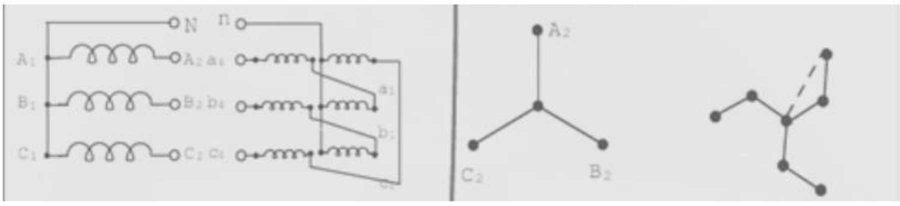


Fig 4.20. Yz1 connection and phasor diagram

The connection of Yz11 is shown in fig. 4.21 and the voltages on primary and secondary sides is also shown on the phasor diagram. The phase angle difference between the phase voltage of high voltage side and low voltage side is 30°.

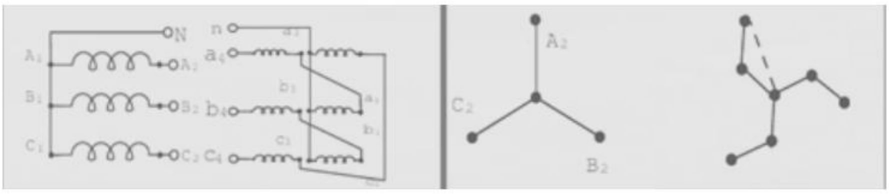


**Fig 4.22 Yz11 connection and phasor diagram**

- These connections are employed where delta connections are weak. Interconnection of phases in zigzag winding effects a reduction of third harmonic voltages and at the same time permits unbalanced loading.
- This connection may be used with either delta connected or star connected winding either for step-up or step-down transformers. In either case, the zigzag winding produces the same angular displacement as a delta winding, and at the same time provides a neutral for earthing purposes.
- The amount of copper required from a zigzag winding in 15% more than a corresponding star or delta winding. This is extensively used for earthing transformer.
- Due to **zigzag** connection (interconnection between phases), third harmonic voltages are reduced. It also allows unbalanced loading. The zigzag connection is employed for LV winding. For a given total voltage per phase, the zigzag side requires 15% more turns as compared to normal phase connection. In cases where delta connections are weak due to large number of turns and small cross sections, then zigzag star connection is preferred. It is also used in rectifiers.

## 4.5 Scott connection

There are two main reasons for the need to transform from three phases to two phases,

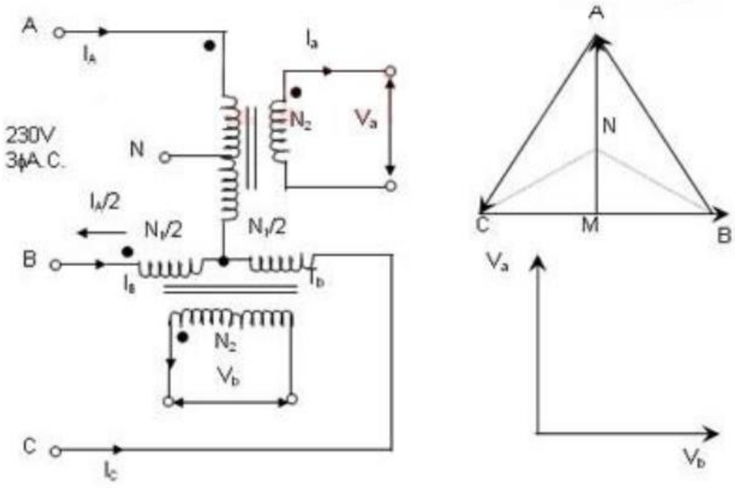
1. To give a supply to an existing two phase system from a three phase supply.

2. To supply two phase furnace transformers from a three phase source.

Two-phase systems can have 3-wire, 4-wire, or 5-wire circuits. It is needed to be considering that a two-phase system is not 2/3 of a three-phase system. Balanced three-wire, two-phase circuits have two phase wires, both carrying approximately the same amount of current, with a neutral wire carrying 1.414 times the currents in the phase wires. The phase-to-neutral voltages are 90° out of phase with each other.

Two phase 4-wire circuits are essentially just two ungrounded single-phase circuits that are electrically 90° out of phase with each other. Two phase 5-wire circuits have four phase wires plus a neutral; the four phase wires are 90° out of phase with each other.

A Scott-T transformer (also called a Scott connection) is a type of circuit used to derive two-phase power from a three-phase source or vice-versa. The Scott connection evenly distributes a balanced load between the phases of the source. Scott T Transformers require a three phase power input and provide two equal single phase outputs called Main and Teaser. The MAIN and Teaser outputs are 90 degrees out of phase. The MAIN and the Teaser outputs must not be connected in parallel or in series as it creates a vector current imbalance on the primary side. MAIN and Teaser outputs are on separate cores. An external jumper is also required to connect the primary side of the MAIN and Teaser sections. The schematic of a typical Scott T Transformer is shown below:



**4.23 Connection diagram of Scott-connected transformer and vector relation of input and output**

From the phasor diagram it is clear that the secondary voltages are of two phases with equal magnitude and 90° phase displacement.

Scott T Transformer is built with two single phase transformers of equal power rating. Assuming the desired voltage is the same on the two and three phase sides, the Scott-T transformer connection consists of a center-tapped 1:1 ratio main transformer, T1, and an 86.6% (0.5√3) ratio teaser transformer, T2. The center-tapped side of T1 is connected between two of the phases on the three-phase side. Its center tap then connects to one end of the lower turn count side of T2, the other end connects to the remaining phase. The other side of the transformers then connects directly to the two pairs of a two-phase four-wire system.

If the main transformer has a turn's ratio of 1: 1, then the teaser transformer requires a turn's ratio of 0.866: 1 for balanced operation. The principle of operation of the Scott connection can be most easily seen by first applying a current to the teaser secondary windings, and then applying a current to the main secondary winding, calculating the primary currents separately and superimposing the results.

The primary three-phase currents are balanced; i.e., the phase currents have the same magnitude and their phase angles are 120° apart. The apparent power supplied by the main transformer is greater than the apparent power supplied by the teaser transformer. This is easily verified by observing that the primary currents in both transformers have the same magnitude; however, the primary voltage of the teaser transformer is only 86.6% as great as the primary voltage of the main transformer. Therefore, the teaser transforms only 86.6% of the apparent power transformed by the main.

- The total real power delivered to the two phase load is equal to the total real power supplied from the three-phase system, the total apparent power transformed by both transformers is greater than the total apparent power delivered to the two-phase load.
- The apparent power transformed by the teaser is 0.866 X IH1= 1.0 and the apparent power transformed by the main is 1.0X IH2 =1.1547 for a total of 2.1547 of apparent power transformed.
- The additional 0.1547 per unit of apparent power is due to parasitic reactive power owing between the two halves of the primary winding in the main transformer.
- Single-phase transformers used in the Scott connection are specialty items that are virtually impossible to buy ‘‘off the shelf ’’ nowadays. In an emergency, standard distribution transformers can be used.

If desired, a three phase, two phase, or single phase load may be supplied simultaneously using scott-connection. The neutral points can be available for grounding or loading purposes. The Scott T connection in theory would be suitable for supplying a three, two and single phase load simultaneously, but such loads are not found together in modern practice.

### *The Scott T would not be recommended as a connection for 3 phase to 3 phase applications for the following reasons:*

The loads of modern buildings and office buildings are inherently unbalanced and contain equipment that can be sensitive to potential voltage fluctuations that may be caused by the Scott T design.

A properly sized Scott T transformer will have to be a minimum of 7.75% larger than the equivalent Delta-Wye transformer. Properly sized, it would be a bulkier and heavier option and should not be considered a less expensive solution.

## 4.6 Open Delta or V-Connection

As seen previously in connection of three single phase transformers that if one of the transformers is unable to operate then the supply to the load can be continued with the remaining two transformers at the cost of reduced efficiency. The connection that obtained is called V-V connection or open delta connection.

Consider the Fig. 4.24 in which 3 phase supply is connected to the primaries. At the secondary side three equal three phase voltages will be available on no load.

The voltages are shown on phasor diagram. The connection is used when the three phase load is very very small to warrant the installation of full three phase transformer.

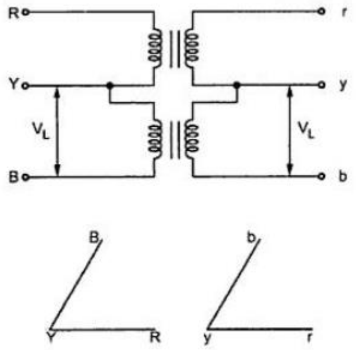


**Fig. 4.24 Open delta connection of transformer at noload**

If one of the transformers fails in Δ - Δ bank and if it is required to continue the supply even though at reduced capacity until the transformer which is removed from the bank is repaired or a new one is installed then this type of connection is most suitable.

When it is anticipated that in future the load increase, then it requires closing of open delta. In such cases open delta connection is preferred. It can be noted here that the removal of one of the transformers will not give the total load carried by V - V bank as two third of the capacity of Δ - Δ bank.

The load that can be carried by V - V bank is only 57.7% of it.

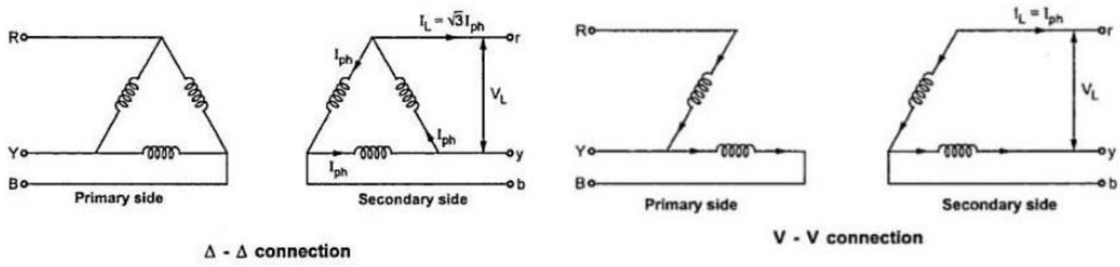


**Fig. 4.25 Delta-delta and V-V connection**

It can be seen from the Fig. 4.25 of delta delta connection that

$$\Delta - \Delta \text{ capacity} = \sqrt{3}\, V_L I_L = \sqrt{3}\, V_L (\sqrt{3}\, I_{ph})$$
$$\Delta - \Delta \text{ capacity} = 3\, V_L I_{ph}$$

It can also be noted from the Fig. 4.25 V-V connection that the secondary line current $I_L$ is equal to the phase current $I_{ph}$.

$$\text{V- V capacity} = \sqrt{3}\, V_L I_L = \sqrt{3}\, V_L I_{ph}$$
$$\text{So, } \frac{\text{V-V capacity}}{\Delta - \Delta \text{ capacity}} = \frac{\sqrt{3} V_L I_{ph}}{3 V_L I_{ph}} = \frac{1}{\sqrt{3}} = 0.577 \simeq 58\%$$

Thus the three phase load that can be carried without exceeding the ratings of the transformers is 57.5 percent of the original load. Hence it is not 66.7 % which was expected otherwise.

The reduction in the rating can be calculated as {(66.67 - 57.735)/(57.735)}x 100 = 15.476

Suppose that we consider three transformers connected in Δ - Δ fashion and supplying their rated load. Now one transformer is removed then each of the remaining two transformers will be overloaded. The overload on each transformer will be given as,

$$\frac{\text{Total load in V-V}}{\text{VA rating of each transformer}} = \frac{\sqrt{3} V_L I_{ph}}{V_L I_{ph}} = \sqrt{3} = 1.732$$

This overload can be carried temporarily if provision is made to reduce the load otherwise overheating and breakdown of the remaining two transformers would take place.

- <u>The limitation with V -V connection are given below :</u>

The average p.f. at which V- V bank is operating is less than that with the load . This power p.f is 86.6 % of the balanced load p.f.

- The two transformers in V -V bank operate at different power factor except for balanced unity p.f .load.
- The terminals voltages available on the secondary side become unbalanced. This may happen even though load is perfectly balanced.
- Thus in summary we can say that if tow transformers are connected in V - V fashion and are loaded to rated capacity and one transformer is added to increase the total capacity by √3 or 173.2 %. Thus the increase in capacity is 73.2 % when converting from a V - V system to a Δ-Δ system.
- With a bank of tow single phase transformers connected in V-V fashion supplying a balanced 3 phase load with cosΦ asp.f., one of the transformer operate at a p.f. of cos (30-Φ) and other at cos (30+Φ). The powers of tow transformers are given by,

$$P_1 = KVA \cos(30-\Phi)$$

$$P_2 = KVA \cos(30+\Phi)$$

## **4.7Oscillating Neutral**

In addition to the operation of transformers on the sinusoidal supplies, the harmonic behavior becomes important as the size and rating of the transformer increases. The effects of the harmonic currents are

1. Additional copper losses due to harmonic currents
2. Increased core losses
3. Increased electro-magnetic interference with communication circuits.

On the other hand the harmonic voltages of the transformer cause

1. Increased dielectric stress on insulation
2. Electro static interference with communication circuits.

3. Resonance between winding reactance and feeder capacitance.

In the present times a greater awareness is generated by the problems of harmonic voltages and currents produced by non-linear loads like the power electronic converters. These combine with non-linear nature of transformer core and produce severe distortions in voltages and currents and increase the power loss. Thus the study of harmonics is of great practical significance in the operation of transformers.

In the case of single phase transformers connected to form three phase bank, each transformer is magnetically decoupled from the other. The flow of harmonic currents are decided by the type of the electrical connection used on the primary and secondary sides. Also, there are three fundamental voltages in the present case each displaced from the other by 120 electrical degrees. Because of the symmetry of the a.c. wave about the time axis only odd harmonics need to be considered. The harmonics which are triplen (multiples of three) behave in a similar manner as they are co-phasal or in phase in the three phases. The non-triplen harmonics behave in a similar manner to the fundamental and have ±120° phase displacement between them.

When the connection of the transformer is Yy without neutral wires both primary and secondary connected in star no closed path exists. As the triplen harmonics are always in phase, by virtue of the Y connection they get canceled in the line voltages. Non-triplen harmonics like fundamental, become 0 times phase value and appear in the line voltages. Line currents remain sinusoidal except for non-triplen harmonic currents. Flux wave in each transformer will be flat topped and the phase voltages remain peaked. The potential of the neutral is no longer steady. The star point oscillates due to the third harmonic voltages. This is termed as "oscillating neutral".

## 4.8 Tertiary winding

Apart from the Primary & Secondary windings, there sometimes placed a third winding in power transformers called "Tertiary Winding". Its purpose is to provide a circulating path for the harmonics (especially third harmonics) produced in the transformers along with power frequency (50Hz. third harmonic means 150 Hz oscillations). In delta-delta, delta-star and star-delta transformers all voltages are balanced and there is no floating of neutral or oscillating neutral. The floating of neutral is developed in the case star-star connection only. The transformers are sometimes constructed with three windings. The main windings are connected to form star-star connection and the third winding known as tertiary winding is used to make a closed delta connection to stabilize the neutrals of both primary and secondary circuits. The tertiary winding carries the third-harmonic currents.

## 4.9 Three Winding Transformers

Thus far we have looked at transformers which have one single primary winding and one single secondary winding. But the beauty of transformers is that they allow us to have more than just one winding in either the primary or secondary side. Transformers which have three winding are known commonly as **Three Winding Transformers**.

The principal of operation of a *three winding transformer* is no different from that of an ordinary transformer. Primary and secondary voltages, currents and turns ratios are all calculated the same, the difference this time is that we need to pay special attention to the voltage polarities of each coil winding, the dot convention marking the positive (or negative) polarity of the winding, when we connect them together.

Three winding transformers, also known as a three-coil, or three-winding transformer, contain one primary and two secondary coils on a common laminated core. They can be either a single-phase transformer or a three-phase transformer, (three-winding, three-phase transformer) the operation is the same.

**Three Winding Transformers** can also be used to provide either a step-up, a step-down, or a combination of both between the various windings. In fact a three winding transformers have two secondary windings on the same core with each one providing a different voltage or current level output.

As transformers operate on the principal of mutual induction, each individual winding of a three winding transformer supports the same number of volts per turn, therefore the volt-ampere product in each winding is the same, that is $N_P/N_S = V_P/V_S$ with any turns ratio between the individual coil windings being relative to the primary supply.

In electronic circuits, one transformer is often used to supply a variety of lower voltage levels for different components in the electronic circuitry. A typical application of three winding transformers is in power supplies and Triac Switching Converters. So a transformer have two secondary windings, each of which is electrically isolated from the others, just as it is electrically isolated from the primary. Then each of the secondary coils will produce a voltage that is proportional to its number of coil turns.

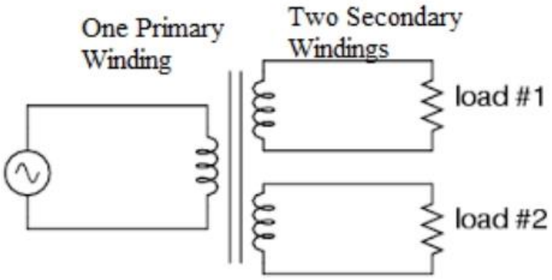


**Fig. 4.27 A three winding transformer**

The secondary windings can be connected together in various configurations producing a higher voltage or current supply. It must be noted that connecting together transformer windings is only possible if the two windings are electrically identical. That is their current and voltage ratings are the same.

## 4.10 Parallel operation of three phase transformer

### 4.10.1Advantages of using transformers in parallel

1. To maximize electrical power system efficiency: Generally electrical power transformer gives the maximum efficiency at full load. If we run numbers of transformers in parallel, we can switch on only those transformers which will give the total demand by running nearer to its full load rating for that time. When load increases, we can switch none by one other transformer connected in parallel to fulfill the total demand. In this way we can run the system

with maximum efficiency.

2. To maximize electrical power system availability: If numbers of transformers run in parallel, we can shut down any one of them for maintenance purpose. Other parallel transformers in system will serve the load without total interruption of power.
3. To maximize power system reliability: If any one of the transformers run in parallel, is tripped due to fault of other parallel transformers is the system will share the load, hence power supply may not be interrupted if the shared loads do not make other transformers over loaded.
4. To maximize electrical power system flexibility: There is always a chance of increasing or decreasing future demand of power system. If it is predicted that power demand will be increased in future, there must be a provision of connecting transformers in system in parallel to fulfill the extra demand because, it is not economical from business point of view to install a bigger rated single transformer by forecasting the increased future demand as it is unnecessary investment of money. Again if future demand is decreased, transformers running in parallel can be removed from system to balance the capital investment and its return.

### 4.10.2 Conditions for parallel operation

Certain conditions have to be met before two or more transformers are connected in parallel and share a common load satisfactorily. They are,

1. The voltage ratio must be the same.
2. The per unit impedance of each machine on its own base must be the same.
3. The polarity must be the same, so that there is no circulating current between the transformers.
4. The phase sequence must be the same and no phase difference must exist between the voltages of the two transformers.

- **Same voltage ratio :** Generally the turns ratio and voltage ratio are taken to be the same. If the ratio is large there can be considerable error in the voltages even if the turns ratios are the same. When the primaries are connected to same bus bars, if the secondaries do not show the same voltage, paralleling them would result in a circulating current between the secondaries. Reflected circulating current will be there on the primary side also. Thus even without connecting a load considerable current can be drawn by the transformers and they produce copper losses. In two identical transformers with percentage impedance of 5 percent, a no-load voltage difference of one percent will result in a circulating current of 10 percent of full load current. This circulating current gets added to the load current when the load is connected resulting in unequal sharing of the load. In such cases the combined full load of the two transformers can never be met without one transformer getting overloaded.

- **Per unit impedance:** Transformers of different ratings may be required to operate in parallel. If they have to share the total load in proportion to their ratings the larger machine has to draw more current. The voltage drop across each machine has to be the same by virtue of their connection at the input and the output ends. Thus the larger machines have smaller impedance and smaller machines must have larger ohmic impedance. Thus the impedances must be in the inverse ratios of the ratings. As the voltage drops must be the same the per unit impedance of each transformer on its own base, must be equal. In addition if active and reactive power are required to be shared in proportion to the ratings the impedance angles also must be the same. Thus we have the requirement that per unit resistance and per unit reactance of both the transformers must be the same for proper load sharing.
- **Polarity of connection:** The polarity of connection in the case of single phase transformers can be either same or opposite. Inside the loop formed by the two secondaries the resulting voltage must be zero. If wrong polarity is chosen the two voltages get added and short circuit results. In the case of polyphase banks it is possible to have permanent phase error between the phases with substantial circulating current. Such transformer banks must not be connected in parallel. The turns ratios in such groups can be adjusted to give very close voltage ratios but phase errors cannot be compensated. Phase error of 0.6 degree gives rise to one percent difference in voltage. Hence poly phase transformers belonging to the same vector group alone

must be taken for paralleling.

Transformers having —30° angle can be paralleled to that having +30° angle by reversing the phase sequence of both primary and secondary terminals of one of the transformers. This way one can overcome the problem of the phase angle error.

- **Phase sequence**- The phase sequence of operation becomes relevant only in the case of poly phase systems. The poly phase banks belonging to same vector group can be connected in parallel. A transformer with +30° phase angle however can be paralleled with the one with —30° phase angle, the phase sequence is reversed for one of them both at primary and secondary terminals. If the phase sequences are not the same then the two transformers cannot be connected in parallel even if they belong to same vector group. The phase sequence can be found out by the use of a phase sequence indicator.

## 4.11 Load Sharing

When the transformers have equal voltage ratios, the magnitudes of secondary no-load voltages are equal. Further if the primary leakage impedance drops due to exciting currents are also equal, then $\overline{E}_a = \overline{E}_b$ and the circulating current at no load is zero.

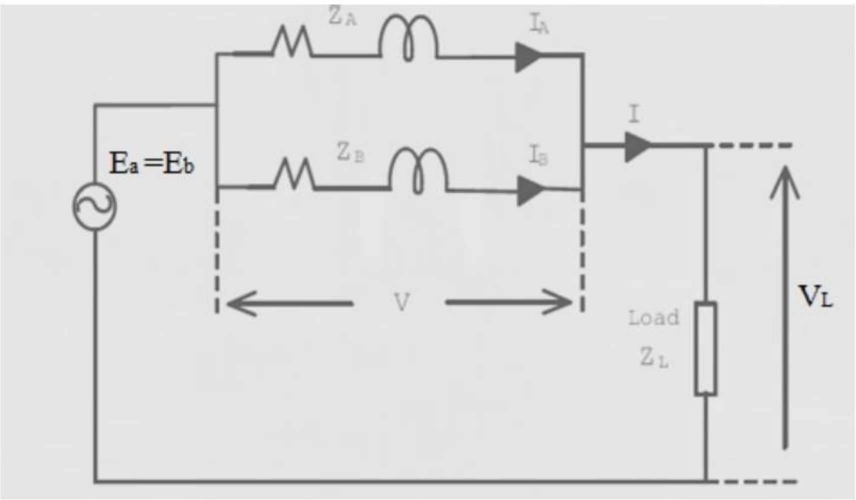


**Fig. 4.28 Circuit modelling of two transformer in parallel**

The equivalent circuit of two three phase transformer connected in parallel connected with a load of

$Z_L$ impedance on per phase basis is drawn in fig 4.28. In this figure transformer A and B are operating in parallel. $I_A$ and $I_B$ are the load current of the two transformer.

The voltage equation of transformer A is

$$\overline{E}_a - \overline{I}_a\overline{Z}_a = \overline{V}_L = \overline{I}\,\overline{Z}_L$$

Since $\overline{E}_a = \overline{E}_b$; $\overline{E}_b - \overline{I}_a\overline{Z}_a = \overline{V}_L = \overline{I}\,\overline{Z}_L$

The voltage equation of transformer B is

$$\overline{E}_b - \overline{I}_b\overline{Z}_b = \overline{V}_L = \overline{I}\,\overline{Z}_L$$

$$\overline{E}_b - \overline{I}_a\overline{Z}_a = \overline{E}_b - \overline{I}_b\overline{Z}_b$$

$$\overline{I}_a\overline{Z}_a = \overline{I}_b\overline{Z}_b$$

According to the voltage drops across the two equivalent leakage impedance Za and Zb are equal.

According to KCL we can write

$$\overline{I} = \overline{I}_a + \overline{I}_b = \overline{I}_a + \frac{\overline{I}_a\overline{Z}_a}{\overline{Z}_b}$$

$$\overline{I}_a = \overline{I}\frac{\overline{Z}_b}{\overline{Z}_a + \overline{Z}_b}$$

similarly, $\overline{I}_b = \overline{I}\dfrac{\overline{Z}_a}{\overline{Z}_a + \overline{Z}_b}$

Multiplying both the current equations by terminal voltage we get,

$$\overline{S}_a = \overline{S}\frac{\overline{Z}_b}{\overline{Z}_a + \overline{Z}_b}$$

similarly, $\overline{S}_b = \overline{S}\dfrac{\overline{Z}_a}{\overline{Z}_a + \overline{Z}_b}$

Thus the power sharing in between two transformer is given in above equation in VA rating.

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However apart from this lecture note students/readers are strongly recommended to follow the below mentioned books in the references and above all confer with the faculty for thorough knowledge of this authoritative subject of electrical engineering.

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**Best of Luck to All the Students**