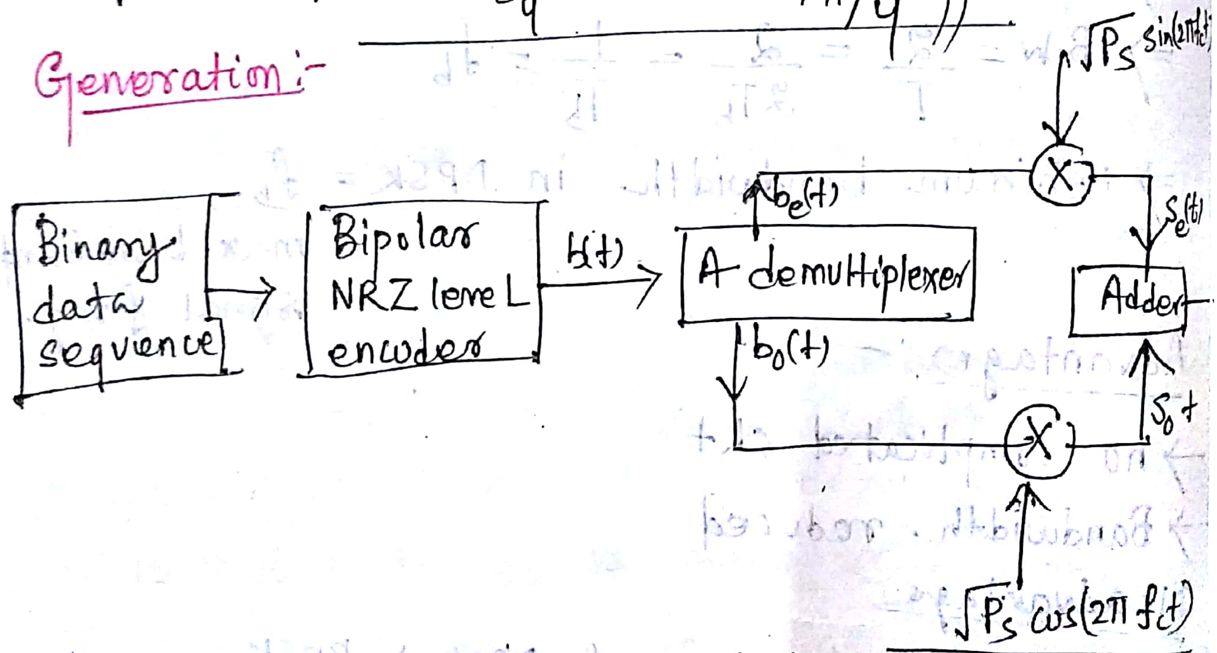
QPSK (Quadrature PSK):-

→ channel bandwidth depends upon bit rate or signalling rate 'f_b'.

→ In QPSK, two successive bits in data sequence are grouped together. This reduces bit rate or signalling rate & reduces bandwidth of channel.

I/P	Bits	symbol	phase shift in carrier
1	0	S_1	$\pi/4$
0	0	S_2	$3\pi/4$
0	1	S_3	$5\pi/4$
1	1	S_4	$7\pi/4$

Generation:-



→ Binary sequence is converted to bipolar type of signal. This signal is denoted by $b(t)$. representing binary '1' by $+1$ V & binary '0' by -1 V

→ Demultiplexer divided $b(t)$ into 2 separate bit streams of odd numbered & even numbered bits. sequences. ($b_o(t)$) ($b_e(t)$)

→ Even numbered bit sequence $b_e(t)$ starts with delay of one bit period 'T_b' w.r.t. 1st symbol of $b_o(t)$. This delay of 'T_b' is known as 'offset'. Hence modulation is known as offset QPSK.

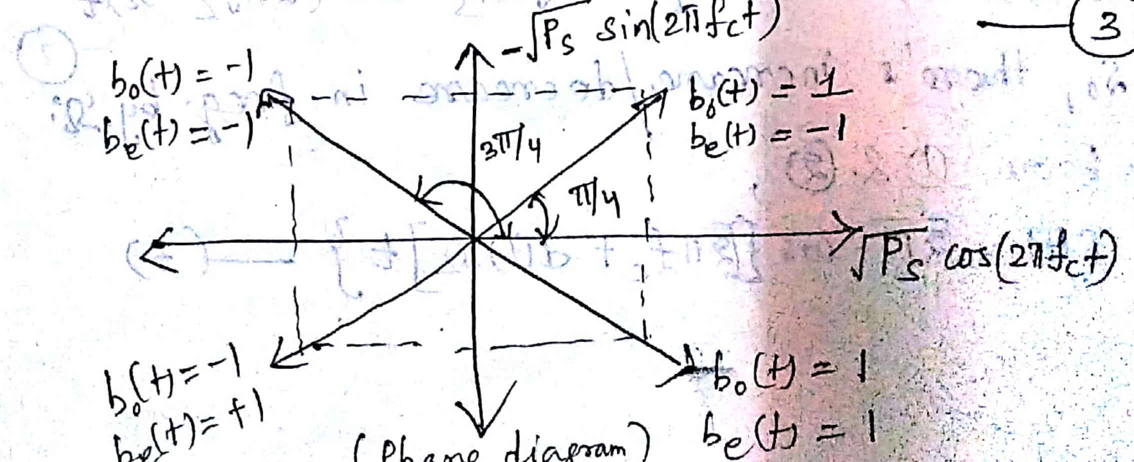
→ Bit stream $b_o(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_c t)$ & $b_e(t)$ modulates $\sqrt{P_s}\sin(2\pi f_c t)$. These carriers are quadrature carriers.

$$\begin{cases} s_e(t) = b_e(t)\cdot\sqrt{P_s}\cdot\sin(2\pi f_c t) & \text{—— (1)} \\ s_o(t) = b_o(t)\cdot\sqrt{P_s}\cdot\cos(2\pi f_c t) & \text{—— (2)} \end{cases}$$

QPSK signal → $s(t)$

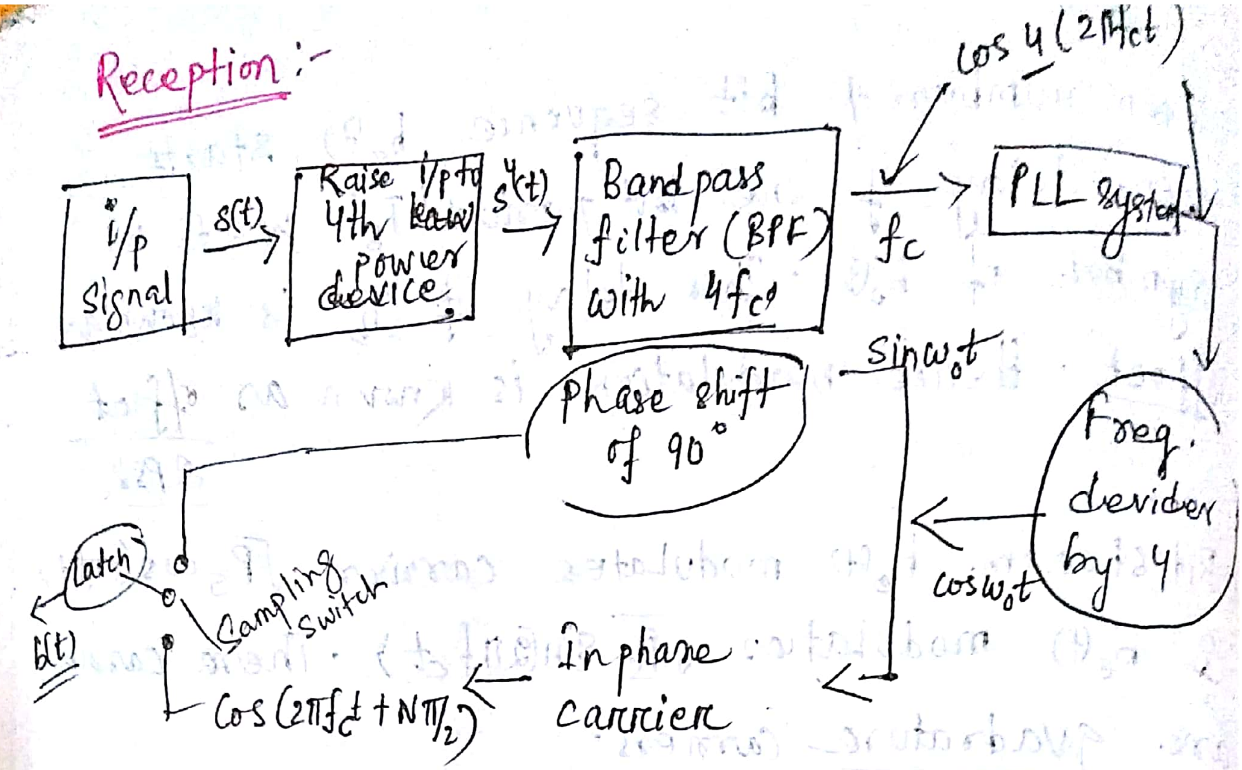
→ Adder adds two signals $b_e(t)$ & $b_o(t)$.
O/p of adder is QPSK signal.

$$s(t) = s_o(t) + s_e(t)$$
$$= b_o(t)\cdot\sqrt{P_s}\cos(2\pi f_c t) + b_e(t)\cdot\sqrt{P_s}\sin(2\pi f_c t) \quad \text{—— (3)}$$



(Phase diagram)

Reception :-



→ Recieved signal $s(t)$ is raised by 4 th power ie $s^4(t)$. After that, it's allowed to pass through BPF which is entered around around $4f_c$.

→ $\boxed{B.W = f_b}$

Binary Freq. Shift Keying (BFSK):-

→ Freq. of carrier is shifted according to binary symbol. But phase of carrier is unaffected.

→ Let there be freq. shift by 'Ω'.

if $b(t) = 1$, then $S_H(t) = \sqrt{2P_s} \cdot \cos(2\pi f_c + \Omega)t$ —①

$b(t) = 0$, then $S_L(t) = \sqrt{2P_s} \cdot \cos(2\pi f_c - \Omega)t$ —②

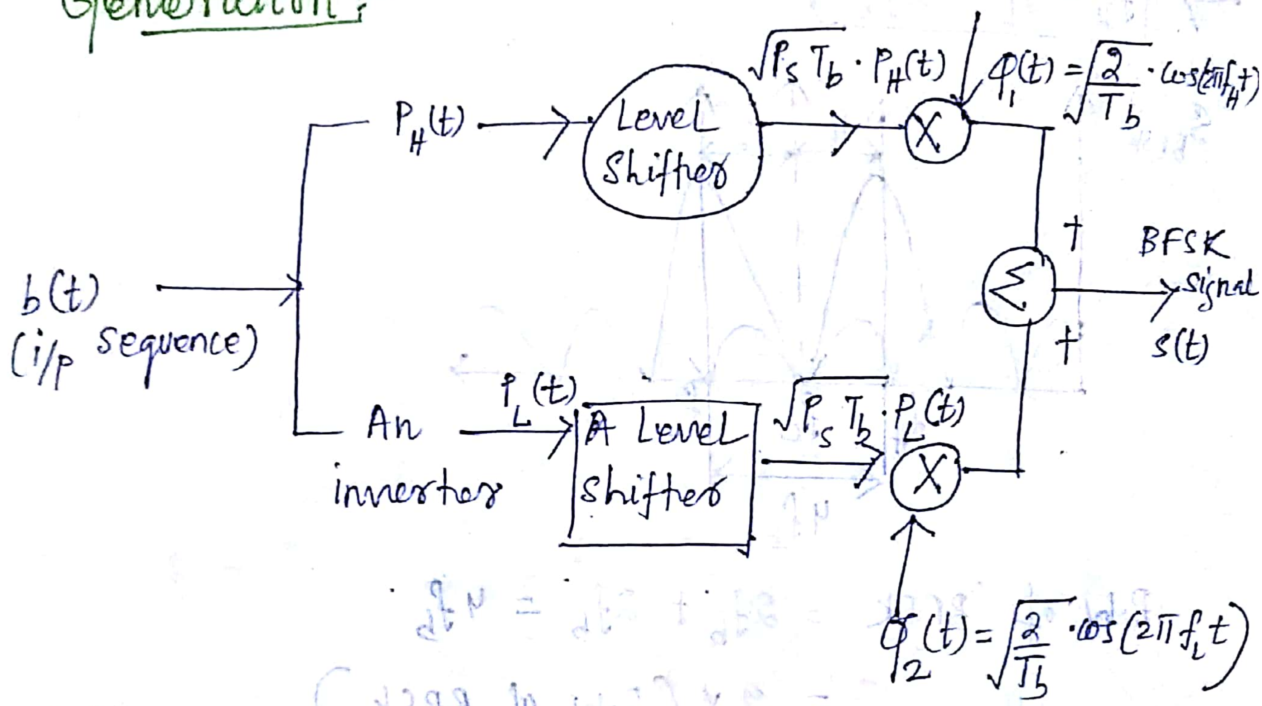
So, there's increase/decrease in freq. by 'Ω'.

→ From ① & ②,

$$s(t) = \sqrt{2P_s} \cdot \cos\{[2\pi f_c + d(t)\,\Omega]t\} \quad \text{—(3)}$$

→ If symbol '1' is to be transmitted, carrier freq. will be '$f_c + \left(\frac{\Omega}{2\pi}\right)$' $= f_H$, otherwise f_c will be '$f_c - \left(\frac{\Omega}{2\pi}\right)$' $= f_L$

Generation :-



→ Two carrier signals are used $\phi_1(t)$ & $\phi_2(t)$, two carrier signals are orthogonal to each other.

Spectrum :-

→ BFSK signal can be represented as,

$$s(t) = \sqrt{2P_s} \cdot P_H(t) \cdot \cos(2\pi f_H t) + \sqrt{2P_s} \cdot P_L(t) \cdot \cos(2\pi f_L t) \quad -(1)$$

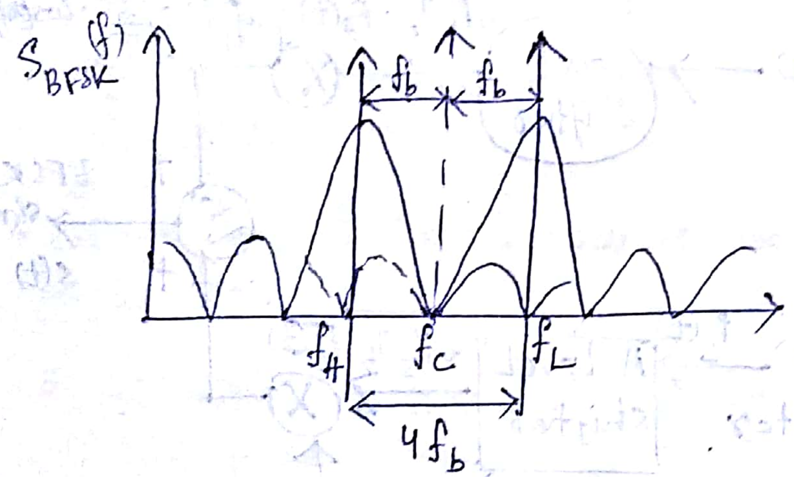
→ $P_H(t) = \frac{1}{2} + \frac{1}{2} P_H'(t)$

$P_L(t) = \frac{1}{2} + \frac{1}{2} P_L'(t)$ $\left\{ P_H'(t), P_L'(t) \text{ are bipolar } (+1) \text{ or } (-1) \right.$

→ PSD of B·F·S·K as,

$$S(f) = \sqrt{\frac{P_S}{2}}\left[\delta(f - f_H) + \delta(f + f_L) + \frac{P_S T_b}{2}\left\{\frac{\sin(\pi f_H T_b)}{\pi f_H T_b}\right\}^2 + \frac{P_S T_b}{2}\left\{\frac{\sin(\pi \cdot f_L \cdot T_b)}{\pi f_L \cdot T_b}\right\}^2\right]$$

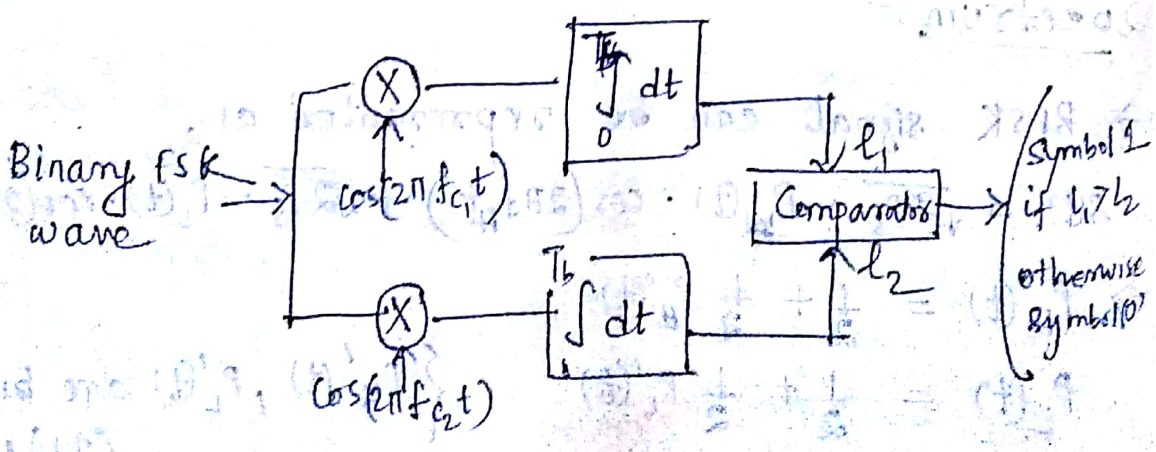
→ $\boxed{f_H - f_L = 2f_b}$



B·W of BFSK $= 2f_b + 2f_b = 4f_b$

" $= 2 \times (\text{B·W of BPSK})$

Detection:-



Advantage:-	Disadvantage
Generation of BFSK is easy	B·W is greater, ~~distance~~ Error rate is more.

MSK (Minimum Shift Keying):- (QFSK)

→ In MSK, o/p waveform is continuous in phase, so there's no abrupt changes in amplitude. Sidelobes of MSK are very small, hence band pass filter (BPF) is not required to avoid interchannel interference.

→ Transmitted MSK signal is,

$$s(t) = \sqrt{2P_s}\left[b_e(t)\cdot\sin\left(2\pi t/4T_b\right)\right]\cdot\cos\left(2\pi f_c t\right) + \sqrt{2P_s}\left[b_o(t)\cdot\cos\left(2\pi t/4T_b\right)\right]\cdot\sin\left(2\pi f_c t\right) \quad \text{--- (1)}$$

$$\Rightarrow s(t) = \sqrt{2P_s}\left[\frac{b_o(t)+b_e(t)}{2}\right]\cdot\sin 2\pi\left[f_c + \frac{1}{4T_b}\right]t + \sqrt{2P_s}\cdot\left[\frac{b_o(t)-b_e(t)}{2}\right]\cdot\sin 2\pi\left[f_c - \frac{1}{4T_b}\right]\cdot t \quad \text{--- (2)}$$

$$= \sqrt{2P_s}\cdot\left[\frac{b_o(t)+b_e(t)}{2}\right]\cdot\sin 2\pi\left(f_c + \frac{f_b}{4}\right)t + \sqrt{2P_s}\left[\frac{b_o(t)-b_e(t)}{2}\right]\cdot\sin 2\pi\left(f_c - \frac{f_b}{4}\right)t \quad \text{--- (3)}$$

Let $\frac{b_o(t)+b_e(t)}{2} = C_H(t)$, $\left(f_c + \frac{f_b}{4}\right) = f_H$

$\frac{b_o(t)-b_e(t)}{2} = C_L(t)$, $\left(f_c - \frac{f_b}{4}\right) = f_L$ --- (4)

$$\boxed{s(t) = \sqrt{2P_s}\cdot C_H(t)\cdot\sin\left(2\pi f_H t\right) + \sqrt{2P_s}\cdot C_L(t)\cdot\sin(2\pi f_L t)} \quad \text{--- (5)}$$

→ Frequencies 'f_H' & 'f_L' are chosen such that $\sin(2\pi f_H t)$ & $\sin(2\pi f_L t)$ are orthogonal over interval period (T_b).

Scanned by CamScanner

$$\int_0^{T_b} \sin(2\pi f_H t) \cdot \sin(2\pi f_L t) \cdot dt = 0 \quad \text{— (6)}$$

→ This relⁿ is satisfied if we have integers 'm' & 'n'

where
$$\begin{cases} 2\pi (f_H - f_L) \cdot T_b = n\pi \\ 2\pi (f_H + f_L) \cdot T_b = m\pi \end{cases} \quad \text{—— (7)}$$

→ $2\pi (f_H - f_L) \cdot T_b = n\pi$

$$\Rightarrow 2\pi \left(\left\{ f_c + \frac{f_b}{4} \right\} - \left\{ f_c - \frac{f_b}{4} \right\} \right) \cdot T_b = n\pi$$

$$\Rightarrow 2\pi \left(\frac{f_b}{2} \right) \cdot T_b = n\pi$$

$$\Rightarrow f_b \cdot T_b = n$$

$$\Rightarrow f_b \cdot \frac{1}{f_b} = n \Rightarrow \boxed{n = 1} \quad \text{—— (8)}$$

→ Again, $2\pi (f_H + f_L) \cdot T_b = m\pi$

$$\Rightarrow 2\pi \left(\left\{ f_c + \frac{f_b}{4} \right\} + \left\{ f_c - \frac{f_b}{4} \right\} \right) \cdot T_b = m\pi$$

$$\Rightarrow 2\pi (2f_c) \cdot T_b = m\pi$$

$$\Rightarrow 4 f_c \cdot T_b = m$$

$$\Rightarrow 4 f_c \cdot \frac{1}{f_b} = m \Rightarrow \left(f_c = \frac{m}{4} \cdot f_b \right) \text{—— (9)}$$

When $\boxed{n = 1}$,

$2\pi (f_H - f_L) \cdot T_b = 1 \cdot (\pi)$ [from eqⁿ (7)]

$$\Rightarrow \boxed{(f_H - f_L) = \frac{1}{2T_b} = \frac{f_b}{2}} \quad \text{—— (10)}$$

* (Here $n = 1$) means, difference betⁿ 'f_H' & 'f_L' is minimum & at same time they are orthogonal. This technique is called MSK

Spectrum :-

→ Base band signal of eqⁿ modulating $\sin(2\pi f_c t)$

$$p(t) = \sqrt{2P_s}\cdot\left[b_0(t)\cdot\cos\left(2\pi t/4T_b\right)\right]$$

$$= \sqrt{2P_s}\, b_0(t)\cdot\cos\left(\pi f_b t/2\right) \quad \text{—— (1)}$$

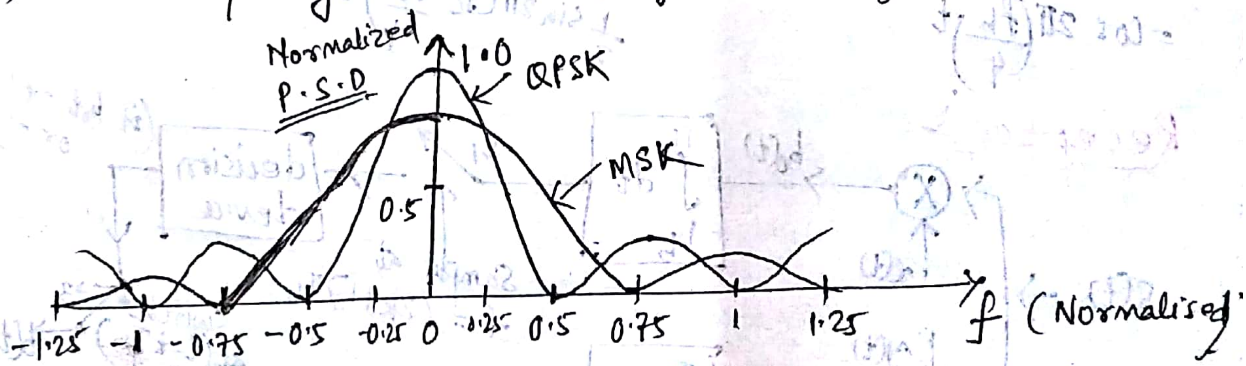
→ power spectral density of above eqⁿ is

$$S_p(f) = \frac{32E_b}{\pi^2}\left[\frac{\cos(2\pi f T_b)}{1-(4fT_b)^2}\right]^2 \quad \text{—— (2)}$$

→ When signal modulates carrier 'f_c', total power spectral density (P.S.D) of baseband signal is divided by '4' & placed at $\pm f_c$ i.e

$$S(f) = \frac{8E_b}{\pi^2}\left\{\frac{\cos 2\pi(f-f_c)\cdot T_b}{1-[4(f-f_c)\cdot T_b]}\right\}^2 + \frac{8E_b}{\pi^2}\left\{\frac{\cos 2\pi(f+f_c)T_b}{1-4(f+f_c)\cdot T_b}\right\}^2 \quad \text{—— (3)}$$

→ Above eqⁿ gives P.S.D of MSK signal.



(P.S.D of MSK & QPSK)

→ Fig. shows normalized P.S.D of MSK & QPSK.
(max. amplitude of signals are scaled w.r.t. '1')

→ Above plot shows main lobe in MSK is wider than QPSK. Side lobes in MSK very small compared to QPSK.

✱ Bandwidth :-

$f T_b = \pm 0.75$

$\Rightarrow \left(f = \pm 0.75 f_b \right)$

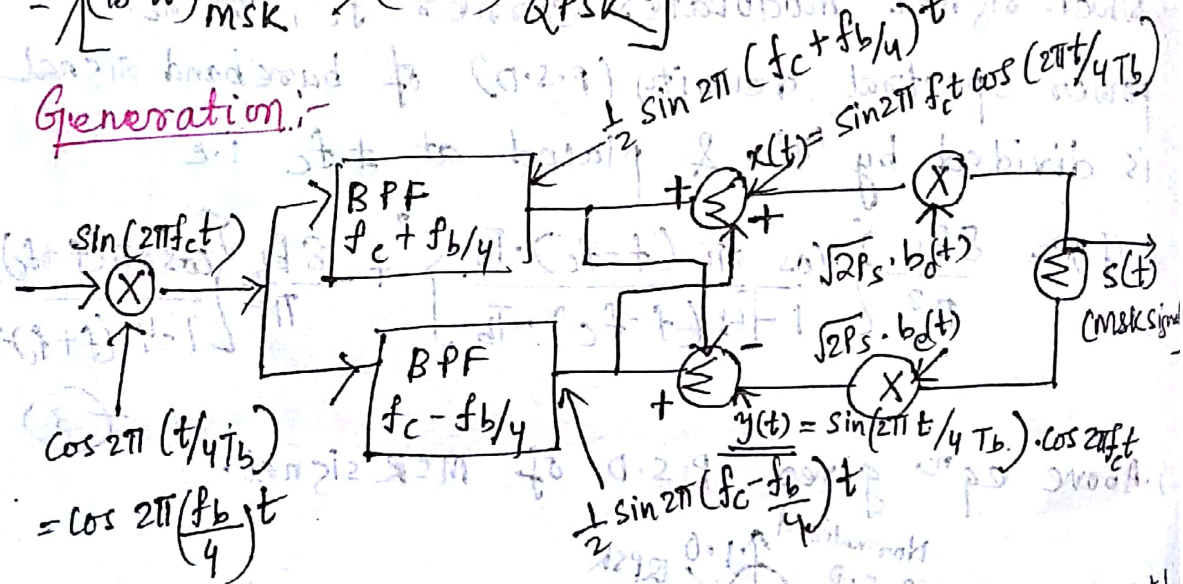
→ B.W will be equal to width of main lobe i.e

$B.W = \pm 0.75 f_b - (-0.75 f_b)$

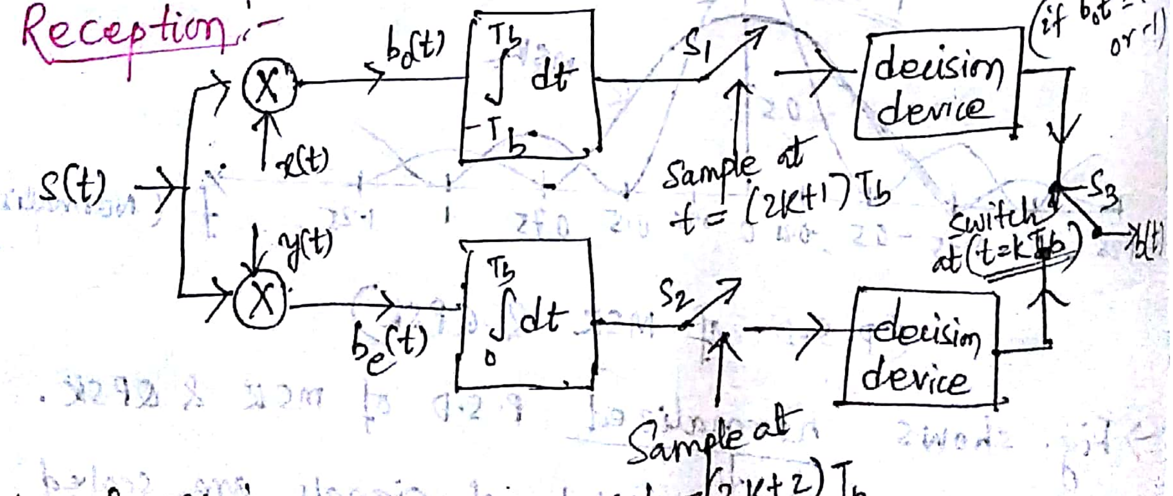
$= 1.5 f_b$

$\Rightarrow \left[(B.W)_{MSK} > (B.W)_{QPSK} \right]$

Generation :-



Reception :-



Advantages :-

1. MSK is smoother, continuous phase
2. Sidelobes is smaller, avoid inter channel interference

Disadvantages

1. BW of MSK > B.W of QPSK
$(1.5 f_b) > (f_b)$
2. Generation & detection is complex

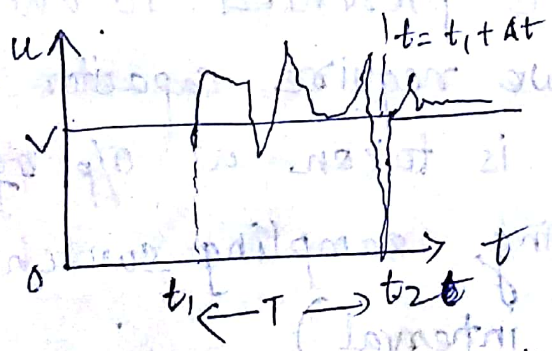
M-IV

(Data Transmission):-

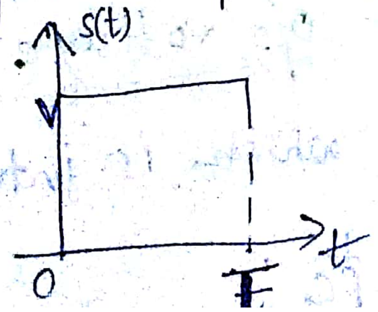
* [Base band signal Reciever]:-

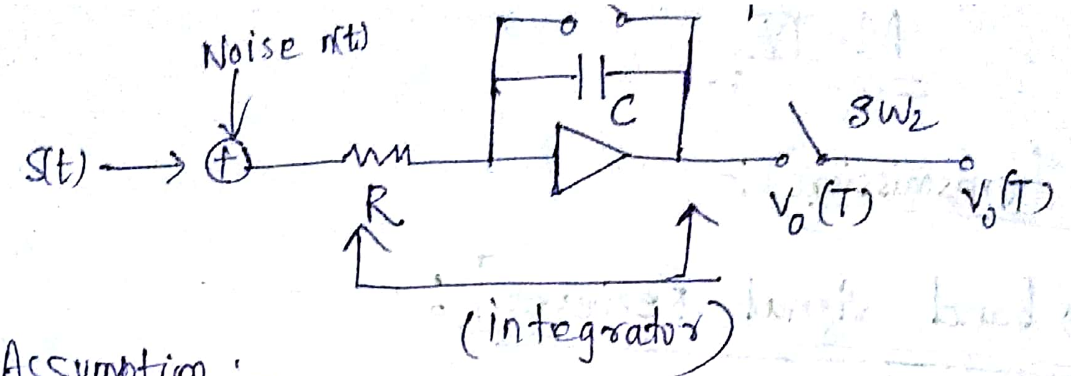
→ consider a binary signal varies from sequence of voltage +V to -V. If there's guard internal between the bits, signal forms a sequence of +ve & -ve pulses.

→ Suppose noise is guassian & so noise voltage has a probability density which is entirely symmetrical w.r.t zero volts. So probability that noise has decreased sampled value.



→ At sampling time, noise voltage larger than V & polarity opposite to parality of transmitted bit. Noise is superimposed on level +V, so that voltage 'V' represents recieved signal & noise.





Assumption :-

Waveform s(t) before t = 0 & after (t = T) has not been indicated, because operation of reciever during each bit interval is independent of waveform during past & future bit intervals.

Operation:-

→ Signal s(t) with added white gaussian noise n(t) of P.S.D $\left(\frac{\eta}{2}\right)$ is presented to an integrator

→ At time (t = 0), we require capacitor to be uncharged. Sample is taken at o/p of integrator by closing sampling switch SW_2 at t = T (end of interval)

→ Total set up is called (integrator & dump filter).
(dump → abrupt discharge of capacitor in each sampling)

Peak signal to RMS noise o/p voltage ratio:-

→ integrator yields an o/p which is integral of its i/p multiplied by $\frac{1}{RC}$

Using $(\tau = RC)$,

$$V_o(T) = \frac{1}{\tau}\int_0^T V_o(t)\,dt \quad \text{——— (1)}$$

$$= \frac{1}{\tau}\int_0^T [s(t)+n(t)]\,dt \quad \text{——— (2)}$$

$$= \underbrace{\frac{1}{\tau}\int_0^T s(t)\,dt}_{(a)} + \underbrace{\frac{1}{\tau}\int_0^T n(t)\,dt}_{(b)} \quad \text{——— (3)}$$

(a) $\frac{1}{\tau}\int_0^T s(t)\,dt$

$$= \frac{1}{\tau}\int_0^T V\cdot dt = \frac{V}{\tau}\int_0^T dt = \frac{V}{\tau}(t)_0^T = \frac{V}{\tau}\cdot(T) \quad \text{——— (4)}$$

(b) $\frac{1}{\tau}\int_0^T n(t)\,dt$

As, $G_{n_o}(f) = G_{n_i}(f)\,|H(f)|^2$

o/p spectral density   square of transfer fun^n.

$$H(f) = \frac{1}{j\omega\tau} - \frac{e^{-j\omega T}}{j\omega\tau} = \frac{1-e^{-j\omega T}}{j\omega\tau}$$

$$= \frac{1-(\cos\omega T - j\sin\omega T)}{j\omega\tau}$$

$$= \frac{1-\cos\omega T}{j\omega\tau} + \frac{\sin\omega T}{\omega\tau}$$

$$\Rightarrow |H(f)|^2 = \left(\frac{1-\cos\omega T}{\omega\tau}\right)^2 + \left(\frac{\sin\omega T}{\omega\tau}\right)^2$$

$$= \frac{\sin^2\frac{\omega T}{2}}{\frac{\omega^2\tau^2}{4}} = \frac{1}{\tau^2}\cdot T^2\,\frac{\sin^2\left(\frac{\omega T}{2}\right)}{\frac{\omega^2 T^2}{4}}$$

$$= \frac{T^2}{\tau^2}\left[\frac{\sin\left(\frac{\omega T}{2}\right)}{\frac{\omega T}{2}}\right]^2$$

As $w = 2\pi f \Rightarrow |H(f)|^2 = \left(\frac{T}{\tau}\right)^2\left[\frac{\sin \pi T f}{\pi T f}\right]^2$

$$G_{ni}(f) = \frac{\eta}{2}$$

$$\Rightarrow G_{no}(f) = \left(\frac{\eta}{2}\right)\left\{\left[\left(\frac{T}{\tau}\right)^2\left[\sin\frac{\pi T f}{\pi T f}\right]^2\right]\right\}$$

$$\Rightarrow N_0 = \int_{-\infty}^{\infty} G_{no}(f)\, df$$

$$= \int_{-\infty}^{\infty}\left(\frac{T}{\tau}\right)^2\left[\frac{\sin \pi T f}{\pi T f}\right]^2 \frac{\eta}{2}\, df.$$

$$= \left(\frac{T}{\tau}\right)^2\left(\frac{\eta}{2}\right)\int_{-\infty}^{\infty}\left(\frac{\sin \pi T f}{\pi T f}\right)^2 df$$

Let $\pi T f \Rightarrow x$. As $f \to \infty,\ x \to \infty$

$\Rightarrow \pi T\, df = dx \qquad f \to -\infty,\ x \to -\infty$

$\Rightarrow df = \frac{dx}{\pi T}$

$$= \left(\frac{T}{\tau}\right)^2\left(\frac{\eta}{2}\right)\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 \cdot \frac{dx}{\pi T}$$

$$= \frac{\eta T}{2\tau^2\pi}\underbrace{\int_{-\infty}^{\infty}\left(\frac{\sin x}{x}\right)^2 dx}_{= (\pi)}$$

$$= \frac{\eta T}{2\tau^2\pi} \times (\pi) \Rightarrow \boxed{\frac{\eta T}{2\tau^2} = N_0 = \sigma^2}$$ — 5

→ At end of interval, ramp attains voltage $s_o(T)$ which is $+\frac{VT}{\tau}$ or $-\frac{VT}{\tau}$, depending on 1 or 0.

$$\Rightarrow \boxed{V_0(T) = s_0(T) + n_0(T)} \quad \text{—— (6)}$$

* Signal to Noise ratio (SNR):-

$$= \frac{\text{O/p signal power}}{\text{O/p noise power}} = \frac{[s_0(t)]^2}{[n_0(t)]^2} = \frac{\left(\frac{VT}{\tau}\right)^2}{\left(\frac{\eta T}{2\tau^2}\right)}$$

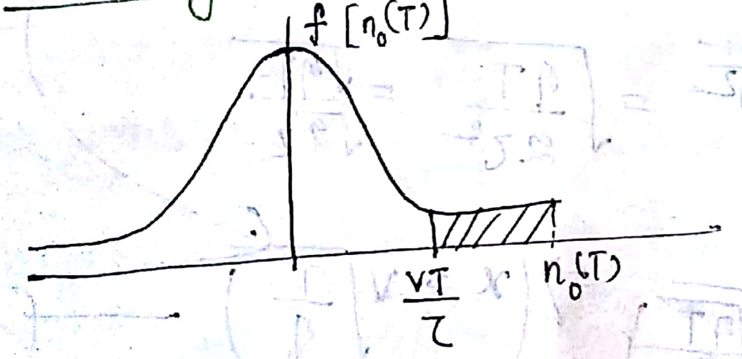
$$\Rightarrow \boxed{SNR = \frac{2}{\eta} \cdot V^2 T}$$

$\Rightarrow SNR \propto T$

$\propto V^2 T$ (Normalized energy of bit signal)

Probability of error:-

→



Probability density of noise sample $n_0(T)$ is gaussian is given by,

$$f[n_0(T)] = \frac{e^{\frac{-n_0^2 T}{2\sigma_0^2}}}{\sqrt{2\pi\sigma_0^2}}, \quad \text{where } \sigma_0^2 = \text{variance} = \overline{n_0^2(T)} \quad \text{—— (1)}$$

→ Total o/p voltage $V_0(T) = S_0(T) + n_0(T)$ ——— (3)

→ +ve sample voltage will result in error i.e misinterpretating probability of $n_0(T) > \frac{VT}{\tau}$ is given by shaded area.

→ probability of error $(P_e) = \int_{VT/\tau}^{\infty} f[n_0(T)]\, d\, n_0(T)$ ——— (4)

$$= \int_{\frac{VT}{\tau}}^{\infty} \frac{e^{\frac{-n_0^2(T)}{2\sigma_0^2}}}{\sqrt{2\pi\sigma_0^2}} \cdot d\, n_0(T)$$

$$= \frac{1}{\sqrt{2\pi\sigma_0^2}} \int_{VT/\tau}^{\infty} e^{-\left(\frac{n_0 T}{\sqrt{2}\sigma_0}\right)^2} \cdot d\, n_0(T)$$

Let $\frac{n_0(T)}{\sqrt{2\sigma_0^2}} = x \Rightarrow dx = \frac{d\, n_0(T)}{\sqrt{2\sigma_0^2}} \Rightarrow d\, n_0(T) = \sqrt{2}\sigma_0\, dx$ ——— (5)

As $n_0(T) \to \frac{VT}{\tau}$, $x \to \frac{\frac{VT}{\tau}}{\sqrt{2\sigma_0^2}}$

$$\sigma_0 = \sqrt{\sigma_0^2}$$

$$= \sqrt{\overline{n_0(T)^2}} = \sqrt{\frac{\eta T}{2\tau^2}} = \frac{\sqrt{\eta T}}{\sqrt{2}\tau}$$

$$\Rightarrow x = \frac{\frac{VT}{\tau}}{\sqrt{2}\left(\frac{\sqrt{\eta T}}{\sqrt{2}\tau}\right)} \Rightarrow \left(x = V\sqrt{\frac{T}{\eta}}\right)$$ ——— (6)

As $n_0(T) \to \infty$, $x \to \infty$

$$\Rightarrow \frac{1}{\sqrt{2\pi\sigma_0^2}} \int_{\sqrt{T/\eta}}^{\infty} e^{-x^2} \cdot (\sqrt{2}\sigma_0\, dx)$$

$$= \frac{\sqrt{2}\sigma_0}{\sqrt{2\pi\sigma_0^2}} \int_{V\sqrt{T/\eta}}^{\infty} e^{-x^2} dx = \frac{1}{\sqrt{\pi}} \int_{V\sqrt{\frac{T}{\eta}}}^{\infty} e^{-x^2} dx$$

As entire curve concerned from $\frac{VI}{\tau}$ to ∞,

so error function $\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du$

$$\Rightarrow \text{erfc}(u) = 1 - \text{erf}(u)$$

$$= \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-u^2} du \quad \text{---(7)}$$

$$u = V\sqrt{\frac{I}{\eta}} = x$$

$$= \frac{1}{2} \text{erfc}\ V\sqrt{\frac{I}{\eta}}$$

$$\Rightarrow \left[P_e = \frac{1}{2} \text{erfc} \left[\frac{(V^2 T)}{\eta} \right] \right]$$

$$\Rightarrow \boxed{P_e = \frac{1}{2} \text{erfc} \sqrt{\frac{(E_s)}{\eta}}}$$

(E_s = energy of signal)

Conclusion :-

(i) 'P_e' decreases rapidly as $\frac{E_s}{\eta}$ increases.

(ii) Max. value of $P_e = \frac{1}{2} = 0.5$

$$P_{e(min)} = 0$$

So, if signal is entirely lost in noise, reciever can't be wrong more than 50% of avg. value.

Principles of Digital Data Transmission (Line Coding)

CHAPTER 5

The digital data can be transmitted by various transmission or line codes such as ON-OFF, POLAR, BIPOLAR etc. This is called **"Line coding"**.

5.1 Properties

(i) Transmission bandwidth should be as small as possible.

(ii) The transmitted power of line codes should be as small as possible.

(iii) It must be possible to detect and preferably correct detection occurs.

(iv) It must be possible to extract timing or clock information from the signal.

(v) It must be possible to transmit a digital signal correctly regardless of pattern of '1' s and '0' s.

5.2 Types of Line Coding

(i) Non-return to zero(NRZ) and return to zero (RZ) unipolar format.

(ii) NRZ and RZ polar formats

(iii) Non-Return to bipolar format

(iv) Manchester format

(v) Polar quaternary NRZ format.

Unipolar →	ON-OFF
NRZ →	Non-Return to zero
RZ →	Return to zero

5.3. Description

5.3.1. Unipolar RZ (ON-OFF RZ) Format :

(i) The waveform has a single polarity.

(ii) In unipolar Rz form, the waveform has zero value when symbol '0' is transmitted and waveform has 'A' volts when '1' is transmitted.

(iii) In Rz form, the 'A' volts is present for $T_b/2$ period if the symbol '1' is transmitted and for remaining $\frac{T_b}{2}$ wave form returns to zero value.

Mathematically,

Symbol-1 transmitted.

$$m(t) = \begin{cases} A \text{ for } 0 \le t \le T_b/2 & \text{(Half)} \\ 0 \text{ for } T_b/2 \le t \le T_b & \text{(Rest half)} \end{cases}$$

$\boxed{T_b = \text{Bit duration}}$

Let the bit sequence 1 1 0 1 0 1 1

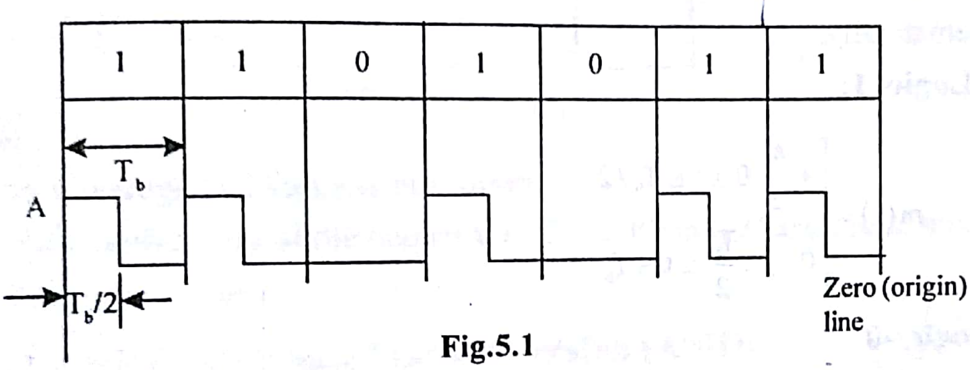


Fig.5.1

5.3.2. Unipolar NRZ (On-OFF NRZ)

(i) The waveform has a single polarity.

(ii) In unipolar NRz form, the waveform has zero value when symbol '0' is transmitted and waveform has 'A' volt when '1' is transmitted.

(iii) In NRz form, the 'A' volts is present for T_b (Full) period if symbol–1 is transmitted.

Mathematically,

$m(t) = A$ for $0 \le t \le T_b$ [For logic –1 symbol]

$m(t) = 0$ for $0 \le t \le T_b$ [For logic– 0 symbol]

Let the binary sequence 1 1 0 1 0 1 1

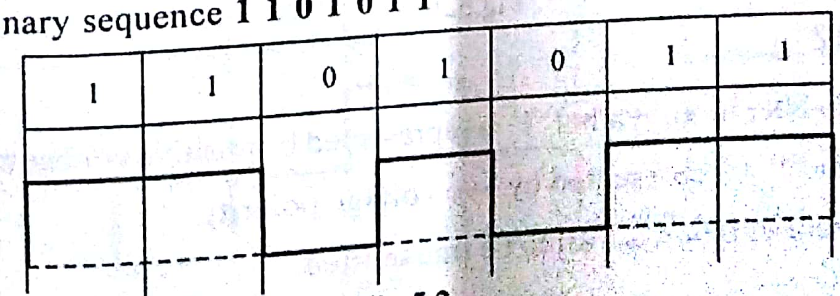


Fig.5.2

Scanned by CamScanner

5.3.2.1. Advantages of NRZ over RZ

(i) There is no separation between the pulses, therefore the receiver needs synchronization to detect unipolar NRz.

(ii) Pulse width is more.

(iii) Energy of pulse is more.

5.3.3. Polar RZ

(i) In the polar Rz format, symbol '1' is represented by positive voltage polarity whereas symbol '0' is represented by negative voltage polarity.

(ii) The pulse transmitted only for half duration.

Mathematically,

For Logic–1:

$$m(t) = \begin{cases} +\frac{A}{2}, & 0 \le t \le T_b/2 \\ 0, & \frac{T_b}{2} \le t \le T_b \end{cases}$$

For logic -0

$$m(t) = \begin{cases} -\frac{A}{2}, & 0 \le t \le T_b/2 \\ 0, & \frac{T_b}{2} \le t \le T_b \end{cases}$$

Example : **1 1 0 1 0 1 1**

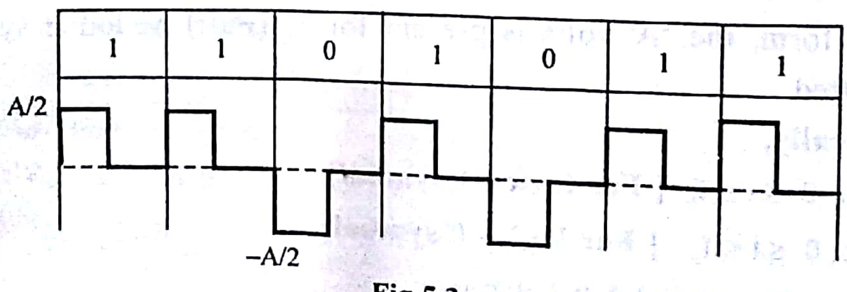


Fig.5.3

5.3.4. Polar NRZ

(i) In polar NRz form, symbol '1' is represented by positive voltage polarity whereas symbol '0' is represented by –ve voltage polarity.

(ii) The pulse duration is Full to be transmitted.

Mathematically,

For logic –1, $m(t) = +\frac{A}{2}$ for $0 \le t \le T_b$

For logic –0, $m(t) = -\frac{A}{2}$ for $0 \le t \le T_b$

Example :- **1 1 0 1 0 1 1**

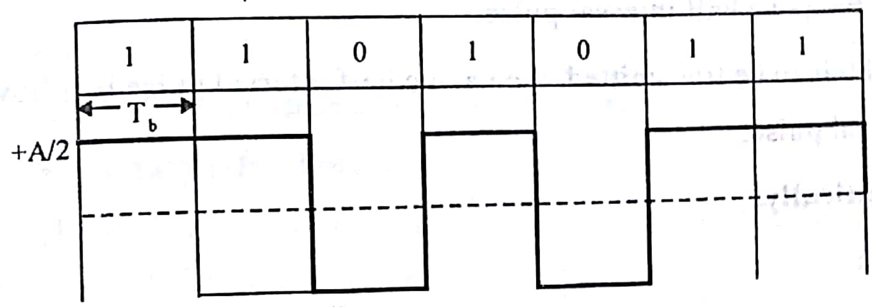


Fig.5.4

Note :

(i) The average DC value is minimum

(ii) If the probability of the occurance of the '0' and '1' are same, then the average DC value is zero.

5.3.5. Bipolar NRZ [Alternate Mark Inversion (AMI)] OR [Pseudoternary Signal]

Alternate Mark Inversion or Bipolar NRZ

(i) It is also known as Alternate Mark Inversion (AMI)

(ii) In this format, the successive '1's are represented by pulses with alternate polarity and '0' s are represented by No pulse (zero)

(iii) For even number of 1's , the average DC value is zero, because, the alternate 1's are is opposite polarity, so they cancel each other and the Final DC output is zero.

(iv) The ambiguities due to transmission sign inversion are eliminated.

Example:- **1 1 0 1 0 1 1**

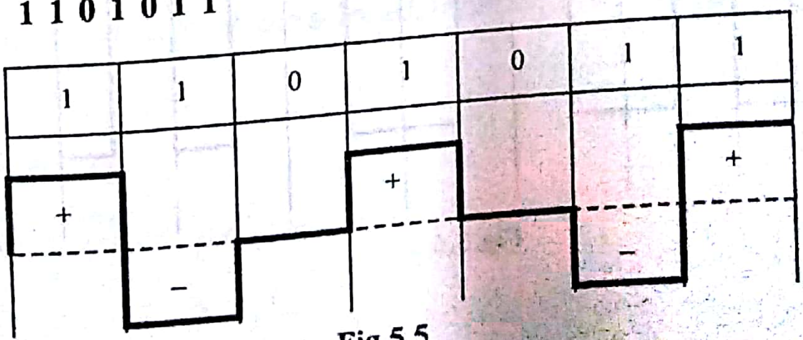


Fig.5.5

5.4 POWER SPECTRAL DENSITY OF LINE CODES

→ A line code in general is represented by

$$S(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT_S)$$

where a_n = channel coded random data

$g(t)$ = symbol pulse shape

T_s = symbol duration

Power spectral density of a line code is defined as

$$S_{psd}(f) = \frac{1}{T_S} |G(f)|^2 S_A(f)$$

where G (f) = Fourier Transform (FT) of pulse shape.

$S_A(f)$ = PSD of data sequence a_n

→ The PSD of data sequence is defined in terms of autocorrelation function as

$$S_A(f) = \sum_{m=-\infty}^{\infty} R_\tau(m) e^{-j2\pi f m T_S} \quad = \frac{1}{T_S}\sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{T_b}\right) \text{ (By poission expression)}$$

Fourier series expansion of $\sum \delta(t - mT_b) = \left(\frac{1}{T_b}\right)\sum e^{-j2\pi f_b m t}$

So taking fourier transform both sides

$$F\left[\sum \delta(t - mT_b)\right] = F\left[\frac{1}{T_b} + \sum e^{-j2\pi f_b m t}\right]$$

$$\Rightarrow \sum e^{-j2\pi f m T_b} = \frac{1}{T_b}\sum f\left[e^{-j2\pi f_b m t}\right] \quad \begin{cases} \because F[\delta(t)] = 1 \\ \Rightarrow F[\delta(t-a)] = e^{-j2\pi f a}, \\ \& \; F\left[e^{-j(2\pi f_0)t}\right] = \delta(f - f_0) \text{ here, } f_0 = mf \end{cases}$$

$$\Rightarrow \boxed{\sum e^{-j2\pi f_m T_b} = \frac{1}{T_b}\sum\left[\delta\left(f - \frac{m}{T_b}\right)\right]} \quad \{f_b = 1/T_b\}$$

$$\Rightarrow \boxed{\sum e^{-j2\pi f m T_b} = \left(e^{-j\omega m T_b}\right) = \frac{1}{T_b} \times \left[2\pi\delta\left(\omega - \frac{2\pi m}{T_b}\right)\right]} \quad \{\because \delta(f) = 2\pi\delta(\omega)\}$$

where $R_\tau(m)$ = auto correlation function.

$$= \boxed{\sum_{i=1}^{\ell} (a_n a_{m+n})_i \, P_i}$$

where P_i = Probability of occurance of a particular symbol.

Procedure for PSD Calculation:

Step -1. (a) Find out autocorrelation function for m = 0 (no time shift) for data sequence

$$R_\tau(0) = \Sigma(a_n a_n) P_i$$

(b) Find out correlation function for $m \neq 0$ data sequence.

$$R_\tau(m) = \sum_{i=1}^{\ell} (a_n a_{m+n}) P_i$$

Step -2. Find out PSD of the data sequence.

$$S_A(f) = \sum_{m=-\infty}^{\infty} R_\tau(m) e^{-j2\pi f m T_S}$$

Step -3. Find out power spectral density (PSD) of the line code.

$$S_{psd}(f) = \frac{1}{T_S}|G(f)|^2 S_A(f)$$

Step -4. Find out average power of the line code

$$P_n = \frac{1}{T_0}\int_0^{T_0}\left[S^2(t)\right]dt \qquad \text{where } T_0 = \text{Period.}$$

Step - 5. Find out normalized PSD of the line code

$$\overline{\overline{S_{psd}(f)}} = \left[\frac{\text{Psd of the line code}}{\text{average power of the line code}}\right]$$

[If it is asked that (i) simply find psd, so proceed upto step -3 (ii) if normalized PSD proceed upto step -5]