TED (Transfer Electron Devices)

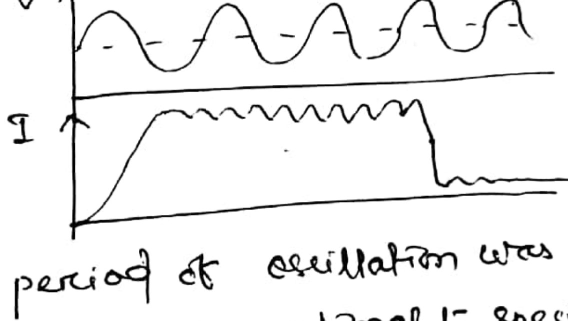
- The common characteristics of all active two terminal solid state devices are their negative resistance.
- In a negative resistance device, the current and voltage are out of phase by 180°. So the voltage drop across a negative resistance is negative and a power of $-I^2R$ is generated. In otherwords the positive resistances absorb power (passive devices) and negative resistances generate power (active devices)

MW Transistors.	TED.
* Operates with either junctions or gates	* These are bulk devices having no junctions or gates.
* fabricated with Si or Ge (elemental semiconductor)	* fabricated from compound semiconductors [GaAs, InP (indium phosphide), CdTe (cadmium Telluride)]
* Operates with 'warm' electrons whose energy is not much greater than thermal energy at room temperature (0.026 eV) of semiconductor.	* Operates with 'Hot' electrons having much larger energy.

* Gunn Effect — (Discovered by J. B. Gunn in 1963)

Gunn's observations –

- Above some critical voltage (≈ E field 2000–4000 Volts/cm) the current becomes a fluctuating function of time. (oscillation)



- The period of oscillation was determined by specimen (GaAs) and inversely proportional to specimen length and approximately equal to transit time of electrons between the electrodes
- The carrier drift velocity is linearly increased from zero to maximum when the electric field is varied from zero to a threshold value (3000 V/cm for n-type GaAs). In negative resistance region this velocity decreases.
- This threshold electric field E_{th} varied with length & type of materials. $E_{th} = 2810$ V/m.

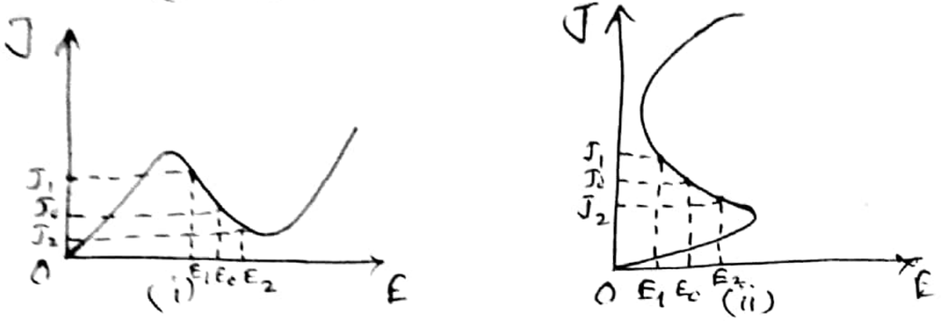
– Negative Resistance Theory –

(A) Ridley-Watkins-Hilsum (RWH) Theory –

(a) Differential Negative Resistance

When a voltage (or E field) or a current is applied to bulk of solid state compound then we have two modes.

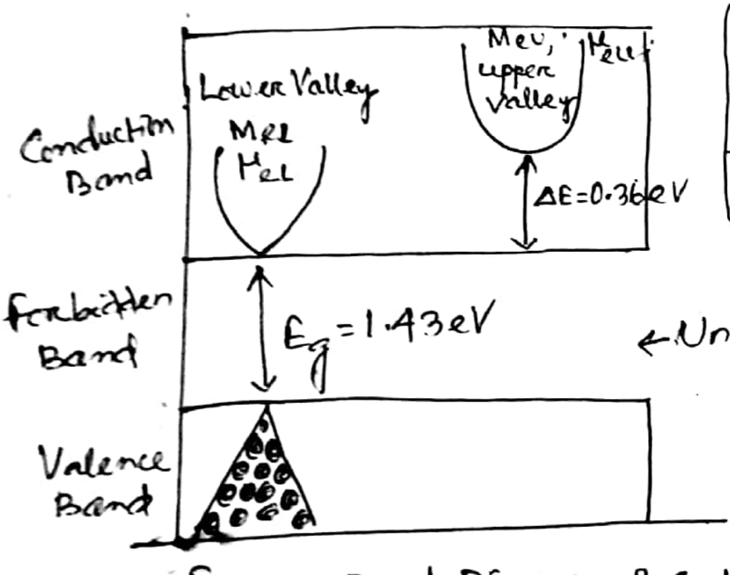
(i) Voltage controlled mode
(ii) Current controlled mode



The negative resistance is mathematically expressed as

$$\boxed{\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative Resistance}}$$

(b) Two Valley Model Theory.



Energy Band Diagram of GaAs

Valley	Effective Mass Me	Mobility μ (cm²/V-sec)	Separation ΔE (eV)
Lower	$M_{el} = 0.068$	$\mu_{el} = 8000$	0.36
Upper	$M_{eu} = 1.2$	$\mu_{eu} = 180$	0.36

← Under Equilibrium Condition.

(a) (b) & (c) are the conditions for applied Electric field.

(a) $E < E_l$ (b) $E_l < E < E_u$ (c) $E_u < E$

- The conductivity of a n-type GaAs

$$\boxed{\sigma = e(\mu_l n_l + \mu_u n_u)} \quad \text{— (a)}$$

n_l and n_u → Electron density in lower & upper valley.

- The n & μ are both functions of E. So

$$\frac{d\sigma}{dE} = \frac{d\{e(\mu_l n_l + \mu_u n_u)\}}{dE} = e\left\{\frac{d(\mu_l n_l)}{dE} + \frac{d(\mu_u n_u)}{dE}\right\}$$

$$= e\left\{\left(\mu_l \frac{dn_l}{dE} + n_l \frac{d\mu_l}{dE}\right) + \left(\mu_u \frac{dn_u}{dE} + n_u \frac{d\mu_n}{dE}\right)\right\}$$

$$= e\left(\mu_l \frac{dn_l}{dE} + \mu_u \frac{dn_u}{dE}\right) + e\left(n_l \frac{d\mu_l}{dE} + n_u \frac{d\mu_u}{dE}\right) \quad \text{— (1)}$$

Total electron density $n = n_l + n_u$

Assumed that μ_l and $\mu_u \propto E^P$ (P = constant)

Then $\dfrac{d(n_l + n_u)}{dE} = \dfrac{dn}{dE} = 0 \Rightarrow \boxed{\dfrac{dn_l}{dE} = -\dfrac{dn_u}{dE}}$ — (2)

and $\dfrac{d\mu}{dE} \propto \dfrac{dE^P}{dE} = PE^{P-1} \Rightarrow P\dfrac{E^P}{E} \propto P\dfrac{\mu}{E} = \mu\dfrac{P}{E}$ — (3) (Rearranging) $P\dfrac{\mu}{E}$

Substituting Eq^n (2) & (3) in (1)

$$\frac{d\sigma}{dE} = e\left(\mu_l \frac{dn_l}{dE} - \mu_u \frac{dn_l}{dE}\right) + e\left(n_l \cdot \mu_l \frac{P}{E} + n_u \mu_u \frac{P}{E}\right)$$

$$= e(\mu_l - \mu_u)\frac{dn_l}{dE} + e(n_l\mu_l + n_u\mu_u)\frac{P}{E} \quad \text{— (4)}$$

87 015 5710

We know that Ohm's Law $\boxed{J = \sigma E}$

$$\Rightarrow \frac{dJ}{dE} = \sigma\frac{dE}{dE} + E\frac{d\sigma}{dE} \Rightarrow \frac{dJ}{dE} = \sigma + E\frac{d\sigma}{dE}$$

$$\Rightarrow \boxed{\frac{1}{\sigma}\frac{dJ}{dE} = 1 + \frac{d\sigma/dE}{\sigma/E}} \quad \text{— (5)}$$

for negative resistance the condition can be derived from the eq^n (5) that is

$$\boxed{-\frac{d\sigma/dE}{\sigma/E} > 1} \quad \text{— (6)}$$

substitution of Eq^n (a) and (4) and $f = n_u/n_l$ results in

$$\boxed{\left[\left(\frac{\mu_l - \mu_u}{\mu_l + \mu_u f}\right)\left(-\frac{E}{n_l}\frac{dn_l}{dE}\right) - P\right] > 1} \quad \text{— (7)}$$

This condition must be satisfied for negative resistance.

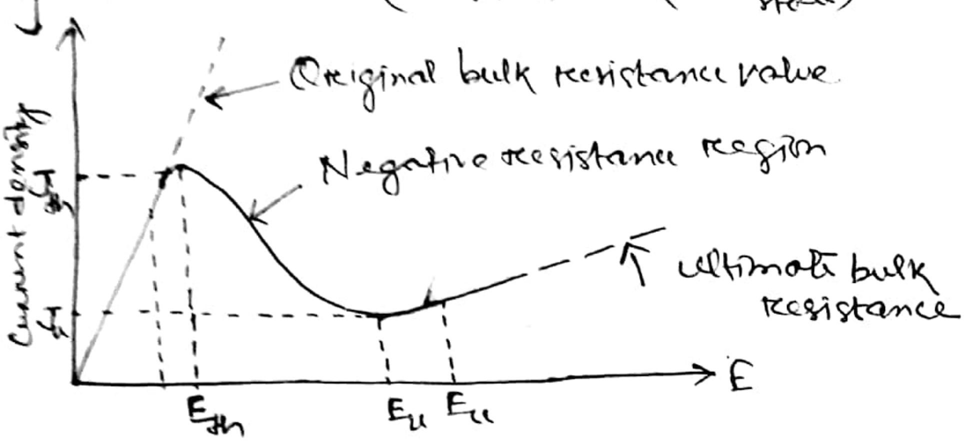
This result depends on differences between electron densities, electron temperature and gap energies in the two valleys.

- In order to exhibit negative resistance the band structure of the semiconductor must satisfy 3 criteria

(a) $\Delta E > 0.026\ eV$ $\quad \Delta E$ = Energy separation of bottom of lower valley to bottom of upper valley

(b) $\Delta E < E_g$

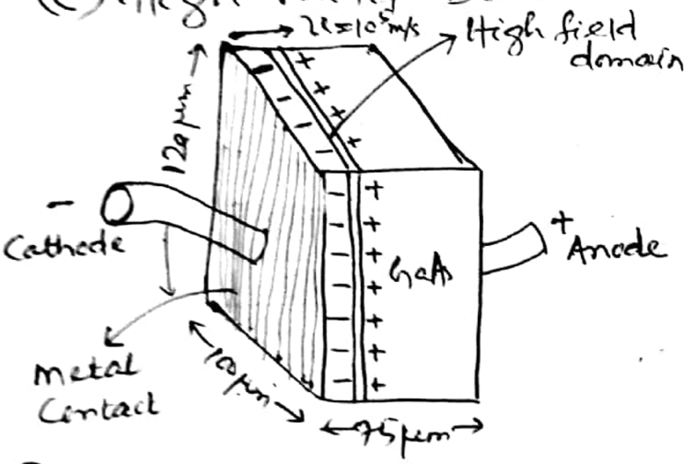
(c) $\mu_l > \mu_u$ (mobility), $M_{el} < M_{eu}$ (Effective mass), $n_l < n_u$ (density of state), $v_l > v_u$ (velocities)



Electric field E

~~* Modes of Operation of Gunn Diode~~ –

(C) High field Domain –



When applied voltage is above the threshold value (≥ 3000 V/cm), a high field domain is formed near the cathode that reduces the electric field in rest of the material and causes the current to drop to about two-thirds of its maximum value. As

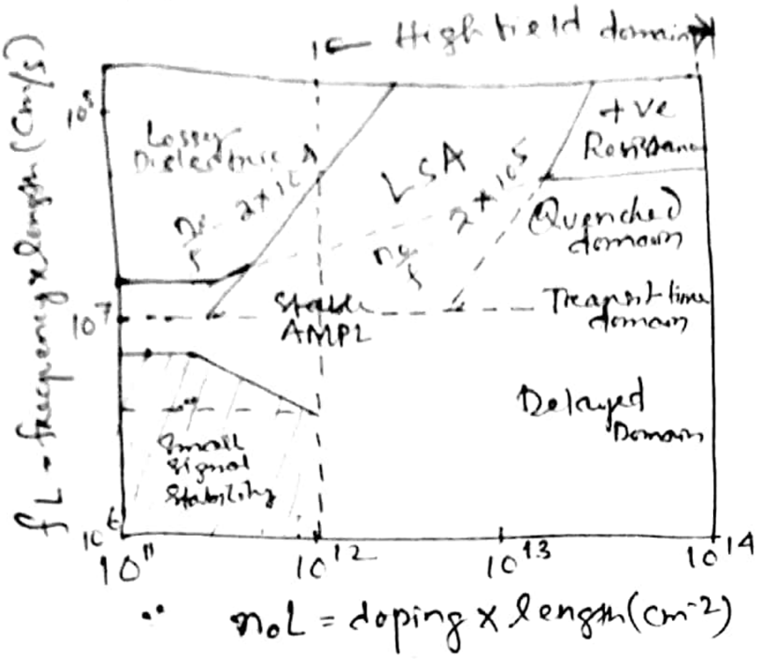
$$\boxed{V = -\int_0^L E_x\, dx}$$

For a constant Voltage V an increase in Electric field within the specimen must be accompanied by a decrease in the electric field in rest of the diode. This high field drifts with the carrier stream across the electrodes and disappears at the anode contact. When the electric field increases the electron drift velocity decreases and GaAs exhibits negative resistance.

* Modes of Operation of Gunn diode —

(1) Gunn Oscillation Mode —

– This mode is defined in the region where



(a) $fL \approx 10^7$ cm/s

(b) $n_0 L > 10^{12}$/cm^2

– In this region the device is unstable because cyclic formation of either the accumulation layer or the high field domain.

– frequency of oscillation

$$\boxed{f = \frac{v_{dom}}{L_{eff}}}$$

v_{dom} = Domain velocity

L_{eff} = Effective length travelled by the domain when it is formed and upto a new domain begins to form.

– This negative conductivity (resistance) devices are used in resonant cavities having high-Q.

– The normal Gunn Oscillation mode is operated with a electric field $E > E_{th}$.

– Electron drift velocity v varies with electric field. Depending on that there are three Gunn Oscillation modes

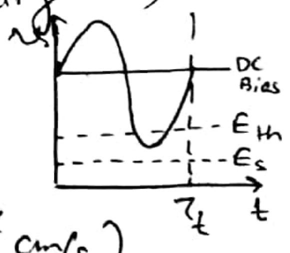
(i) Transit time Domain Mode ($fL \approx 10^7$ cm/s)

$\boxed{\text{when } v_d = v_s}$ (v_d = Drift velocity of electron, v_s = Sustaining velocity)

– then the high field domain is stable

– then $\boxed{\tau_0 = \tau_t}$ τ_0 = Oscillation period, τ_t = transit time

– Efficiency 10%



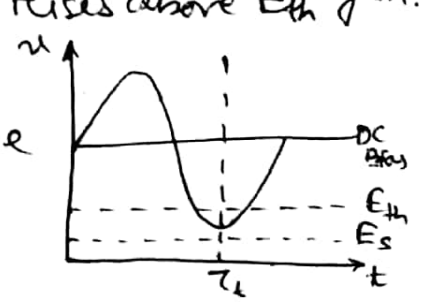
(ii) Delayed Domain Mode (10^6 cm/s $< fL < 10^7$ cm/s)

– When the domain is collected $E < E_{th}$ then a new domain cannot be formed until the field rises above E_{th} again.

– In this domain $\boxed{\tau_0 > \tau_t}$

– It is also called inhibited mode

– Efficiency 20%.

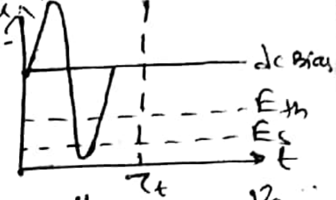


(iii) Quenched Domain Mode ($fL > 2 \times 10^7$ cm/s)

– If the bias field drops below sustaining field E_s during the negative half cycle then the domain collapses before it reaches the anode.

– When bias field swings back above E_{th} a new domain is nucleated and the process repeats. So the oscillations occur at the frequency of resonant ckt rather than at the transit time frequency.

– Efficiency 13%. $\boxed{\tau_0 < \tau_t}$

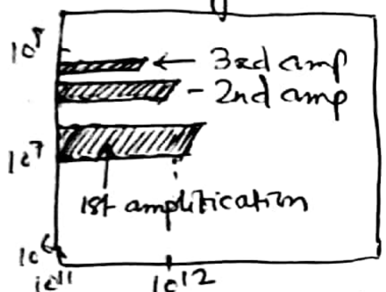


(2.) Stable Amplification mode -

– Defined in the region $fL \approx 10^7$ cm/s and $10^{11} < n_0 L < 10^{12}$ /cm²

– The device exhibits amplification at the transit-time frequency rather than spontaneous oscillation.

– The negative conductance is utilized without domain formation



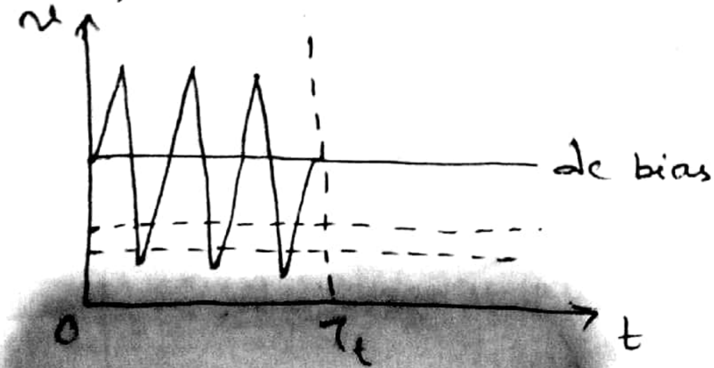
(3) LSA Oscillation Mode - (Limited Space charge Accumulation)

– When the frequency is very high, the domains donot have sufficient time to form while the field is above threshold. As a result most of the domains are maintained in the –ve conductance state during a large fraction of voltage cycle.

– Any accumulation of electrons near the cathode has time to collapse while the signal is below threshold.

– The current in the device is then proportional to ~~the applied~~ the drift velocity.

– Efficiency 20%



$\tau_0 < \tau_t$

$\tau_t = 3\tau_d$

τ_d = Dielectric Relaxation time

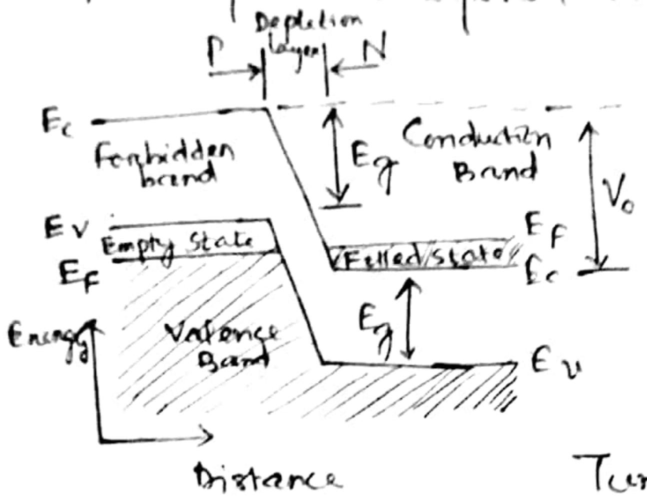
τ_0 = Oscillation period

τ_t = transit time

Microwave Tunnel Diodes

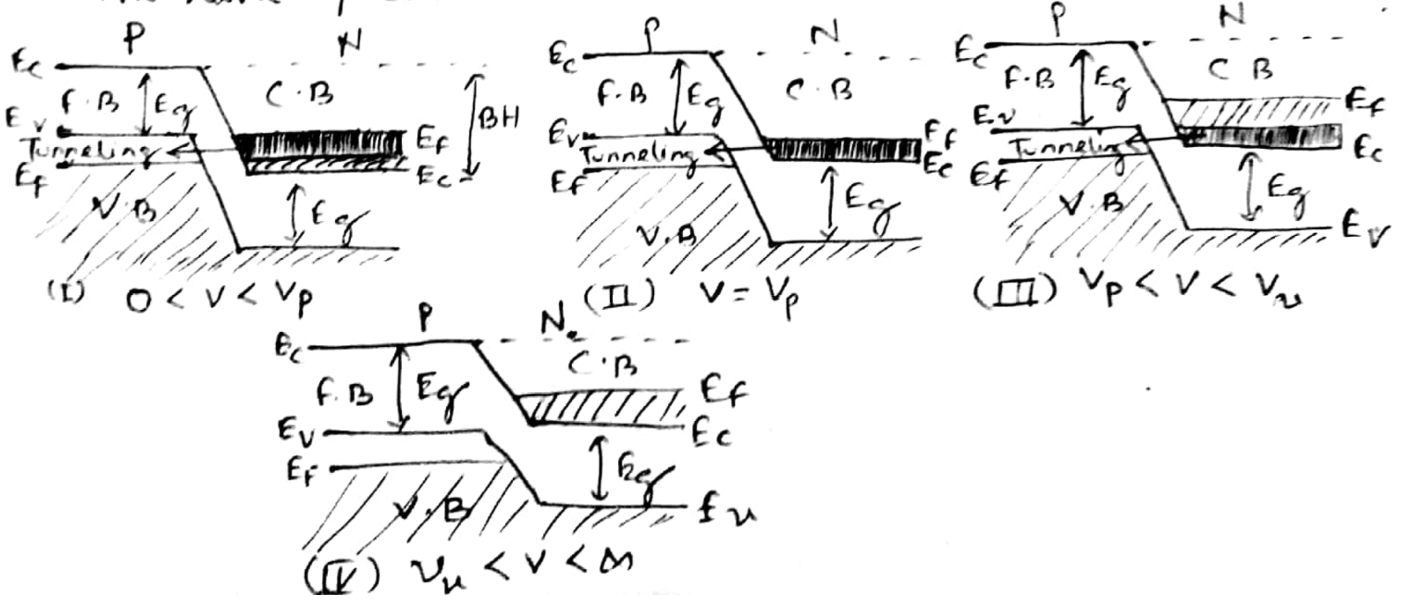
- Tunneling phenomenon was described by Esaki in diodes.
- Tunnel diodes are useful in microwave amplification, mw oscillation and binary memory.

*** Principle of Operation –**



Tunnel Diode under zero bias equilibrium.

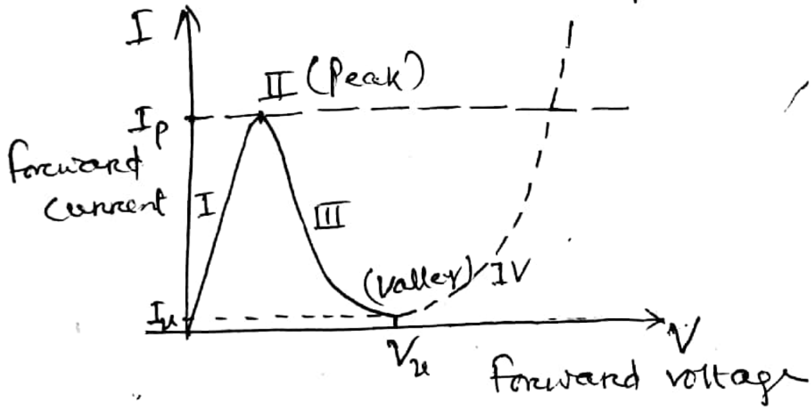
- Tunnel diode is a negative resistance semiconductor pn junction diode.
- Doping of both p & n regions are very high with impurity concentration 10^{19} to 10^{20} atoms/cm³. So with very thin depletion region at junction in the order 100 Å or 10^{-6} cm.
- Those carriers having higher energy than the potential barrier
- According to Quantum mechanics if the barrier width is less than 3 Å then carriers can tunnel through the potential barrier even though they don't have enough Kinetic Energy.
- Another condition for tunneling is as shown in Energy band diagram that is there must be a filled state from which particles will tunnel and an empty state on the other side.
- We have 4 condition for a forward bias case



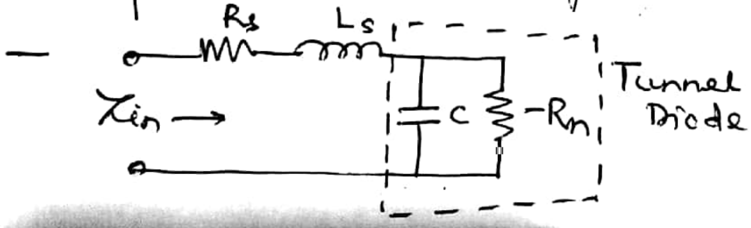
- In open ckt condⁿ filled state and empty state are not in same energy level. So there is no flow of charge across the junction and the current is zero.
- In ordinary diodes the Fermi level exists in the forbidden band. But as the tunnel diode is heavily doped, the Fermi level exists in the valence band in p-type and in the conduction band in n-type semiconductors.
- When $0 < V < V_p$ where V = applied voltage, V_p = ~~peak of~~ voltage which produce peak tunneling current I_p.

  there are filled states in conduction band of n-type at the same energy level as allowed empty states in the valence band of p-type. So tunnel occurs to give rise a forward tunneling current.
- When $V = V_p$ a maximum number of electrons can tunnel through the barrier giving rise to a peak current I_p.
- At $V_p < V < V_v$ (V_v = valley voltage)

  Tunneling current decreases as the Fermi level in n-side moves up from the empty state level in p-side.
- Beyond $V > V_v$ there is no conduction ~~as there is no level~~ as Fermi level in n-side is completely above the level of empty state in p-side. So $I_p = 0$.



- Tunnel diode is used as microwave oscillators and amplifier due to a negative resistance region.
-



R_n = Magnitude of Negative Resistance

⇐ Equivalent ckt of tunnel diode in negative resistance region.

R_s & L_s = Resistance & Inductance of packaging circuit of tunnel diode

C = Junction capacitance at valley point.

Then

$$Z_{in} = R_s + j\omega L_s + R_{eq}.$$

$$= R_s + j\omega L_s + \frac{R_n(j/\omega C)}{-R_n - j/\omega C}$$

(where
$$R_{eq} = \frac{-R_n \times \frac{1}{j\omega C}}{-R_n + \frac{1}{j\omega C}} = \frac{-R_n \times \frac{-j}{\omega C}}{-R_n - \frac{j}{\omega C}}$$
)

Applying conjugate and separation Real & imaginary parts

$$Z_{in} = R_s - \frac{R_n}{1+(\omega R_n C)^2} + j\left[\omega L_s - \frac{\omega R_n^2 C}{1+(\omega R_n C)^2}\right]$$

for Resistive cutoff frequency $Z_{in_{Real}} = 0$

$$\Rightarrow R_s - \frac{R_n}{1+(\omega R_n C)^2} = 0 \Rightarrow R_s = \frac{R_n}{1+(\omega R_n C)^2}$$

$$\Rightarrow \frac{R_n}{R_s} = 1 + (\omega R_n C)^2 \Rightarrow (\omega R_n C)^2 = \frac{R_n}{R_s} - 1 \Rightarrow \omega R_n C = \sqrt{\frac{R_n}{R_s} - 1}$$

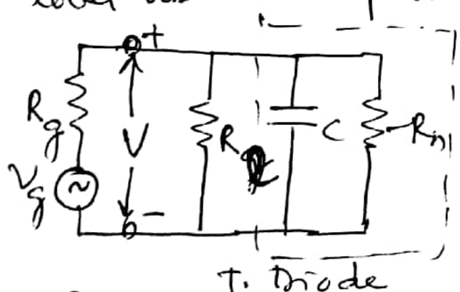
$$\Rightarrow \boxed{f_c = \frac{1}{2\pi R_n C}\sqrt{\frac{R_n}{R_s} - 1}}$$

for Self resonance frequency, $Z_{in_{img}} = 0$

$$\omega L_s - \frac{\omega R_n^2 C}{1+(\omega R_n C)^2} = 0 \Rightarrow \omega L_s = \frac{\omega R_n^2 C}{1+(\omega R_n C)^2}$$

$$\Rightarrow 1 + (\omega R_n C)^2 = \frac{\omega R_n^2 C}{\omega L_s} \Rightarrow \boxed{f_r = \frac{1}{2\pi R_n C}\sqrt{\frac{R_n^2 C}{L_s} - 1}}$$

- Tunnel diode can be connected in parallel or series with a resistive load as an amplifier.

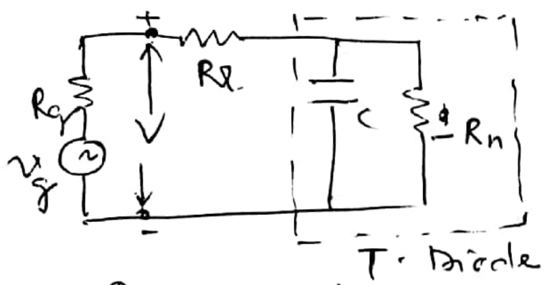


Parallel loading

$$\boxed{\text{Gain } A = \frac{R_n}{R_n - R_L}}$$

when $R_n \approx R_L \Rightarrow A \approx \infty$

$\Rightarrow$ System goes into Oscillation



Series loading

$$\text{Gain} = \boxed{A = \frac{1}{1 - R_n/R_L}}$$

If $R_L < R_n \Rightarrow$ Device remains stable in −ve resistance region without switching.

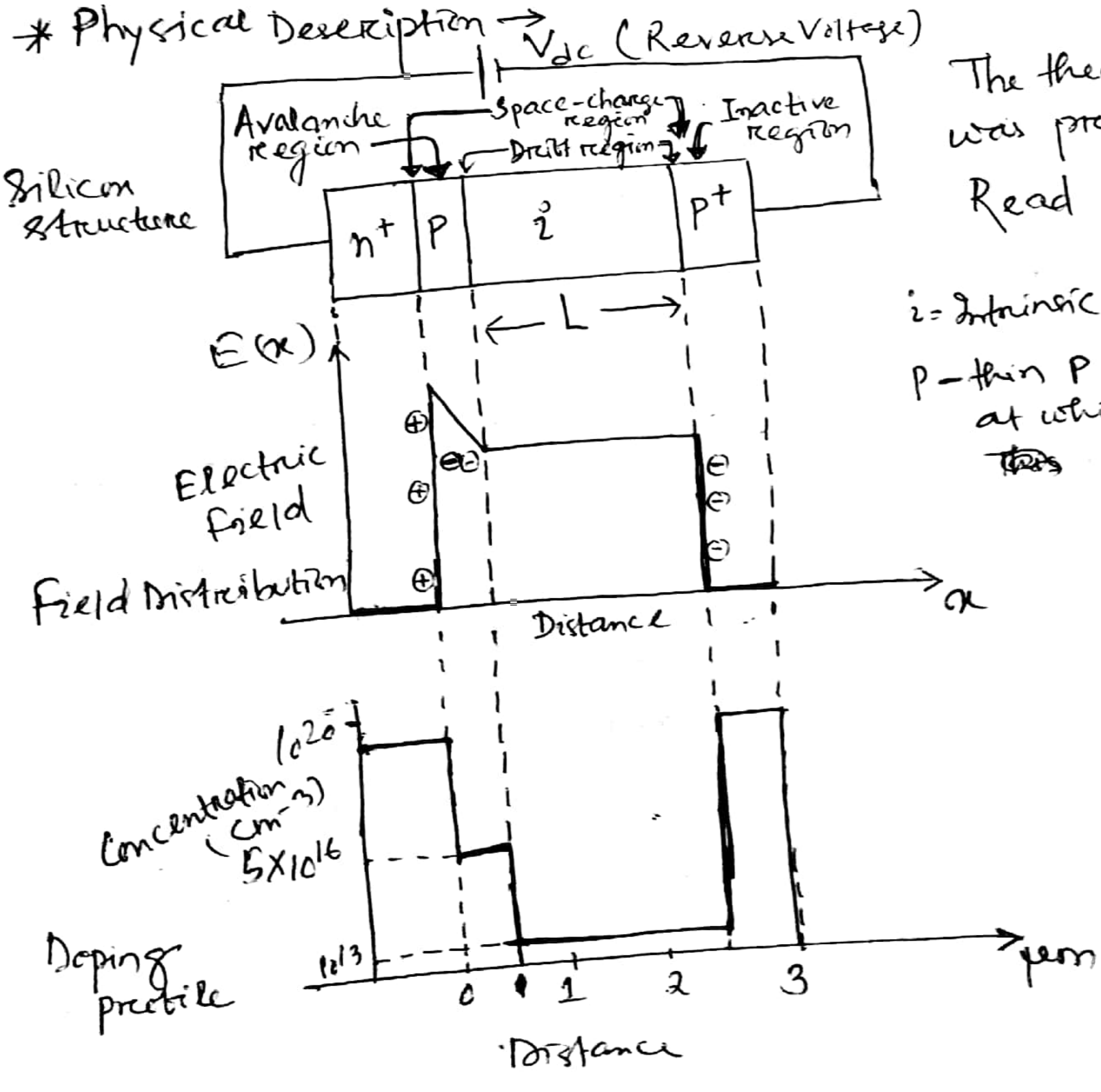
Avalanche Transit Time Devices →

- These devices are used as microwave Oscillators.
- The avalanche diode oscillator uses carrier ionization and drift in the high-field region of a semiconductor junction to produce a negative resistance at microwave frequencies.
- Two modes of avalanche oscillator have been observed.

(i) IMPATT mode →
- Impact ionization avalanche transit-time mode.
- In this mode the typical dc-to-RF conversion efficiency is 5 to 10%,
- Frequencies are as high as 100GHz with Silicon diodes.

(ii) TRAPATT mode →
→ Trapped plasma avalanche triggered transit mode.
→ Conversion efficiency is from 20 to 60%.

* Physical Description →



The theory of this device was presented by Read in 1958.

i = Intrinsic material.

P – thin P region at which avalanche ~~this~~

- The thin p-region is also called the high field region or the avalanche region.
- The i region through which the generated holes must drift in moving to the p^+ contact. This region is also called the intrinsic region or the drift region.
- The space between the n^+-p junction and the i-p^+ junction is called the space-change region.
- This device can produce a negative ac resistance which delivers power from the dc bias to the oscillation.
- The reverse bias voltage increases the maximum field in n^+-p junction to about several hundred kV/cm. The carriers (holes) moving in this high field near n^+-p junction acquire energy to knock valence electrons into the conduction band thus producing hole-electron pairs. This rate of avalanche multiplication is a nonlinear function of the field.
- The generated electrons move into the n^+ region and the holes drift through the space-change region to the p^+ region with a constant velocity v_d of about 10^7 cm/s for silicon.
- The field throughout the space-change region is ≈ 5 kV/cm
- The transit time of the hole across the drift-i region is

$$\boxed{\tau = \frac{L}{v_d}} \quad -\ (1)$$

& $$\boxed{M = \frac{1}{1-(V/V_b)^n}} \quad -\ (2)$$ Avalanche

where

V = applied voltage

V_b = Avalanche breakdown voltage

$n = 3-6$ for Si (factor depending on p^+-n or n^+-p junction)

and $$\boxed{|V_b| = \frac{\rho_n \mu_n \epsilon_s |E_{max}|_b^2}{2}}$$

ρ_n = Resistivity

μ_n = electron mobility

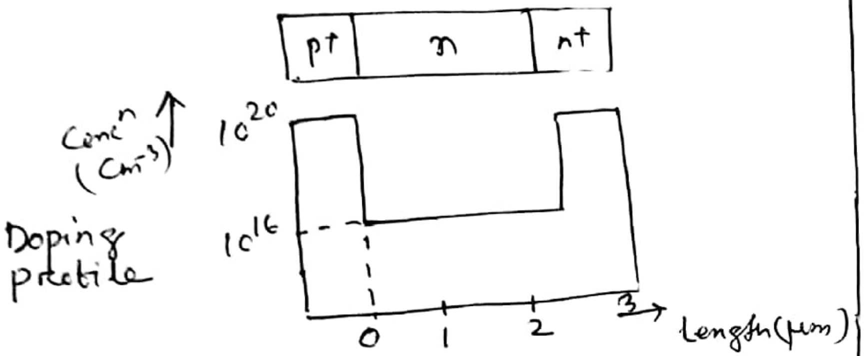
ϵ_s = Semiconductor permittivity

E_{max} = Electric field for breakdown

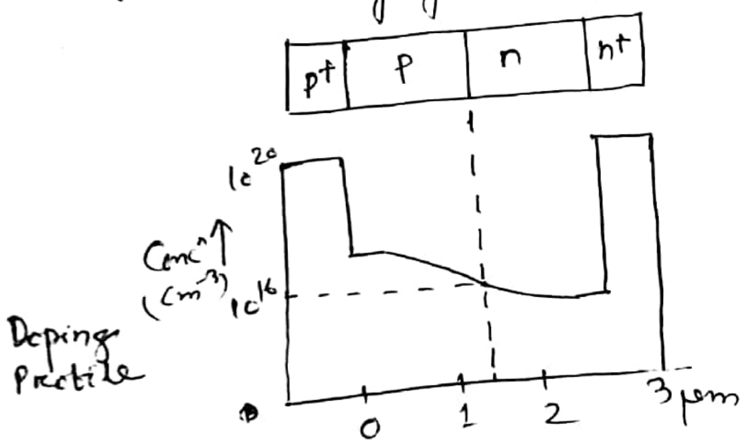
* IMPATT DIODES —

- These Impact ionization avalanche transit time diodes exhibit a differential negative resistance by two effects.
  1. (1) The impact ionization avalanche effect which causes the carrier current $I_o(t)$ and the ac voltage to be out of phase by $90°$.
  2. (2) The transit-time effect, which further delays the external current $I_e(t)$ relative to the ac voltage by $90°$.
- There are three types of silicon IMPATT diodes —

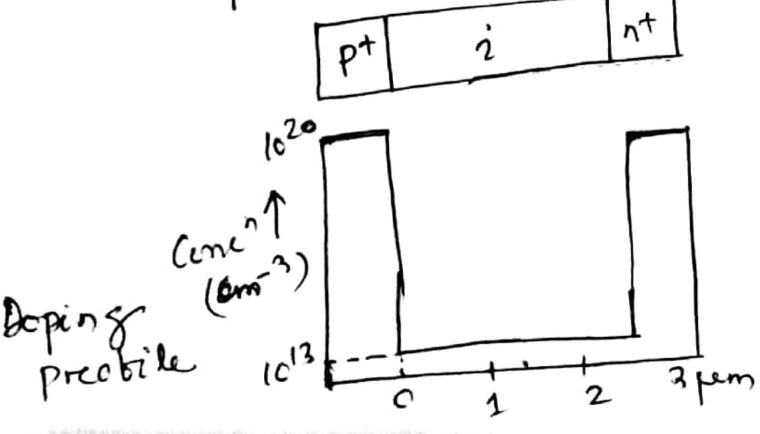
(a) Abrupt p-n junction



(b) Linearly graded PN junction



(c) p-i-n diode



* Transit angle

$$\theta = \omega\tau = \omega\frac{L}{v_d}$$

ω = angular frequency of ac-signal

* Resonant Frequency

$$\omega_r = \left(\frac{2\alpha' v_d I_0}{\epsilon_s A}\right)^{1/2}$$

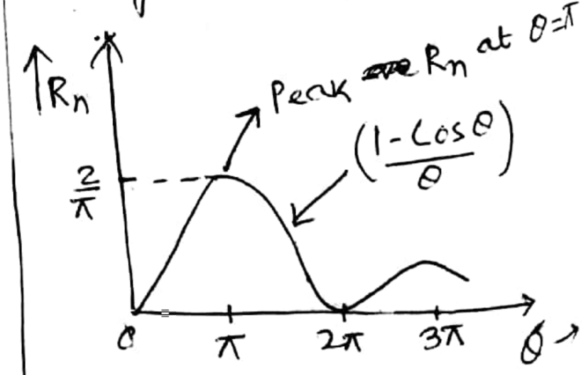
I_0 = Carrier current

α' = Ionization Coefficient w.r.t electric field.

A = Diode cross section

v_d = Drift velocity

* Negative Resistance Vs θ



For practical purpose

$\theta = \pi$

$$\Rightarrow f = \frac{1}{2\tau} = \frac{v_d}{2L}$$

- Power O/P & efficiency -

For a uniform avalanche, the maximum voltage that can be applied across the diode

$$\boxed{V_m = E_m L}$$

L = Depletion length

E_m - Maximum Electric field

The maximum current

$$I_m = J_m A = \sigma E_m A = \frac{\epsilon_s}{\tau} E_m A = \frac{v_d \epsilon_s E_m A}{L}$$

$$\Rightarrow \boxed{P_{max} = I_m V_m = E_m^2 v_d \epsilon_s A}$$

The capacitance across the space charge region is

$$C = \frac{\epsilon_s A}{L}$$, and we know $\tau = \frac{v_d}{2L}$

$f = \frac{1}{2\tau} \Rightarrow$ Applying $2\pi f \tau = 1$

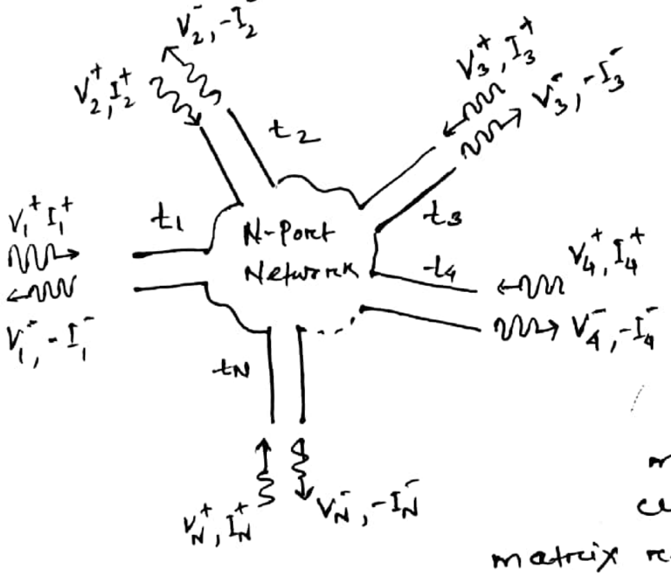
$$\Rightarrow \epsilon_s A = LC$$

substituting these in P_{max}

$$\boxed{P_m f^2 = \frac{E_m^2 v_d^2}{4\pi^2 X_c}}$$ Power frequency limitation

Efficiency $\eta = \frac{P_{ac}}{P_{dc}} \approx 30\%$

* The Scattering Matrix :-



Incident ~~voltage~~ waves = (V_n^+, I_n^+)
Reflected " = $(V_n^-, -I_n^-)$

Scattering matrix gives ideas of incident, reflected and transmitted waves in high frequency networks. (Microwave frequency).

Impedance (Z) matrix and admittance (Y) matrix relate the total voltages and total currents at the ports, but the scattering matrix relates the voltage waves incident on the ports and reflected from the ports.

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & & & \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

or $\boxed{[V^-] = [S][V^+]}$

$$\boxed{S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+ = 0 \text{ for } k \neq j}}$$

V_j^+ = Incident wave at driving port j

V_i^- = Reflected wave at port i

Incident waves on all ports should be ~~terminated~~ set to zero by terminating in matched loads to avoid reflections.

→ S_{ij} = The transmission co-efficient from j to port i when all other ports are terminated in matched loads.

For Two port

S_{11} = reflection co-efficient $= \left.\dfrac{V_1^-}{V_1^+}\right|_{V_2^+ = 0} = \left.\Gamma^{(1)}\right|_{V_2^+ = 0}$

$S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+ = 0}$ = applying Incident wave at port 1 and measuring the outcoming wave at port 2.

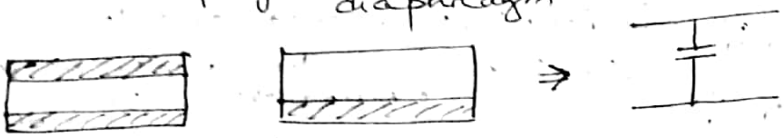
S_{21} = Transmission co-efficient from port 1 to port 2

$V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ - S_{12}V_2^-$

$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ - S_{22}V_2^-$

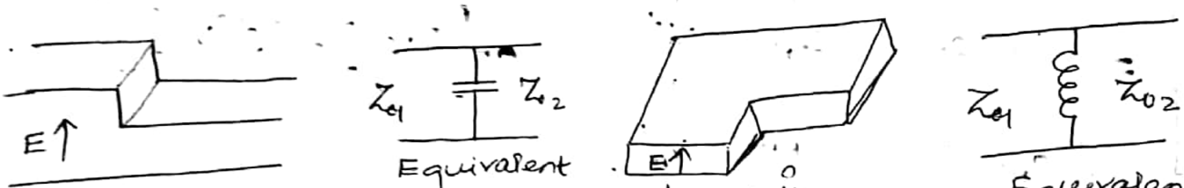
* Rectangular Waveguide discontinuities —

Symmetrical inductive diaphragm — Assymmetrical inductive diaphragm — Equivalent ckt:

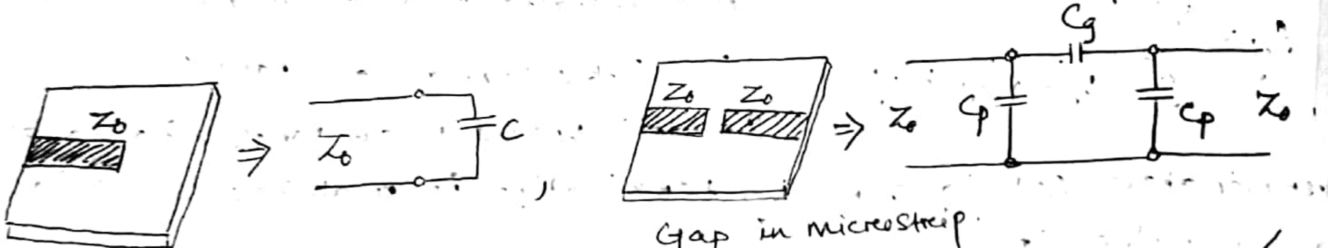


Symmetrical Capacitive Diaphragm — Asymmetrical Capacitive Diaphragm — Equivalent ckt

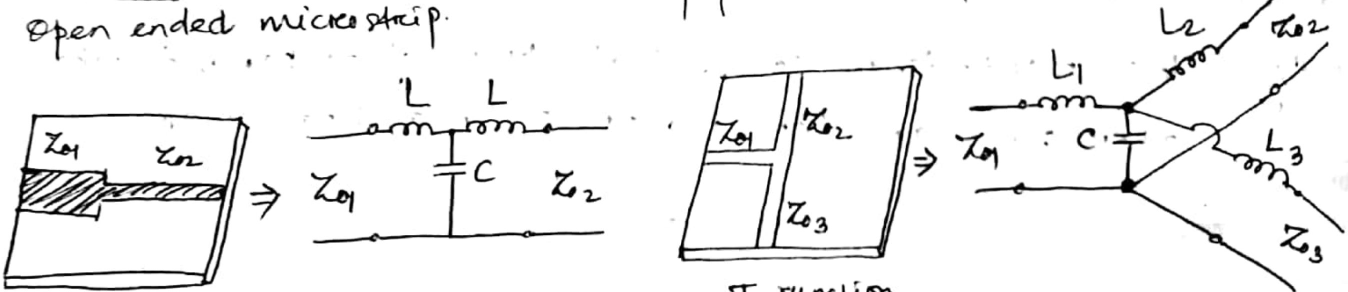
Rectangular Resonant iris — Circular Resonant iris — Equivalent circuit



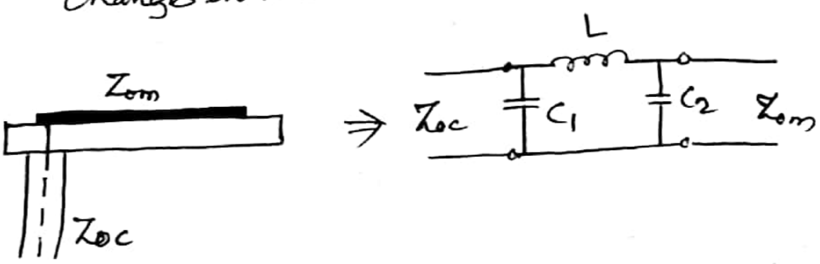
Change in Height — Equivalent circuit — Change in width — Equivalent circuit



Open ended microstrip. — Gap in microstrip.

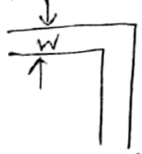


Change in width — T-junction



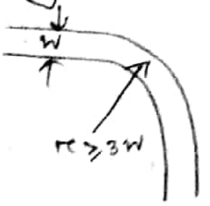
Co-axial microstrip junction

* The straightforward right-angle bend has a parasitic discontinuity capacitance caused by the increased conductor area near the bend.



Right angle Bend.

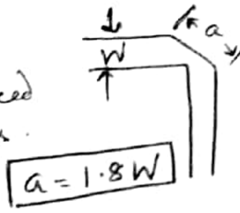
* This effect could be eliminated by making a smooth "swept" bend with a radius $r \geq 3W$. (which requires more space)



swept Bend

<u>Alternate</u>

the right angle bend can be compensated by mitering the corner which will reduce the excess capacitance at bend.



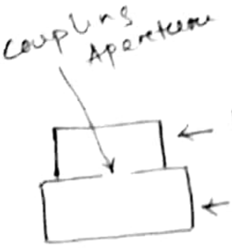
mitered Bends.

The Optimum value of the miter length a, depends on Characteristic impedance and bend angle.

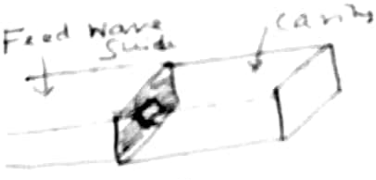
Mitered step.

* Waveguides can be excited by

① Probe feeds
② Loop feeds
③ Aperture coupling. ⇒ Such coupling is used in directional couplers and power dividers, where power from one guide is coupled to another guide through small apertures in a common wall



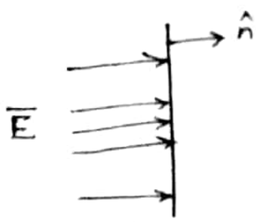
Common broad wall



Aperture in Transverse wall.

Coupling Aperture

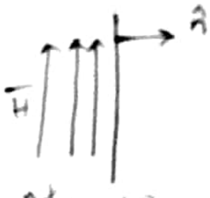
* If a small aperture is cut into the conductor the electric field lines will fringe through and around the aperture.



Normal $\bar{E}$ field at conducting wall

E line around an aperture in a conducting wall

Electric field lines around a Electric polarization current normal to conducting wall



Magnetic field lines near conducting wall

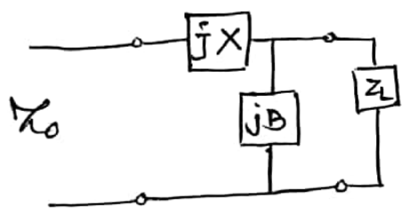
* Importance of Impedance matching or Tuning –

- Maximum power is delivered when the load is matched to line and the power loss in feed line is minimized.
- Impedance matching sensitive receiver components (Antenna, LNA etc) improves the SNR of the system.
- Impedance matching in a power distribution network (like an antenna array feed network) will reduce amplitude and phase error.

* L-Section Matching Networks –

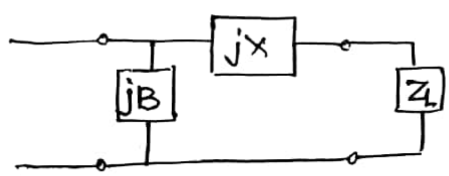
Let $1+jx$

→ If normalized load ~~1+jx circle on~~ impedance is outside the $1+jx$ circle on smith chart then



circuit is used.

→ If normalized load impedance is outside $1+jx$ circle on smith chart then



circuit is used.

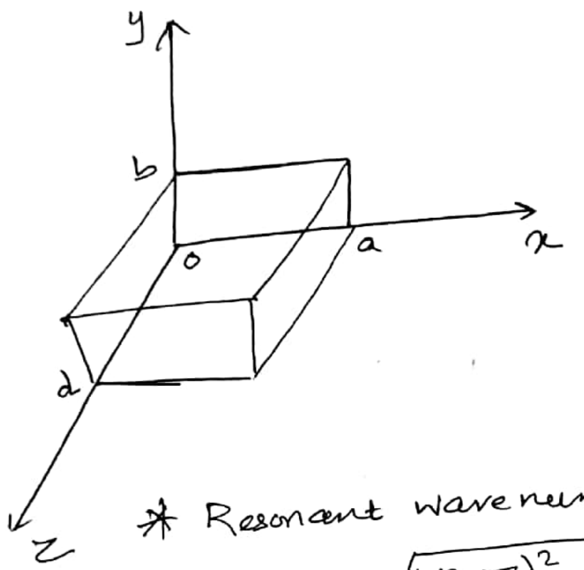
The reactive ~~impedance~~ elements may be either inductors or capacitors depending on the load impedance.

* Rectangular Waveguide cavities →

→ Resonators can also be constructed from closed sections of waveguide.

→ Waveguide resonator cavities store Electric & magnetic energy.

→ Coupling to the resonator can be by a small aperture or a small probe or loop.



Boundary condⁿs.

$$\left.\begin{array}{l} x=0 \\ x=a \end{array}\right\} E_y = 0$$

$$\left.\begin{array}{l} y=0 \\ y=b \end{array}\right\} E_x = 0$$

$$\left.\begin{array}{l} z=0 \\ z=d \end{array}\right\} E_x = E_y = 0$$

* Resonant wavenumber for rectangular cavity can be defined as

$$K_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

m, n, l = refer to the number of variations in the standing wave pattern in the x, y, z directions respectively.

* The Resonant frequency of TE_{mnl} or TM_{mnl} mode is given by

$$f_{mnl} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

if $b < a < d$

the dominant mode will be TE_{101}

& TM_{110}

* Quality factor of TE_{10l} mode. (Q. factor)

Q. Can be calculated by finding the stored electric & magnetic energies and power loss in conducting walls and dielectrics.

Stored Electric Energy

$$W_e = \frac{\epsilon abd}{16} E_0^2 \quad \left(\text{as } W_e = \frac{\epsilon}{4}\int_V E_y E_y^* \, dv\right)$$

Stored Magnetic Energy

$$W_m = \frac{\mu abd}{16} E_0^2 \underbrace{\left(\frac{1}{Z_{TE}^2} + \frac{\pi^2}{k^2 \eta^2 a^2}\right)}_{\Downarrow \; \epsilon/\mu}$$

$$\Rightarrow \boxed{W_m = \frac{\epsilon abd}{16} E_0^2 = W_e}$$

$$\left(\text{As } W_m = \frac{\mu}{4}\int_V (H_x H_x^* + H_z H_z^*)\, dv \;\; \& \;\; \eta_g = \frac{k\eta}{\beta} = Z_{TE} \;\; \& \;\; \beta = \sqrt{k^2 - (\pi/a)^2}\right)$$

Power lost in conducting walls

$$P_c = \frac{R_s}{2} \int_{\text{walls}} |H_t|^2 \, ds \qquad R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}} = \text{Surface Resistivity of metallic walls}$$

H_t = Tangential magnetic field at surface of wall.

$$= \frac{R_s E_0^2 \pi^2}{8\eta^2}\left(\frac{l^2 ab}{d^2} + \frac{bd}{a^2} + \frac{l^2 a}{2d} + \frac{d}{2a}\right)$$

$$\left(\text{As } P_c = \frac{R_s}{2}\left\{2\int_{y=0}^{b}\int_{x=0}^{a} |H_x(z=0)|^2 \, dx\, dy + 2\int_{z=0}^{d}\int_{y=0}^{b} |H_z(x=0)|^2 \, dy\, dz + 2\int_{z=0}^{d}\int_{x=0}^{a}\left[|H_x(y=0)|^2 + |H_z(y=0)|^2\right] dx\, dz\right\}\right)$$

Q_c = Quality factor for lossy conducting walls (lossless dielectric)

$$\boxed{Q_c = \frac{2\omega_0 W_e}{P_c}} = \frac{(kad)^3 b\eta}{2\pi^2 R_s} \frac{1}{(2l^2 a^3 b + 2bd^3 + l^2 a^3 d + ad^3)}$$

Complex dielectric constant

$$\left(\epsilon = \epsilon' - j\epsilon'' = \epsilon_r \epsilon_0 (1 - j\tan\delta)\right)$$

Power dissipated in the dielectric is

$$P_d = \frac{1}{2}\int_V \bar{J}\cdot\bar{E}^* \, dv = \frac{\omega\epsilon''}{2}\int_V |E|^2 \, dv$$

$$\Rightarrow P_d = \frac{abd\,\omega\epsilon''|E_0|^2}{8}$$

Q_d = Quality factor for lossy dielectric (and ~~lossless~~ perfectly conducting wall)

$$\boxed{Q_d = \frac{2\omega W_e}{P_d} = \frac{\epsilon'}{\epsilon''} = \frac{1}{\tan\delta}}$$

When both wall losses and dielectric losses are present the total power loss is

$$P_t = P_c + P_d$$

Quality factor $Q = \left(\frac{1}{Q_c} + \frac{1}{Q_d}\right)^{-1}$

Or $$\boxed{\frac{1}{Q} = \frac{1}{Q_c} + \frac{1}{Q_d}}$$

Ex A rectangular wave guide cavity is made from a piece of copper WR-187 H band waveguide, with $a = 4.755$ cm and $b = 2.215$ cm. The cavity is filled with polyethylene ($\epsilon_r = 2.25$, $\tan\delta = 0.0004$). If resonance is to occur at $f = 5$ GHz, find the required length, d, and the resulting Q for the $l = 1$ and $l = 2$ resonant modes.

Soln

Wave number $K = \frac{\omega}{v} = \frac{2\pi f}{c/\sqrt{\epsilon_r}} = \frac{2\pi f\sqrt{\epsilon_r}}{c} = 157.08\,m^{-1}$

$$f_{mnl} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

$$\Rightarrow K_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2} \Rightarrow K_{10l}^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{l\pi}{d}\right)^2$$

$$\Rightarrow K_{10l}^2 - \left(\frac{\pi}{a}\right)^2 = \left(\frac{l\pi}{d}\right)^2 \Rightarrow \frac{l\pi}{d} = \sqrt{K_{10l}^2 - \left(\frac{\pi}{a}\right)^2}$$

$$\Rightarrow \boxed{d = \frac{l\pi}{\sqrt{K_{10l}^2 - (\pi/a)^2}}}$$

for $l = 1$, $d = 2.20$ cm

for $l = 2$, $d = 4.40$ cm.

$$R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}} = \sqrt{\frac{2\pi f \mu_0}{2\sigma}} = \sqrt{\frac{\pi \times 5\,GHz}{5.813 \times 10^7\ S/m}} = 1.84 \times 10^{-2}\ \Omega$$

↑ for copper

The intrinsic impedance is

$$\eta = \frac{377}{\sqrt{\epsilon_r}} = 251.3\ \Omega$$

$$Q_c = \frac{2\omega_0 W_e}{P_c} = \frac{k^3 a b d \eta}{4\pi^2 R_s} \frac{1}{[(l^2 ab/d^2) + (bd/a^2) + (l^2 a/2d) + (d/2a)]}$$

for $l = 1$, $Q_c = 8403$
for $l = 2$, $Q_c = 11,898$

$$Q_d = \frac{1}{\tan\delta} = \frac{1}{0.0004} = 2500$$

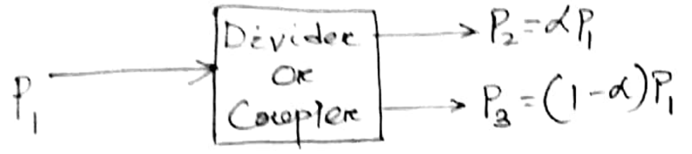
for $l = 1$, $Q = \left(\frac{1}{8403} + \frac{1}{2500}\right)^{-1} = 1927$

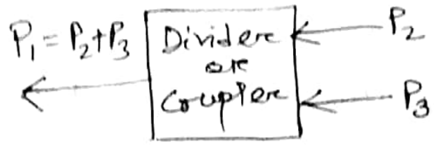
for $l = 2$, $Q = \left(\frac{1}{11898} + \frac{1}{2500}\right)^{-1} = 3065$

*

* Power Divider :→

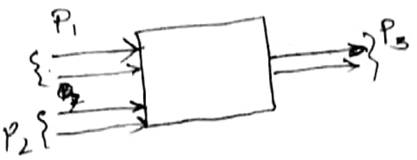
An input signal can be divided by the coupler into two or more signals of lesser power.





* Three-Port Networks (T-Junctions) ⟹

- Two inputs and one output



$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$ Scattering matrix

① If the component is passive and contains no anisotropic materials then it must be Reciprocal ⇒ S matrix is Symmetric

* Symmetric means ($S_{ij} = S_{ji}$)

② It is impossible to construct ~~such~~ a three port lossless reciprocal network that is matched at all ports.

Proof: Ⓐ If All ports are matched then $S_{ii} = 0$

$$\Rightarrow S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \xrightarrow{\text{Reciprocal}} \Rightarrow S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

③ For lossless network ⇒ To apply Energy Conservation property the scattering matrix should be Unitary.

$|S_{12}|^2 + |S_{13}|^2 = 1$, $|S_{12}|^2 + |S_{23}|^2 = 1$, $|S_{13}|^2 + |S_{23}|^2 = 1$ — Cond^n 1

$S_{13}^* S_{23} = 0$, $S_{23}^* S_{12} = 0$, $S_{12}^* S_{13} = 0$ ⇒ At least two of the three Parameters (S_{12}, S_{13}, S_{23}) must be zero. → Cond^n 2

So Cond^n 1 & Cond^n 2 can't be occur ~~at same~~ in a three port network.

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which is a proof to (A).

i.e A three-port network can't be lossless, reciprocal and matched at all ports. (for a physically realizable device)

(4) If the three-port network is non-reciprocal then $S_{ij} \neq S_{ji}$ and the conditions of input matching at all ports and energy conservation can be satisfied. Such device is known as a circulator.

Circulators uses ferrite material (anisotropic material) to achieve non-reciprocal behavior.

Then scattering matrix is

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

If the network is lossless [S] must be unitary i.e

$$S_{31}^* S_{32} = 0, \quad S_{21}^* S_{23} = 0, \quad S_{12}^* S_{13} = 0$$

$$|S_{12}|^2 + |S_{13}|^2 = 1, \quad |S_{21}|^2 + |S_{23}|^2 = 1, \quad |S_{31}|^2 + |S_{32}|^2 = 1$$

Above equations can be satisfied in two ways

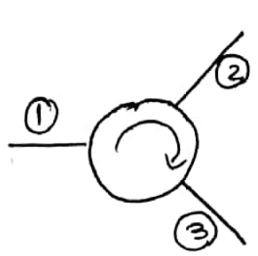
Either $S_{12} = S_{23} = S_{31} = 0$; $|S_{21}| = |S_{32}| = |S_{13}| = 1$ — (A)

Or $S_{21} = S_{32} = S_{13} = 0$, $|S_{12}| = |S_{23}| = |S_{31}| = 1$ — (B)

Then the device is non-reciprocal → Circulator

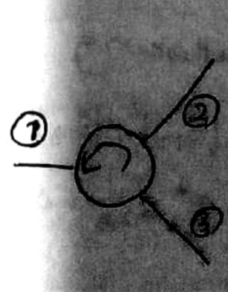
For (A)

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



For (B)

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



* A lossless and reciprocal three-port network can be physically realized if only two of its ports are matched. Then

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

To be lossless, the following unitary cond^n must be satisfied.

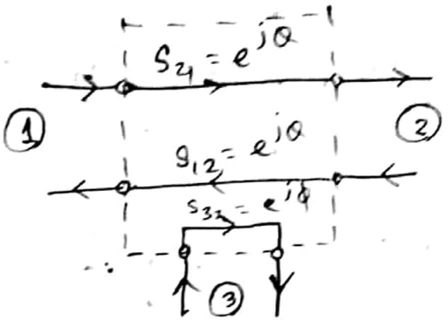
$0 \cdot S_{12} + S_{12}^* \cdot 0 + S_{13}^* \cdot S_{23} = 0 \Rightarrow S_{13}^* S_{23} = 0$

$S_{12}^* S_{13} + 0 \cdot S_{23} + S_{23}^* \cdot S_{33} = 0 \Rightarrow S_{12}^* S_{13} + S_{23}^* S_{33} = 0$

$$S_{13}^* S_{23} = 0 \quad , \quad S_{12}^* S_{13} + S_{23}^* S_{33} = 0 \quad , \quad S_{23}^* S_{12} + S_{33}^* S_{13} = 0$$

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad , \quad |S_{12}|^2 + |S_{23}|^2 = 1 \quad , \quad |S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1$$

$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



A reciprocal, lossless three-port network matched at port 1 and 2.

* Four Port Networks (Directional Couplers)

For a reciprocal four-port network matched at all ports has following ports -

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

If the network is lossless, unitary (energy conservation)

$0 \cdot S_{12} + S_{12}^* \cdot 0 + S_{13}^* S_{23} + S_{14}^* S_{24} = 0$

$\Rightarrow S_{13}^* S_{23} + S_{14}^* S_{24} = 0$ — (a)

$S_{13}^* \cdot S_{13} + S_{24}^* S_{23} + 0 \cdot S_{34}^* + 0 \cdot S_{34} = 0$

$\Rightarrow S_{14}^* S_{13} + S_{24}^* S_{23} = 0$ — (b)

Multiplying S_{24}^* with eq^n (a)

$S_{13}^* S_{23} S_{24}^* + S_{14}^* |S_{24}|^2 = 0$ — (c)

Multiplying S_{13}^* with eq^n (b)

$|S_{13}|^2 S_{14}^* + S_{24}^* S_{23} S_{13}^* = 0$ — (d)

Subtracting (c) from (d)

$S_{14}^* (|S_{13}|^2 - |S_{24}|^2) = 0$ — (e)

Similarly multiplication of row 1 & row 3 and row 4 and row 2

$S_{12}^* S_{23} + S_{14}^* S_{34} = 0$ — (f)

$S_{14}^* S_{12} + S_{34}^* S_{23} = 0$ — (g)

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Multiply eq^n (f) with S_{12} and eq^n (g) with S_{34} and subtract, then

$$S_{28}(|S_{12}|^2 - |S_{34}|^2) = 0 \quad \text{— (h)}$$

eq^n (e) & eq^n (h) will satisfy, if

$$\boxed{S_{14} = S_{23} = 0} \quad \text{— (i)} \rightarrow \text{No flow in backward dir}^n$$

⇒ Results in Directional coupler.

The self product of rows of unitary matrix are

$$|S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \Rightarrow |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{— (j)}$$

$$|S_{12}|^2 + |S_{23}|^2 + |S_{24}|^2 = 1 \Rightarrow |S_{12}|^2 + |S_{24}|^2 = 1 \quad \text{— (k)}$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{34}|^2 = 1 \Rightarrow |S_{13}|^2 + |S_{34}|^2 = 1 \quad \text{— (l)}$$

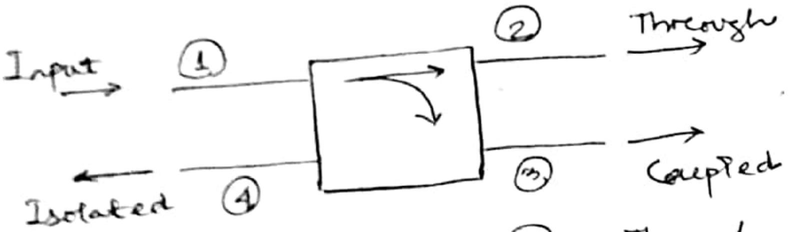
$$|S_{14}|^2 + |S_{24}|^2 + |S_{34}|^2 = 1 \Rightarrow |S_{24}|^2 + |S_{34}|^2 = 1 \quad \text{— (m)}$$

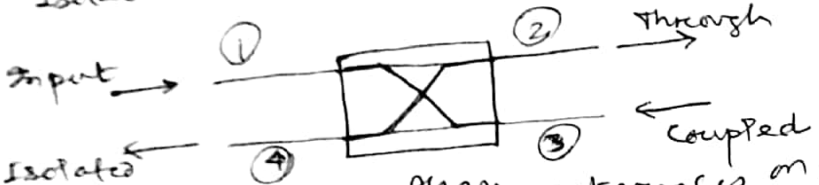
from (j) & (k)

$$|S_{13}|^2 = |S_{24}|^2 \Rightarrow \boxed{|S_{13}| = |S_{24}|}$$

from (l) & (m)

$$\boxed{|S_{12}| = |S_{34}|}$$





Assume ~~references~~ phase references on ports.

$$\boxed{\begin{aligned} S_{12} &= S_{34} = \alpha \\ S_{13} &= \beta e^{j\theta} \\ S_{24} &= \beta e^{j\phi} \end{aligned}}$$

Where

α, β = phase constant = Real

θ, ϕ = " " to be determined

Dot product of rows 2 and 3

$$S_{12}^* S_{13} + S_{24}^* S_{34} = 0$$

$$\Rightarrow \alpha \cdot \beta e^{j\theta} + \beta e^{-j\phi} \cdot \alpha = 0$$

$$\Rightarrow \alpha\beta(e^{j\theta} + e^{-j\phi}) = 0$$

$$\Rightarrow e^{j\theta} + \frac{1}{e^{j\phi}} = 0$$

$$\Rightarrow e^{j(\theta+\phi)} + 1 = 0$$

$$\Rightarrow \alpha\beta\, e^{-j(\theta+\phi)} = 0$$

$$e^{-j(\theta+\phi)} = 0$$

$$\Rightarrow \theta + \phi = \pi \pm 2n\pi$$

* If we ignore $2n\pi$ then

$$\boxed{\theta + \phi = \pi} \quad -(1)$$

Eq (1) has two possible cases

<u>case-I</u> $\theta = \phi = \pi/2$

then it is called Symmetrical coupler

$$\beta e^{j\theta} = \beta e^{j\phi}$$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

<u>case-II</u> $\theta = 0$ & $\phi = \pi$

Then it is called Antisymmetrical coupler

$$\beta e^{j\theta} = -\beta e^{j\phi}$$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$\boxed{\alpha^2 + \beta^2 = 1}$$

- Any reciprocal, lossless, matched four port network is a directional coupler.

$|S_{13}|^2 = \beta^2$ = Coupling factor

$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$ = Directivity.

For Ideal directional coupler, no power is delivered to port 4 (Isolated port)

→ Coupling $= C = 10 \log \frac{P_1}{P_3} = -20 \log \beta$ dB

→ Directivity $= D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|}$ dB

→ Isolation $= I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}|$ dB

→ Coupling factor indicates the fraction of input power that is coupled to the output port.

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→ The directivity is the measure of coupler's ability to isolate forward power and backward waves.

$$\boxed{I = D + C} \text{ dB.}$$

→ For ideal coupler
directivity & isolation are infinite
$$\Rightarrow \boxed{S_{14} = 0}$$

* Hybrid Couplers:

- They are special case of directional couplers.
- Coupling factor $C = 3\,dB$.
- $\alpha = \beta = \frac{1}{\sqrt{2}}$
- They are two types.
  (a) Quadrature Hybrid
  (b) Magic T-Hybrid or Rat-Race hybrid

(a) Quadrature Hybrid.

→ 90° phase shift betⁿ ports 2 & 3 ($\theta = \phi = \pi/2$) when fed at port 1.

→ It is a symmetrical coupler

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{2} & j1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 0 & 0 & j1/\sqrt{2} \\ j1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\ 0 & j1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

(b) Rat Race Hybrid or Magic-T Hybrid.

→ 180° phase difference betⁿ port 2 and 3 when fed at port 4

→ It is an antisymmetrical coupler

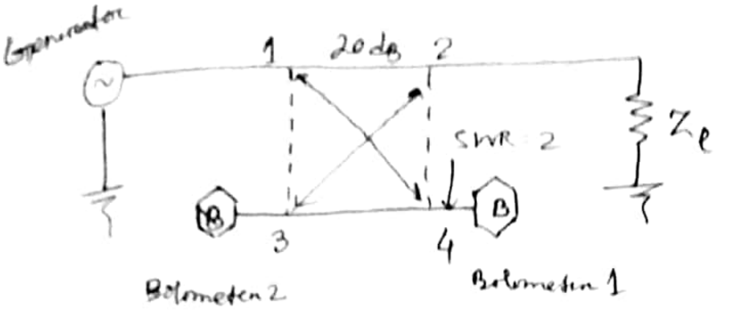
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

*

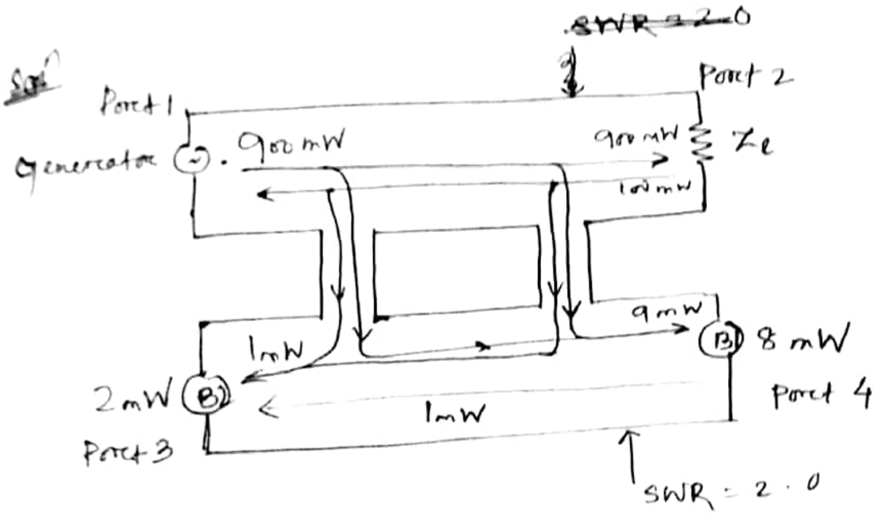
Q. A symmetrical directional coupler with infinite directivity and a forward attenuation of 20 dB is used to monitor the power delivered to a load Z_L. Bolometer 1 introduces a VSWR of 2.0 on arm 4, bolometer 2 is matched to arm 3. ~~If bolometer~~ If bolometer 1 reads 8mW and bolometer 2 reads 2mW, find

(a) Amount of power dissipated in Z_L

(b) the VSWR on arm 2.



Sol.



(a) Power dissipated at Z_L

(1) Reflection co-efficient at port 4 is

$$|\Gamma| = \frac{\rho - 1}{\rho + 1} = \frac{2-1}{2+1} = \frac{1}{3}$$

(2) $$|\Gamma|^2 = \frac{P^-}{P^+} \Rightarrow \frac{1}{9} = \frac{P^-}{P^+} \Rightarrow \left.\begin{matrix} P^- = 1\ mW \\ P^+ = 9\ mW \end{matrix}\right\} \text{ at Port 4}$$

(3) From port 4 1mW is reflected to port 3 side. Bolometer at Port 3 reads 2mW.

(4) $$20\,dB = 10 \log_{10}\left(\frac{P_1}{P_4^+}\right) \Rightarrow 2 = \log_{10}\left(\frac{P_1}{P_4^+}\right) \Rightarrow \frac{P_1}{P_4^+} = 10^2 \Rightarrow \frac{P_1}{P_4^+} = \frac{100}{1}$$

$$P_1 = 100 P_4^+ \Rightarrow P_1 = 100 \times 9\,mW = 900\,mW$$

Power reflected from load is

$$20\,dB = 10 \log_{10} \frac{P_2^-}{P_3} \Rightarrow \frac{P_2^-}{P_3} = 100 \Rightarrow P_2^- = 100 \times 1\,mW = 100\,mW$$

(5) Power dissipated in the load is

$$P_L = P_2^+ - P_2^- = 900 - 100 = 800\,mW$$

(b) The reflection co-efficient is calculated as

$$|\Gamma| = \sqrt{\frac{P_2^-}{P_2^+}} = \sqrt{\frac{100}{900}} = \frac{1}{3}$$

Then VSWR on arm 2 is

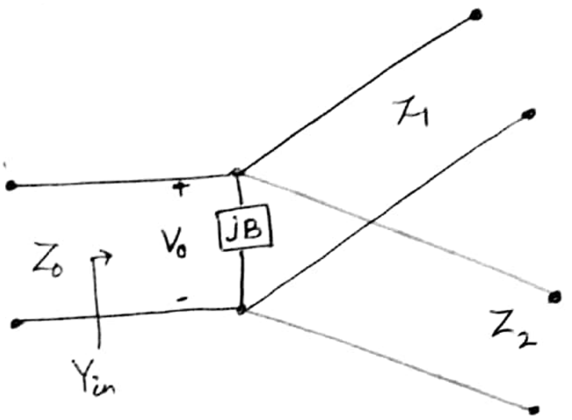
$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/3}{1-1/3} = 2.0$$

* THE T-JUNCTION POWER DIVIDER

→ Three port networks
→ Used for power division or power combining
→ These junctions can't be matched simultaneously at all ports.

* Transmission line Model of a Lossless T-junction:—

(a) Lossless Divider



Modeled as junction of 3 Tx lines.

Divider should be matched with input line

$$Y_{in} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}$$

If Tx lines are ~~lossless~~ $(B=0)$ then

$$\boxed{\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}}$$

If B is not zero, we can add some reactive elements to cancel that susceptance.

Ex A lossless T-junction power divider has a source impedance of $50\,\Omega$. Find the o/p characteristics impedances so that the input power is divided in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.

Sol^n If the voltage at the junction is V_0, the input power to the matched divider is

$$P_{in} = \frac{1}{2}\frac{V_0^2}{Z_0}$$

While the o/p powers are

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{in}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{in}$$

The C/cs impedances are

$$Z_1 = 3Z_0 = 150\,\Omega, \quad Z_2 = \frac{3Z_0}{2} = 75\,\Omega$$

Input impedance to the junction is

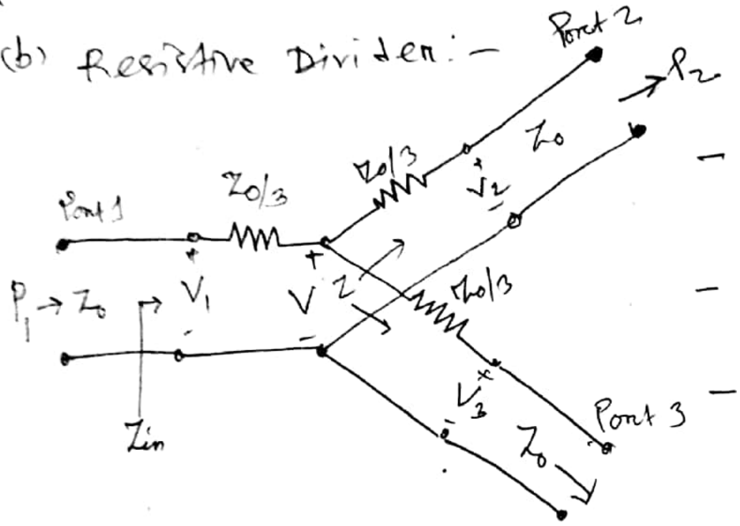
$$Z_{in} = Z_1 \parallel Z_2 = 75 \parallel 150 = 50\,\Omega$$

Looking into the 150 Ω o/p line, we see an impedance of $50 \| 75 = 30\,\Omega$
" " " 75 Ω " " , " " " " " $50 \| 150 = 37.5$

Then $\Gamma_1 = \dfrac{30-150}{30+150} = -0.666$

$\Gamma_2 = \dfrac{37.5-75}{37.5+75} = -0.333$

(b) Resistive Divider :-



- This is an equal-split (−3dB) divider.
- To solve this network we can use circuit theory.
- Assume all ports are terminated in c/s impedance Z_0

$Z = \dfrac{4}{3}\left(\dfrac{Z_0}{3} + Z_0\right) = \dfrac{4Z_0}{3}$

$Z_{in} = Z_0/3 + \dfrac{4Z_0/3}{2}$ $\quad \left(\text{as } \left(Z_0/3 + Z_0\right) \| \left(\dfrac{Z_0}{3} + Z_0\right)\right)$ these two arms

$= Z_0/3 + 2Z_0/3 = Z_0$

As $Z_{in} = Z_0$ this is a matched line network.

$S_{11} = S_{22} = S_{33} = 0$

If voltage at port 1 is V_1 then

$V = V_1 \cdot \dfrac{2Z_0/3}{Z_0/3 + 2Z_0/3} \Rightarrow V = V_1 \dfrac{2Z_0/3}{Z_0} \Rightarrow \boxed{V = \frac{2}{3}V_1}$

Power delivered to input port $= \frac{1}{2}\dfrac{V_1^2}{Z_0} = P_{in}$

Output voltage

$V_2 = V_3 = V \cdot \dfrac{Z_0}{Z_0 + Z_0/3} \Rightarrow \boxed{V_2 = V_3 = \frac{3}{4}V} \Rightarrow \boxed{V_2 = V_3 = \frac{1}{2}V_1}$

Power delivered at o/p ports $P_3 = P_4 = \frac{1}{2}\dfrac{\left(\frac{1}{2}V_1\right)^2}{Z_0} = \frac{1}{8}\dfrac{V_1^2}{Z_0} = \frac{1}{4}P_{in}$

$S_{21} = \dfrac{V_2}{V_1} = \dfrac{1}{2}$

$S_{31} = \dfrac{V_3}{V_1} = \dfrac{1}{2}$

$S_{23} = \dfrac{V_2}{V_3} = 1/2$

The network is reciprocal so [S] is symmetric

$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \dfrac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

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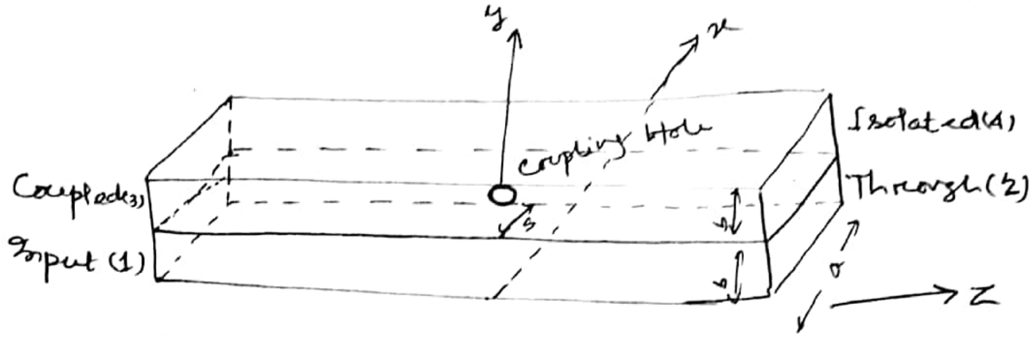
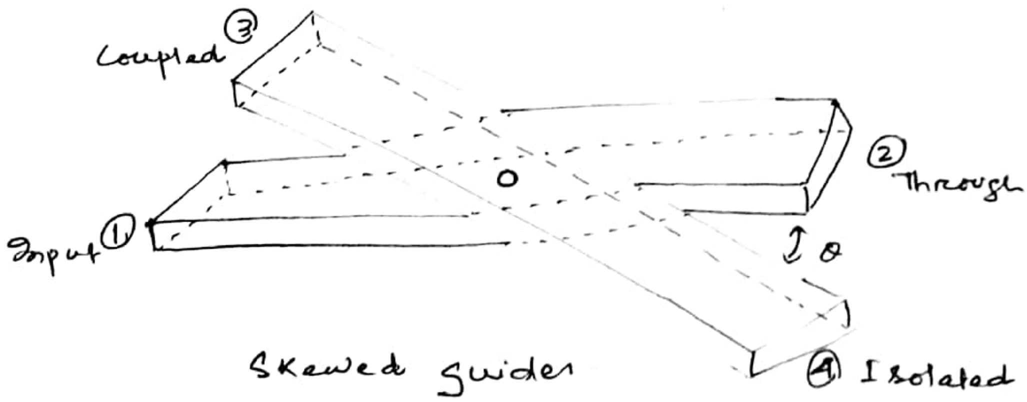
* Bethe Hole Coupler :-

→ Here one waveguide is Coupled to another through a single hole in the common broad wall betn the Two guides.

→ The single hole or aperture can be treated as a source Consisting of electric and magnetic dipole moments.



Parallel Guide.



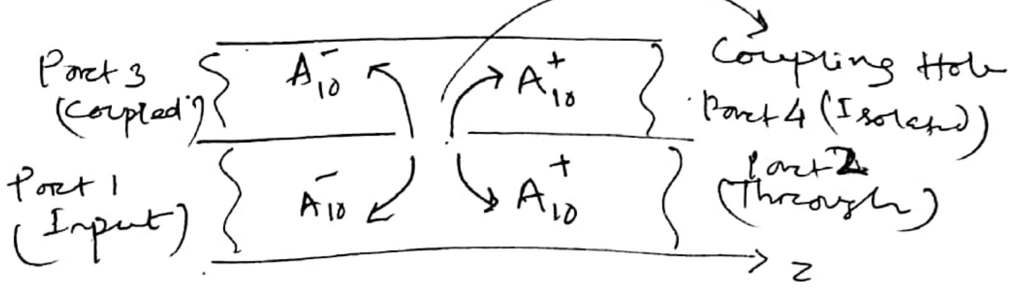
Skewed guides

S = Aperture offset from the side wall of the guide.

The wave amplitudes are controlled by the angle θ.

Let incident TE_{10} mode into port 1

→ 2 wave components add in phase at the coupler port and are cancelled at the isolation port



$E_y = A \sin\frac{\pi x}{a} e^{-j\beta z}$

$H_x = \frac{-A}{Z_{10}} \sin\frac{\pi x}{a} e^{-j\beta z}$

$H_z = \frac{j\pi A}{\beta a Z_{10}} \cos\frac{\pi x}{a} e^{-j\beta z}$

A = amplitude of electric field (V/m)

$Z_0 = \frac{\eta_0}{\sqrt{1-(\lambda_0/2a)^2}}$ = Wave impedance dominate mode

$\beta = k_0\sqrt{1-(\lambda_0/2a)^2}$ = phase constant

$k_0 = 2\pi/\lambda_0$

In the bottom guide the amplitude of the forward scattered wave is given by

$$A_{10}^{+} = -\frac{j\omega A}{P_{10}}\left[\epsilon_0\alpha_e \sin^2\frac{\pi s}{a} - \frac{\mu_0\alpha_m}{Z_{10}^2}\left(\sin^2\frac{\pi s}{a} + \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] \quad -(a)$$

Amplitude of the reversed scattered wave is given by

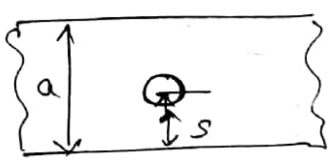
$$A_{10}^{-} = -\frac{j\omega A}{P_{10}}\left[\epsilon_0\alpha_e \sin^2\frac{\pi s}{a} + \frac{\mu_0\alpha_m}{Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] \quad -(b)$$

where

$P_{10} = \frac{ab}{Z_{10}}$ Power normalization constant | $\alpha_e = \frac{2}{3}r_0^2$ Electric Polarizability | $\alpha_m = \frac{4}{3}r_0^2$ Magnetic Polarizability | r_0 = Radius of Aperture

A_{10}^{+} and A_{10}^{-} can be calculated from the electric polarization current ~~and~~ (P_e) and magnetic polarization current (P_m) near ~~an~~ conducting walls around an aperture. P_e and P_m are related to $\bar{J}$ & $\bar{M}$.

If we make $A_{10}^{+} = 0$ then power delivered to ~~From eq (a)~~ port 4 is cancelled.



for making $A_{10}^{+} = 0$

$$\Rightarrow -\frac{j\omega A}{P_{10}}\left[\epsilon_0\alpha_e \sin^2\frac{\pi s}{a} - \frac{\mu_0\alpha_m}{Z_{10}^2}\left(\sin^2\frac{\pi s}{a} + \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] = 0$$

$$\Rightarrow \epsilon_0\cdot\frac{2}{3}r_0^2 \sin^2\frac{\pi s}{a} - \frac{\mu_0}{Z_{10}^2}\frac{4}{3}r_0^2\sin^2\frac{\pi s}{a} - \frac{\mu_0}{Z_{10}^2}\cdot\frac{4}{3}r_0^2\frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a} = 0$$

$$\Rightarrow \frac{r_0^2}{3}\sin^2\frac{\pi s}{a}\left(2\epsilon_0 - \frac{4\mu_0}{Z_{10}^2}\right) = \frac{4\mu_0}{Z_{10}^2}\cdot\frac{r_0^2}{3}\frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}$$

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$$\Rightarrow \left(2\epsilon_0 - \frac{4\mu_0}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} = \frac{4\pi^2\mu_0}{Z_{10}^2\beta^2 a^2}\cos^2\frac{\pi s}{a} \qquad \left(\because Z_{10} = k_0\eta_0/\beta\right)$$

$$\Rightarrow \left(2\epsilon_0 - \frac{4\mu_0\beta^2}{k_0^2\eta_0^2}\right)\sin^2\frac{\pi s}{a} = \frac{4\pi^2\mu_0\beta^2}{k_0^2\eta_0^2\beta^2 a^2}\cos^2\frac{\pi s}{a} \qquad \left(\sqrt{\mu_0/\epsilon_0} = \eta_0\right)$$

$$\Rightarrow \left(2\epsilon_0 k_0^2\eta_0^2 - 4\mu_0\beta^2\right)\sin^2\frac{\pi s}{a} = \frac{4\pi^2\mu_0}{a^2}\cos^2\frac{\pi s}{a}$$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = (1-\sin^2\theta)$

$$\Rightarrow 2\mu_0\left(\frac{\epsilon_0}{\mu_0}k_0^2\eta_0^2 - 2\beta^2\right)\sin^2\frac{\pi s}{a} = \frac{4\pi^2\mu_0}{a^2}\cos^2\frac{\pi s}{a}$$

$$\Rightarrow \left(k_0^2 - 2\beta^2\right)\sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2}\cos^2\frac{\pi s}{a}$$

$$\Rightarrow \left(k_0^2 - 2\beta^2\right)\sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2}\cdot 1 - \frac{2\pi^2}{a^2}\sin^2\frac{\pi s}{a}$$

$$\Rightarrow \left(k_0^2 - 2\beta^2 + \frac{2\pi^2}{a^2}\right)\sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2} \qquad \left(-2\beta^2 = -2k_0^2 + \frac{2\pi^2}{a^2}\right)$$

$\cancel{\Rightarrow k_0^2 - 2}$

$$\Rightarrow \left(\frac{4\pi^2}{a^2} - k_0^2\right)\sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2}$$

$$\Rightarrow \sin\frac{\pi s}{a} = \pi\sqrt{\frac{2}{4\pi^2 - k_0^2 a^2}} \Rightarrow \boxed{\sin\frac{\pi s}{a} = \frac{\lambda_0}{\sqrt{2(\lambda_0^2 - a^2)}}}$$

→ Coupling Factor for single hole Bethe coupler is

$$C = 20\log\left|\frac{A}{A_{10}^-}\right| \text{ (dB)}$$

→ Directivity is

$$D = 20\log\left|\frac{A_{10}^-}{A_{10}^+}\right| \text{ (dB)}$$

For skewed geometry $\quad s = a/2, \ \alpha_m\cos\theta$

$$A_{10}^+ = \frac{-j\omega A}{P_{10}}\left(\epsilon_0\alpha_e - \frac{\mu_0\alpha_m}{Z_{10}^2}\cos\theta\right)$$

$$A_{10}^- = -\frac{j\omega A}{P_{10}}\left(\epsilon_0\alpha_e + \frac{\mu_0\alpha_m}{Z_{10}^2}\cos\theta\right)$$

By setting $A_{10}^+ = 0$

$$\cos\theta = \frac{k_0^2}{2\beta^2}$$

$$C = 20\log\frac{4k_0^2 r_0^3}{3ab\beta} \text{ dB}$$

Ex: Design a Bethe hole coupler for X band waveguide operating at 9 GHz with a coupling of 20 dB. Calculate the coupling and directivity. Assume a round aperture.

Sol: For x-band waveguide at 9 GHz

$a = 2.286$ cm $= 0.02286$ m | $k_0 = 188.5\ m^{-1}$

$b = 1.016$ cm $= 0.01016$ m | $\beta = 129.0\ m^{-1}$

$\lambda_0 = 0.0333$ m | $Z_{10} = 530.9\ \Omega$

$P_{10} = 4.22 \times 10^{-7}\ m^2/\Omega$

$$\sin\frac{\pi s}{a} = \frac{\lambda_0}{\sqrt{2(\lambda_0^2 - a^2)}} = 0.972$$

$$\Rightarrow s = \frac{a}{\pi}\sin^{-1}(0.972) \Rightarrow s = 9.69\ mm$$

$$C = 20\ dB = 20\log\left|\frac{A}{A_{10}^-}\right| \Rightarrow \left|\frac{A}{A_{10}^-}\right| = 10^{20/20} = 10$$

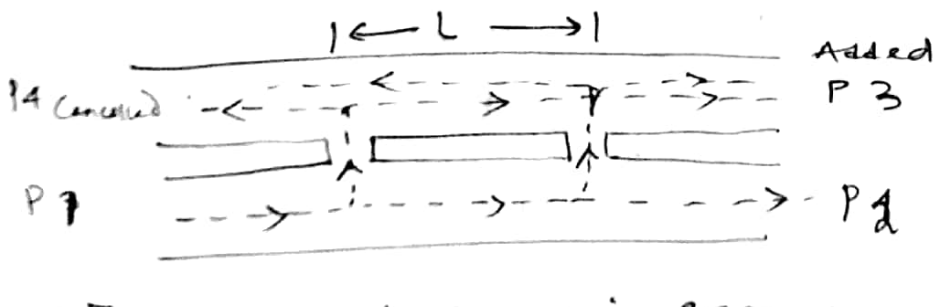
$$\left|\frac{A_{10}^-}{A}\right| = \frac{1}{10} = \frac{\omega}{P_{10}}\left[\left(\epsilon_0\alpha_e + \frac{\mu_0\alpha_m}{Z_{10}^2}\right)(0.944) - \frac{\pi^2\mu_0\alpha_m}{\beta^2 a^2 Z_{10}^2}(0.056)\right]$$

* Multihole couplers has a narrow bandwidth

* Two hole directional couplers

The spacing bet^n the centers of two holes must be

$$L = (2n+1)\lambda_g/4 \qquad n = 1, 2, 3 \ldots$$



- The forward waves in secondary guide are in same phase, regardless of hole space, and are added at port 3.
- Backward waves in secondary guide are out of phase by $(2L/\lambda_g)2\pi$ rad and are canceled at port 4.

Ferrimagnetic compounds →

→ The most practical anisotropic materials for microwave applications are ferrimagnetic compounds such as YIG (Yattrium iron garnet) and ferrites composed of iron oxides with elements like aluminium, cobalt, manganese, and nickel.

→ Ferrimagnetic compounds have high resistivity and a significant amount of anisotropy at microwave frequencies.

→ Magnetic anisotropy of a ferrimagnetic material is actually induced by applying a DC magnetic bias field. This field aligns the magnetic dipoles in the ferrite material to produce a non zero magnetic dipole moment and the magnetic dipoles can be varied at frequency controlled by strength of bias field.

(of Ferrimagnetic material)

→ the sense of polarization changes with direction of propagation. It is utilized in isolators, circulators and gyrators.

→ By adjusting the strength of the bias field the microwave signal propagation can be change inside ferrimagnetic material.

→ the magnetic properties of a material are due to the existance of magnetic dipole moments which arise from electron spin.

→ Ferrite is a family of $MeO \cdot Fe_2O_3$ where M_e is a divalent iron metal.

→ Ferrite material is affected by a dc magnetic field to produce faraday rotation. because it is a non linear material and its permeability is an asymmetric tensor.

$$\boxed{B = \mu H}$$

where $\hat{\mu} = \mu_0 (1 + \hat{\chi}_m)$

$$\hat{\chi}_m = \begin{bmatrix} \chi_m & jK & 0 \\ jK & \chi_m & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

χ_m = Diagonal susceptibility

jK = off diagonal susceptibility

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→ When a dc ~~material~~ magnetic field is applied to a ferrite, the unpaired electrons in the ferrite material tend to align themselves with dc field of their magnetic dipole moment.

The nonreciprocal c/s of unpaired electron in ferrite causes unequal relative permeability due to which the wave in ferrite is circular polarized.

→ Propagation constant for a linearly polarized wave inside the ferrite can be expressed as

$$\gamma^{\pm} = j\omega\sqrt{\epsilon\mu_0(\mu \pm k)}$$

Where $\mu = 1 + \hat{\chi}_m$

$\mu_r^{+} = \mu + k$

$\mu_r^{-} = \mu - k$.

→ Relative permeability μ_r changes with applied dc magnetic field

$$\boxed{\mu_r^{\pm} = 1 + \frac{\gamma_e M_e}{|\gamma_e| H_{dc} \mp \omega}}$$

γ_e = Gyromagnetic ratio of an electron

M_e = Saturation Magnetization

ω = Angular frequency of a microwave field

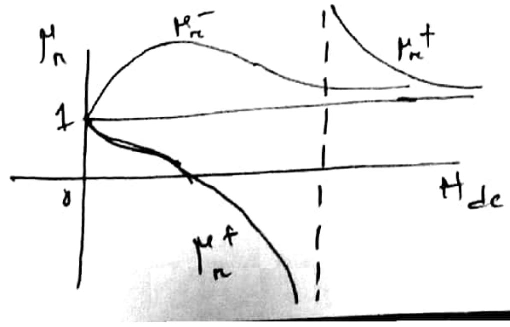
H_{dc} = dc Magnetic field.

μ_r^{+} = Relative Permeability in clockwise direction (Right or positive circular polarization)

μ_r^{-} = Relative Permeability in anticlockwise direction. (Left or negative circular polarization)

→ Gyromagnetic Resonance

If $\omega = |\gamma_e| H_{dc} \Rightarrow \mu_r = \infty$.

→ 

If $\mu_r^+ \gg \mu_r^- \Rightarrow$ Wave in ferrite is rotated in CW direction.

Consequently $\boxed{\omega = (\beta^+ - \beta^-)\ell = \pi/2}$

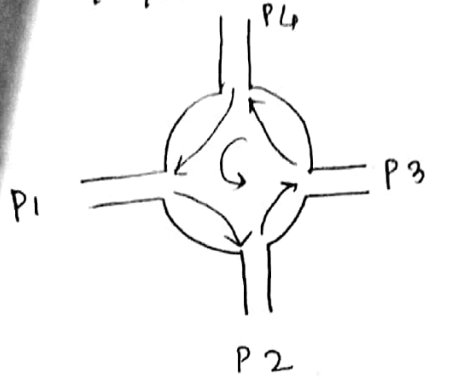
β^+ = ~~Propagation~~ phase constant for forward dirⁿ

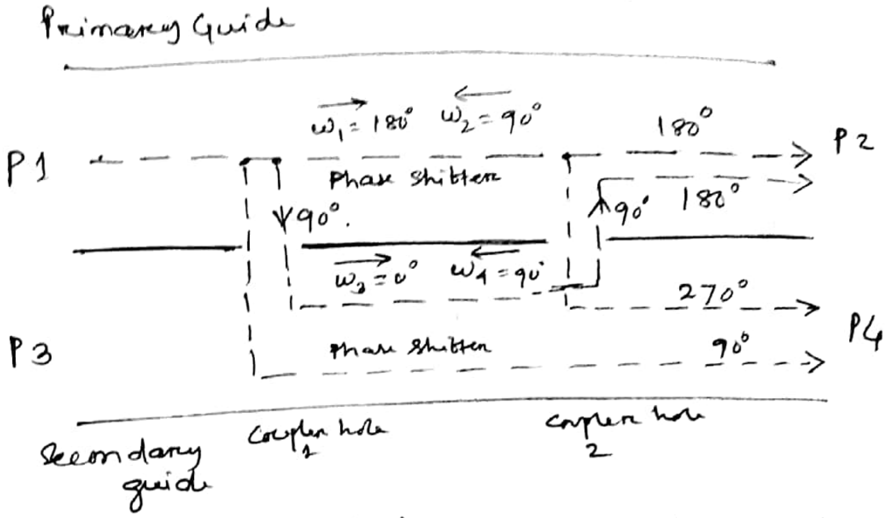
β^- = " " " backward "

ℓ = length of ferrite slab.

Microwave Circulators:-

- It is a multiport waveguide junction in which the wave can flow only from nth port to (n+1)th port in one direction.
- 4 port microwave circulators are a commonly used.





Schematic diagram of 4 port circulat

- When the wave is incident to port 1, the wave is split into t components by coupler 1.
- The through wave at port 2 arrives with a phase shift of 18
- The second wave propagates through the two couplers and secondary guide and arrives at port 2 with a relative phase change of 180°.
- At port 4 one wave has a phase shift of 270° & other has 90°. So net 180° phase shift is there. So at port 4 power transmission from port 1 to port 4 is zero.
- For a perfectly matched, lossless and non reciprocal 4 port circulator has an S-matrix of the form

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$ and using properties of S-parameters

$$\Rightarrow S = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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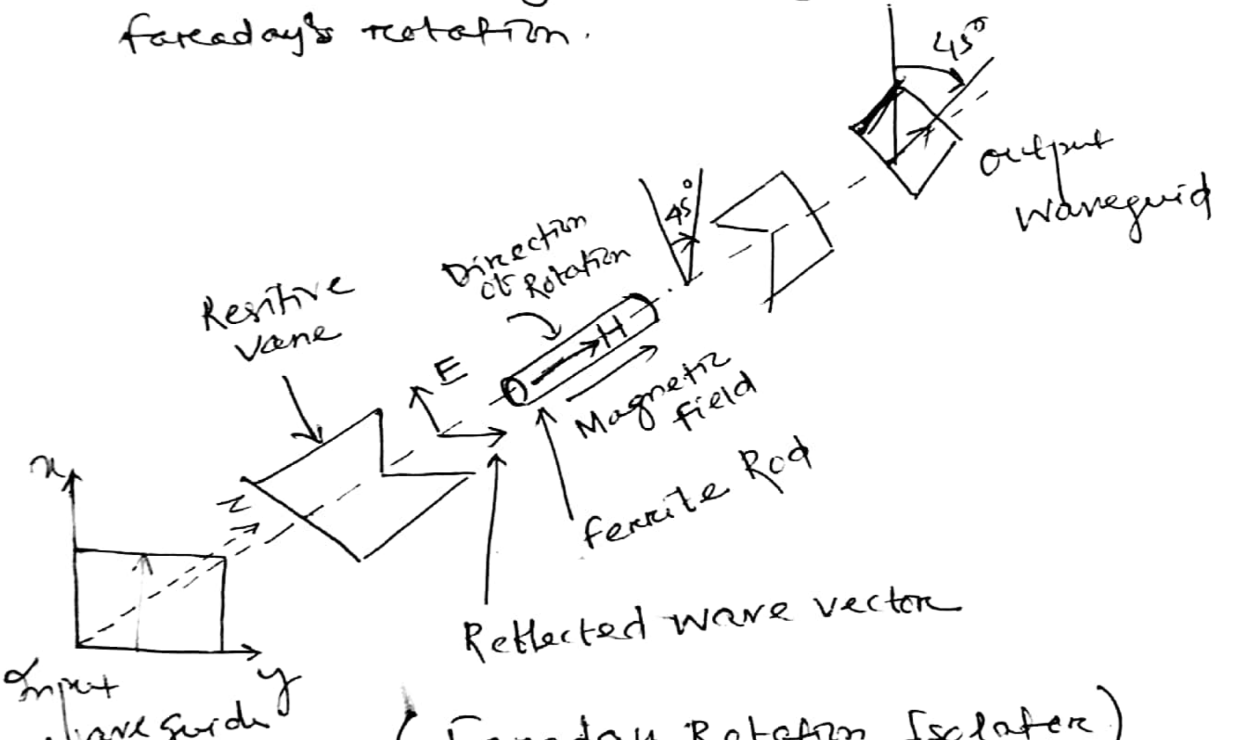
* Microwave Isolators

- An ~~used~~ isolator is a non-reciprocal device that is used to isolate one component from reflections of other components.
- Ideal isolator completely absorbs the power in one direction and provided lossless transmission in opposite direction.
- So it is called Uniline.
- They are used to ~~increase~~ improve frequency stability of microwave generators.
- Isolator can be made by inserting a ferrite rod along the axis of a rectangular waveguide. This uses Principle of faraday's rotation.



(Faraday Rotation Isolator)

- The input resistive card is in YZ plane and at a shift of 45° with respect to output resistive card.
- The dc magnetic field applied longitudinally to ferrite rod rotates the wave plane by 45°. This degree of ~~rotation~~ rotation depends on the length and diameter of ferrite rod and applied magnetic field.
- As the incident wave with TE_{10} mode $\perp^r$ to input resistive card, it passes through the ferrite rod without attenuation. But this wave will be rotated after passing through ferrite rod. So the reflected wave is no longer $\perp^r$ to input resistive card and absorbed ~~by~~ by it.

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CHAPTER 1

INTRODUCTION TO RADAR SYSTEM

1.1 Introduction:-

Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform, a pulse-modulated sine wave for example, and detects the nature of the echo signal. Radar is used to extend the capability of one's senses for observing the environment, especially the sense of vision.

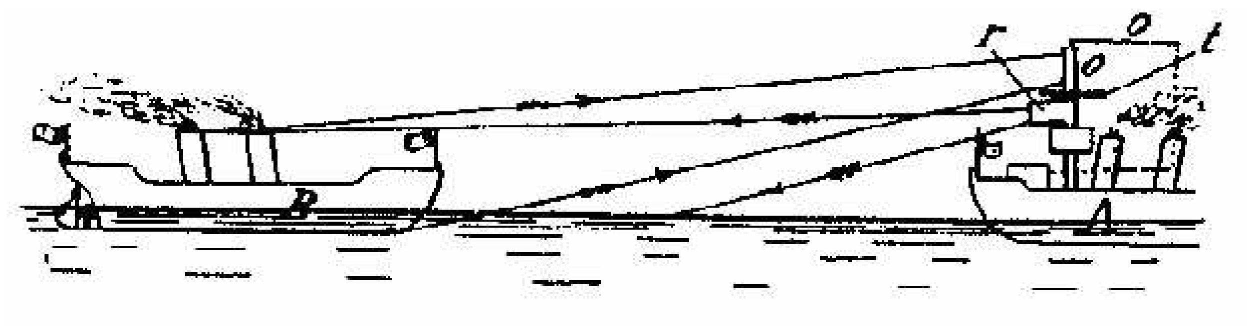
An elementary form of radar consists of a transmitting antenna emitting electromagnetic radiation generated by an oscillator of some sort, a receiving antenna, and an energy-detecting device, or receiver. A portion of the transmitted signal is intercepted by a reflecting object (target) and is reradiated in all directions. It is the energy reradiated in the back direction that is of prime interest to the radar. The receiving antenna collects the returned energy and delivers it to a receiver, where it is processed to detect the presence of the target and to extract its location and relative velocity.

The distance to the target is determined by measuring the time taken for the radar signal to travel to the target and back. The direction, or angular position, of the target may be determined from the direction of arrival of the reflected wave- front. The usual method of measuring the direction of arrival is with narrow antenna beams. If relative motion exists between target and radar, the shift in the carrier frequency of the reflected wave (Doppler Effect) is a measure of the target's relative (radial) velocity and may be used to distinguish moving targets from stationary objects. In radars which continuously track the movement of a target, a continuous indication of the rate of change of target position is also available.

1.2 History Background

James Clerk Maxwell (1831 –1879) - predicted the existence of radio waves in his theory of electromagnetism. In 1886, Hertz experimentally tested the theories of Maxwell and demonstrated the similarity between radio and light waves. Hertz showed that radio waves could be reflected itself. Heinrich Hertz, in 1886, experimentally tested the theories of Maxwell and demonstrated the similarity between radio and light waves. Hertz showed that radio waves could be reflected by metallic and dielectric bodies. Due to these reflections occurred through metallic bodies given a start to the development of radar systems.

In 1903 a German engineer by the name of Hiilsmeyer experimented with the detection of radio waves reflected from ships. He obtained a patent in 1904 in several countries for an radio waves reflected from ships as shown in fig.1.



(a)

(b)

Fig. 1 (a) Detection of wooden ship in 1904 **(b)** Hülsmeyer 1904, who detected the first object through radar

In the autumn of 1922 A. H. Taylor and L. C. Young of the Naval Research Laboratory detected a wooden ship using a CW wave-interference radar with separated receiver and transmitter. The wavelength was 5 m. The first application of the pulse technique to the measurement of distance was in the basic scientific investigation by Breit and Tuve in 1925 for measuring the height of the ionosphere. However, more than a decade was to elapse before the detection of aircraft by pulse radar was demonstrated.

The first detection of aircraft using the wave-interference effect was made in June, 1930, by L. A. tlyland of the Naval Research Laboratory.' It was made accidentally while he was working with a direction-finding apparatus located in an aircraft on the ground. The transmitter at a frequency of 33 MHz was located 2 miles away, and the beam crossed an air lane from L. Hyland of the Naval Research Laboratory. It was made accidentally while he was working with a direction-finding apparatus located in an aircraft on the ground. The transmitter at a frequency of 33 MHz was located 2 miles away, and the beam crossed an air lane from a nearby airfield.

Before the advent of radar, the only practicable means of detection of aircraft was acoustic, and a network of acoustic detectors was built in the 1920s and 1930s around the south and east coast of the UK, some of which still remain. In calm air conditions, detection ranges of up to 25km were achievable.

(a)

(b) (c)

Fig. 2 Different types of Acoustic Radars from 1920-1930

Radar Applications:-

In aviation, aircraft are equipped with radar devices that warn of aircraft or other obstacles in or approaching their path, display weather information, and give accurate altitude readings. The first commercial device fitted to aircraft was a 1938 Bell Lab unit on some United Air Lines aircraft. Such aircraft can land in fog at airports equipped with radar-assisted ground-controlled approach systems in which the plane's flight is observed on radar screens while operators radio landing directions to the pilot.

Marine radars are used to measure the bearing and distance of ships to prevent collision with other ships, to navigate, and to fix their position at sea when within range of shore or other fixed references such as islands, buoys, and lightships. In port or in harbour, vessel traffic service radar systems are used to monitor and regulate ship movements in busy waters.

Normal radar functions:

1. Range (from pulse delay)
2. Velocity (from Doppler frequency shift)
3. Angular direction (from antenna pointing)

Signature analysis and inverse scattering:

4. Target size (from magnitude of return)
5. Target shape and components (return as a function of direction)
6. Moving parts (modulation of the return)
7. Material composition

The complexity (cost & size) of the radar increases with the extent of the functions that the radar performs.

CHAPTER 2: BASIC PRINCIPLES OF RADAR

A radar system has a transmitter that emits radio waves called *radar signals* in moving or stationary target directions. When these come into contact with an object they are usually reflected or scattered in many directions. Radar signals are reflected especially well by materials of considerable electrical conductivity especially by most metals, by seawater and by wet ground. Some of these make the use of radar altimeters possible. The radar signals that are reflected back towards the transmitter are the desirable ones that make radar work. If the object is *moving* either toward or away from the transmitter, there is a slight equivalent change in the frequency of the radio waves, caused by the Doppler effect.

The basic principle of the radar is shown in fig. 2.1. A transmitter generates an electromagnetic signal that is radiated by the antenna into space. A portion of the transmitted electromagnetic energy is reflected back by the target towards the radar. Based on the received target echo signal the receiver made decision for the position, range and direction of the target. The term radar is a contraction of the words radio detection and ranging.

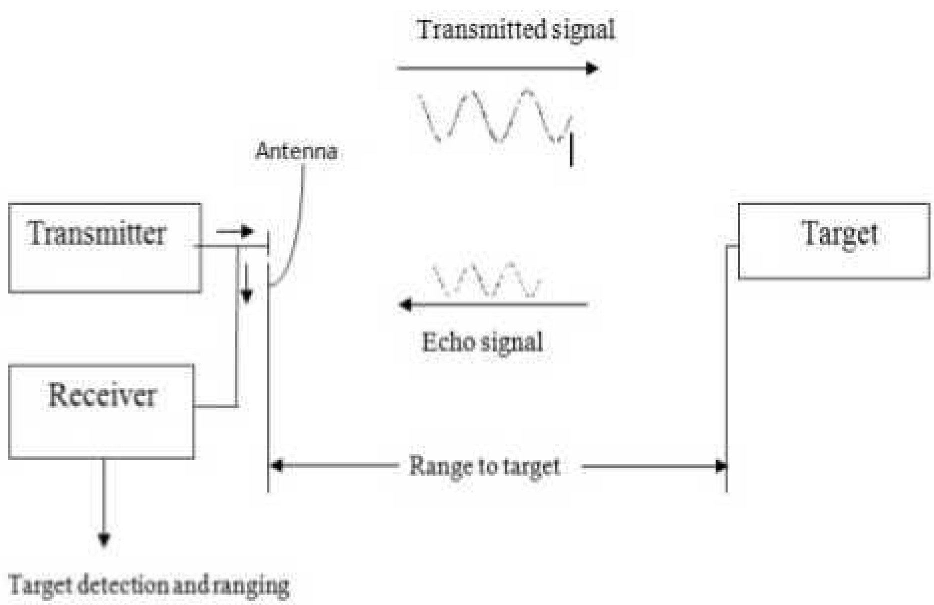


Fig. 2.1 Basic Principles of the Radar

The basic terminology used for radar is discussed as follows.

Range:- The range of the target is observed by measuring the time (T_R) it takes for the radar signal to travel to the target and return back to the radar. Thus the time for the signal to travel to the target located at range (R) and the return back to the radar is $2R/C$. The range of the target can be given as:

$$R = \frac{cT_R}{2} \quad \text{… (1)}$$

with the range in kilometers or in nautical miles, and T in microseconds.

$$\begin{aligned} R(km) &= 0.15T_R\,(\mu s) \\ R(nmi) &= 0.081T_R\,(\mu s) \end{aligned} \quad \text{… (2)}$$

Maximum Unambiguous Range:- Once a signal is radiated into space by a radar, enough time must elapse to allow all echo signal to return to the radar before the transmission of next pulse. The rate at which the pulses are transmitted, is determined by the longest range of the target. If the time between pulses T_P is too short, an echo signal from the long range target might arrive after the transmission of the next pulse. The echo that arrives after the transmission of next pulse is called as *second-time-around-echo (or multiple-time-around-echo*). Such an echo would appear to be at a closer range than actual, this range measurement will be misleading for range calculation, if it is not known that this is second time echo. The range beyond which the target appears as second-time-around-echoes is the *maximum unambiguous range*, R_{un} and is given by

$$\begin{aligned} R_{un} &= \frac{cT_P}{2} = \frac{c}{2f_p} \\ f_p &= \frac{1}{T_P} \\ Duty\,cycle &= \frac{\tau}{T_P} \end{aligned} \quad \text{… (3)}$$

Where T_P is the pulse repletion time and f_p is the pulse repetition frequency.

A problem with pulsed radars and range measurement is how to unambiguously determine the range to the target if the target returns a strong echo. This problem arises because of the fact that pulsed radars typically transmit a sequence of pulses. The radar receiver measures the time between the leading edges of the last transmitting pulse and the echo pulse. It is possible that an echo will be received from a long range target after the transmission of a second transmitting pulse.

In this case, the radar will determine the wrong time interval and therefore the wrong range. The measurement process assumes that the pulse is associated with the second transmitted pulse and declares a much reduced range for the target. This is called range ambiguity and occurs where there are strong targets at a range in excess of the pulse repetition time. The pulse repetition time defines a maximum unambiguous range. To increase the value of the unambiguous range, it is necessary to increase the PRT, this means: to reduce the PRF.

Echo signals arriving after the reception time are placed either into the transmit time where they remain unconsidered since the radar equipment isn't ready to receive during this time, or into the following reception time where they lead to measuring failures (ambiguous returns).

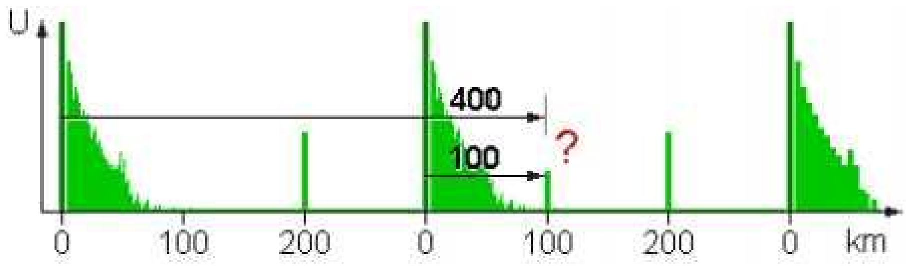


Fig. 2.2 a second-time-arounr-echo in a distance of 400 km assumes a wrong range of 100 km

Pulse Repetition Frequency (PRF):- The rate at which the pulses are transmitted towards the target from the radar is called as the pulse repletion frequency, f_p .

$$f_p = \frac{1}{T_P} \qquad \dots (4)$$

Pulse Repetition Period:- The time interval at which the pulses are periodically transmitted towards the target from the radar is called as the pulse repletion period, T_P is given by in terms of prf.

$$T_p = \frac{1}{f_P} \qquad \dots (5)$$

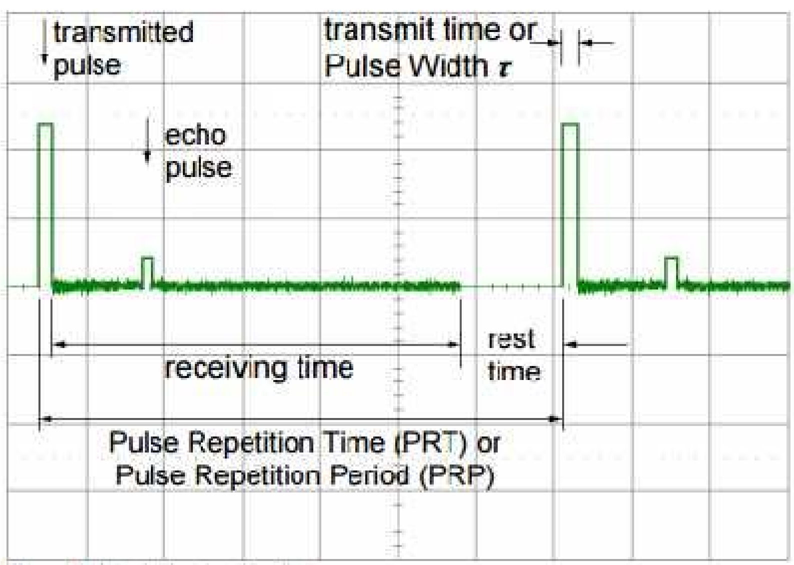


Fig. 2.3 A typical radar time line

Duty Cycle:- The duty cycle of the radar waveform is described as the ratio of the total time the radar is radiating to the total time it could have radiated.

$$Duty\,cycle = \frac{p_{av}}{P_T} \qquad \ldots (6)$$

$$Duty\,cycle = \frac{\tau}{T_P} = \tau f_p \qquad \ldots (7)$$

Where τ is pulse width of the transmitted pulse and T_P is the pulse repetition period.

Peak Power of the Radar:- The maximum power of the radar antenna, that can be transmitted for the maximum unambiguous range target detection in particular direction.

Average Power of the Radar:- The average power of the radar antenna, that can be transmitted for the maximum unambiguous range target detection in all the direction (for isotropic antenna).

Radar Wave forms:- Typical radar utilizes various waveforms for target detection.

- **Pulse waveform:-** A radar uses rectangular pulse wave form with pulse width of 1microsecond, pulse repletion period 1 millisecond.
- **Continuous waveform:-** A very long continuous waveform are required for some long range radars to achieve sufficient energy for small target detection.
-

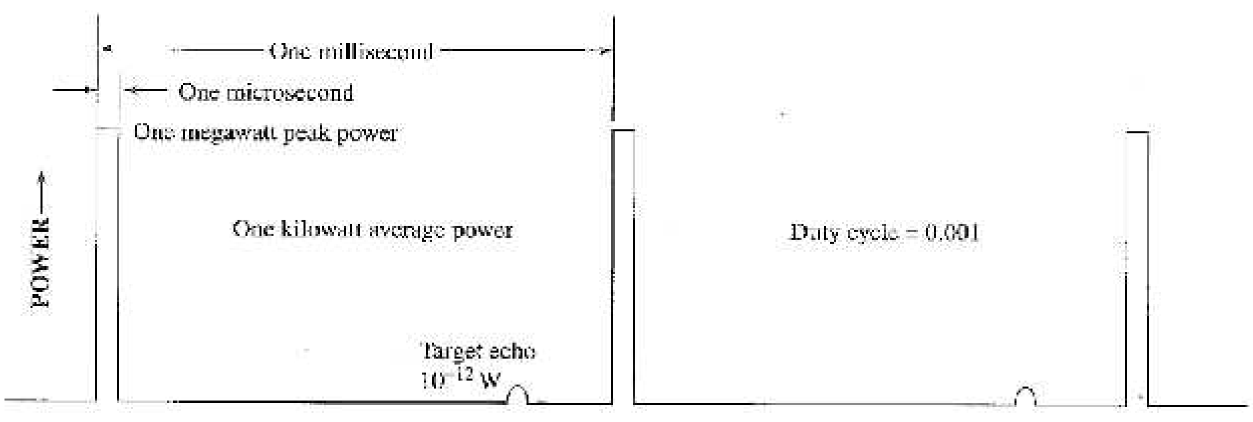


Fig.2.4 Example of typical pulse waveform for medium range air surveillance radar

CHAPTER: 3 RADAR RANGE EQUATION

3.1 Introduction:-

The radar range relates the radar range with the characteristics of transmitter, receiver antenna, target and environment. The radar range equation is useful to understand the maximum range of the radar that can be detected by the radar with their performance parameters. One of the simpler equations of radar theory is the radar range equation.

3.2 BASIC RADAR RANGE EQUATIONS

The transmitted power P_t is radiated by an isotropic antenna, the power density at distance R can be given as:

$$\text{Power demnsity at range R from an isotropic antenna} = \frac{P_t}{4\pi R^2} \text{ (Watt/square meter)} \quad \dots (3.1)$$

The maximum gain of the antenna can be defined as:

$$G = \frac{\text{max power density radiated byan antenna}}{\text{power density radiated by a lossless isotropic antenna}} \quad \dots (3.2)$$

Thus the power density at target from a directive antenna can be given as:

$$\text{Power density at range R from } a \text{ directive antenna} = \frac{P_t G}{4\pi R^2} \quad \dots (3.3)$$

The target receives a portion of the incident energy and reflected it in various directions. Thus the radar cross section of the target determines the power density returned back to the radar.
The reflected power from the target through its cross section (target cross section) can be given as:

$$\text{Reflected power from the target towards the radar} = \frac{P_t G}{4\pi R^2} \bullet \frac{\sigma}{4\pi R^2} \quad \dots (3.4)$$

The radar antenna receives a portion of the reflected power from the target cross section. the received power can be given as:

$$P_r = \frac{P_t G}{4\pi R^2} \bullet \frac{\sigma}{4\pi R^2} \bullet A_e \quad \dots (3.5)$$

$$A_e = \rho_a \bullet A \qquad \dots (3.6)$$

Where A_e is the effective area of the receiving antenna, A is the physical antenna area and ρ_a is the antenna aperture efficiency. The maximum range of the radar (Rmax) can be defined as the maximum distance beyond which radar cannot detect the target. So the received signal power can be given as the minimum detectable signal.

$$S_{\min} = \frac{P_t G}{4\pi R^2} \bullet \frac{\sigma}{4\pi R_{\max}{}^2} \bullet A_e \qquad \dots (3.7)$$

$$R_{\max} = \left[\frac{P_t G}{4\pi} \bullet \frac{\sigma}{4\pi} \bullet \frac{A_e}{S_{\min}} \right]^{1/4} \qquad \dots (3.8)$$

This is the fundamental form of radar range equation. If the antenna is used for both the transmission and receiving purpose, then the transmitted gain (G) can be given in terms of the effective area (A_e).

$$G = \frac{4\pi A_e}{\lambda^2} \qquad \dots (3.9)$$

Now the maximum radar range can be given as follows.

$$R_{\max} = \left[\frac{P_t G^2 \lambda}{(4\pi)^3} \bullet \sigma \bullet \frac{A_e}{S_{\min}} \right]^{1/4} \quad \text{(When G is constant)} \qquad \dots (3.10)$$

$$R_{\max} = \left[\frac{P_t}{(4\pi)^3} \bullet \sigma \bullet \frac{A_e{}^2}{S_{\min}} \right]^{1/4} \quad \text{(When } A_e \text{ is constant)} \qquad \dots (3.11)$$

These three forms of radar range equations [2.8, 2.10 and 2.11] are based on the effective area (A_e) and transmitter antenna gain (G).

3.3 Radar Block Diagram

The operation of a typical pulse radar may be described with the aid of the block diagram shown in Fig. 1.2. The transmitter may be an oscillator. such as a magnetron. that is "pulsed" (turned on and off) by the modulator to generate a repetitive train of pulses. The magnetron has prohahly heen the most widely used of the various microwave generators for radar. A typical radar for the detection of aircraft at ranges of 100 or 200 nmi might employ a peak power of the order of a megawatt. an average power of several kilowatts, a pulse width of several microseconds. and a

pulse repetition frequency of several hundred pulses per second. The waveform generated by the transmitter travels via a transmission line to the antenna.
where it is radiated into space.

A single antenna is generally used for both transmitting and receiving. The receiver must be protected from damage caused by the high power of the transmitter. This is the function of the duplexer. The receiver is usually of the superheterodyne type. The first stage might be a low-noise RF amplifier. such as a parametric amplifier or a low-noise transistor. However. it is not always desirable to employ a low-noise first stage in radar.

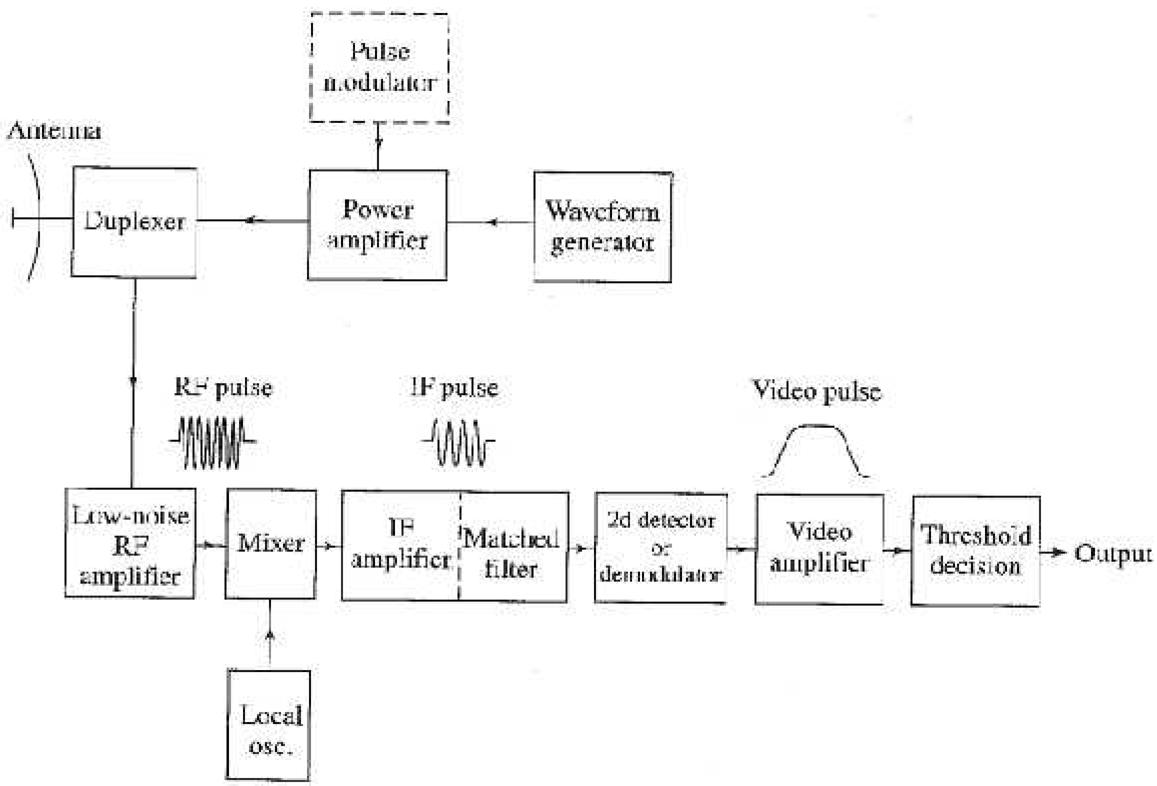


Fig. 3.1 Radar Block Diagram

The mixer and local oscillator (LO) convert the RF signal to an intermediate frequency (IF). A "typical" IF amplifier for an air-surveillance radar might have a center frequency of 30 or 60 MHz and a bandwidth of the order of one megahertz.

The IF amplifier should be designed as a matchted filter; i.e., its frequency-response function H(f) should maximize the peak-signal-to-mean-noise-power ratio at the output.

After maximizing the signal-to-noise ratio in the IF amplifier, the pulse modulation is extracted by the second detector and amplified by the video amplifier to a level where it can be properly displayed, usually on a cathode-ray tube (CRT). Timing signals are also supplied to the indicator to provide the range zero. Angle information is obtained from the pointing direction of the antenna.

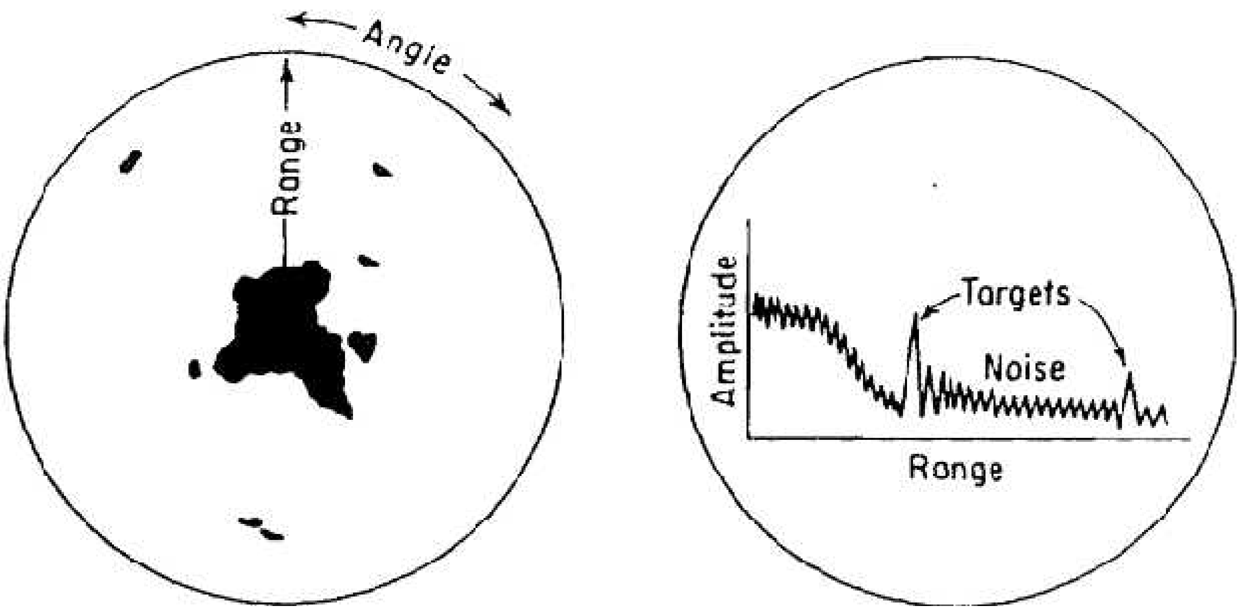


Fig. 3.2 (a) PPI presentation displaying range vs. angle (intensity modulation); (b) A scope presentation displaying amplitude vs. range (deflection modulation).

A common form of radar antenna is a reflector with a parabolic shape, fed (illuminated) from a point source at its focus. The parabolic reflector focuses the energy into a narrow beam, just as does a searchlight or an automobile headlamp. The beam may be scanned in space by mechanical pointing of the antenna. Phased-array antennas have also been used for radar. In a phased array the beam is scanned by electronically varying the phase of the currents across the aperture.

3.3 Radar's Electromagnetic Spectrum

Conventional radars generally have been operated at frequencies extending from about 220 MHz to 35 GHz, a spread of more than seven octaves. These are not necessarily the limits, since radars can be, and have been, operated at frequencies outside either end of this range.Skywave HF over-the-horizon (OTH) radar might be at frequencies as low as 4 or 5 MHz, and Groundwave HF radars as low as 2 MHz. At the other end of the spectrum, millimeter radars Have operated at 94 GHz. Laser radars operate at even higher frequencies.

The place of radar frequencies in the electromagnetic spectrum is shown in Fig. 3.3. Some of the nomenclature employed to designate the various frequency regions is also shown. Early in the development of radar, a letter code such as S, X, L, etc., was employed to Designate radar frequency bands. Although its original purpose was to guard military secrecy, the designations were maintained, probably out of habit as well as the need for some convenient short nomenclature. This usage has continued and is now an accepted practice of radar engineers.

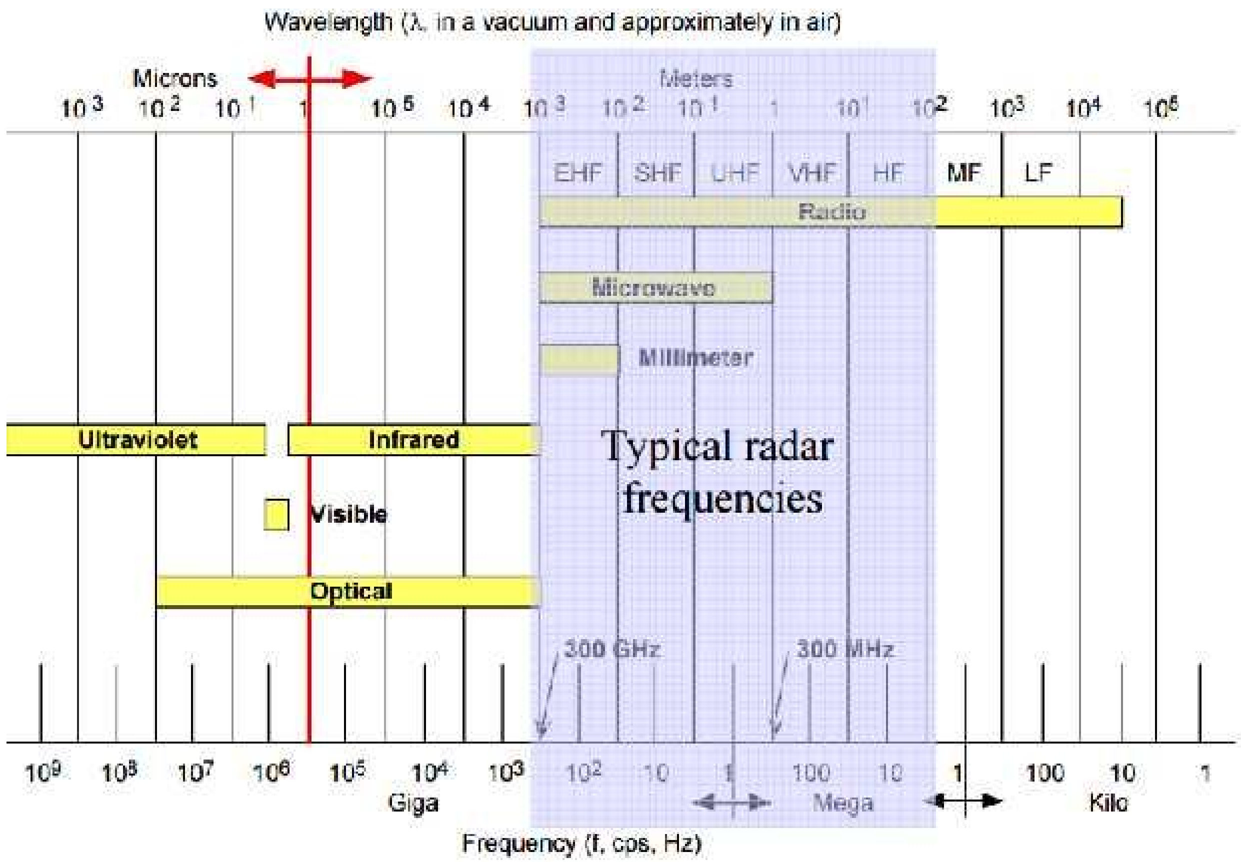


Fig. 3.3 Frequency spectrum for radar frequencies

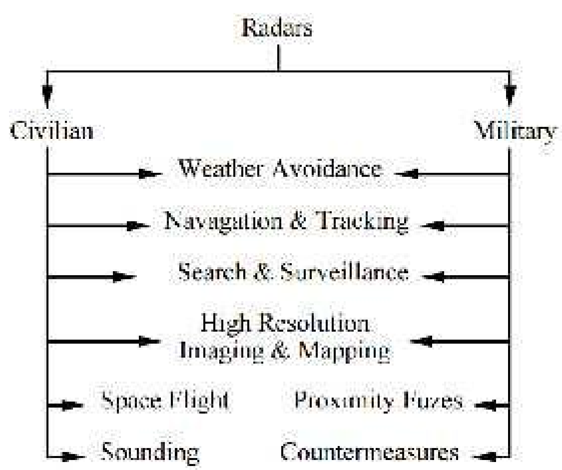
Table 3.1 lists the radar-frequency letter-band nomenclature adopted by the IEEE. These are related to the specific bands assigned by the International Telecommunications Union for radar. For example, although the nominal frequency range for L band is 1000 to 2000 MHz, an L-band radar is thought of as being confined within the region from 1215 to 1400 MHz since that is the extent of the assigned band.

Table 3.1 Radar Bands and their Usage

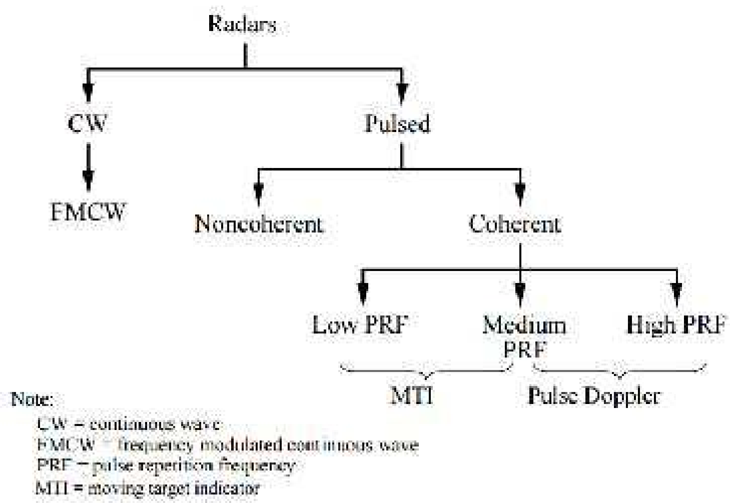
Band Designation	Frequency Range	Usage
HF	3–30 MHz	OTH surveillance
VHF	30–300 MHz	Very-long-range surveillance
UHF	300–1,000 MHz	Very-long-range surveillance
L	1–2 GHz	Long-range surveillance
		En route traffic control
S	2–4 GHz	Moderate-range surveillance
		Terminal traffic control
		Long-range weather
C	4–8 GHz	Long-range tracking
		Airborne weather detection
X	8–12 GHz	Short-range tracking
		Missile guidance
		Mapping, marine radar
		Airborne intercept
K_u	12–18 GHz	High-resolution mapping
		Satellite altimetry
K	18–27 GHz	Little use (water vapor)
K_a	27–40 GHz	Very-high-resolution mapping
		Airport surveillance
millimeter	40–100+ GHz	Experimental

3.4 Radar classification

Radar can be classified based on the function and the waveforms



(a)



(b)

Fig. 3.3 Radar can be classified based on the (a) function and (b) waveforms

In practice, however, the simple radar equation does not predict the range performance of actual radar equipments to a satisfactory degree of accuracy. The predicted values of radar range are usually optimistic. In some cases the actual range might be only half that predicted. Part of this discrepancy is due to the failure of Eq. (3.10) to explicitly include the various losses that can occur throughout the system or the loss in performance usually experienced when electronic equipment is operated in the field rather than under laboratory-type conditions &.another important factor that must be considered in the radar equation is the statistical or unpredictable nature of several of the parameters. The minimum detectable signal Smin and the target cross section (σ) are both statistical in nature and must be expressed in statistical terms.

3.5 MINIMUM DETECTABLE SIGNAL

The ability of a radar receiver to detect a weak echo signal is limited by the noise energy that occupies the same portion of the frequency spectrum as does' the signal energy. The weakest signal the receiver can detect is called. the minimum detectable signal. The specification of the minimum detectable signal is sometimes difficult because of its statistical nature and because the criterion for deciding whether a target is present or not may not be too well defined.

Detection is based on establishing a threshold level at the output of the receiver. If the Receiver output exceeds the threshold, a signal is assumed to be present. This is called threshold detection.

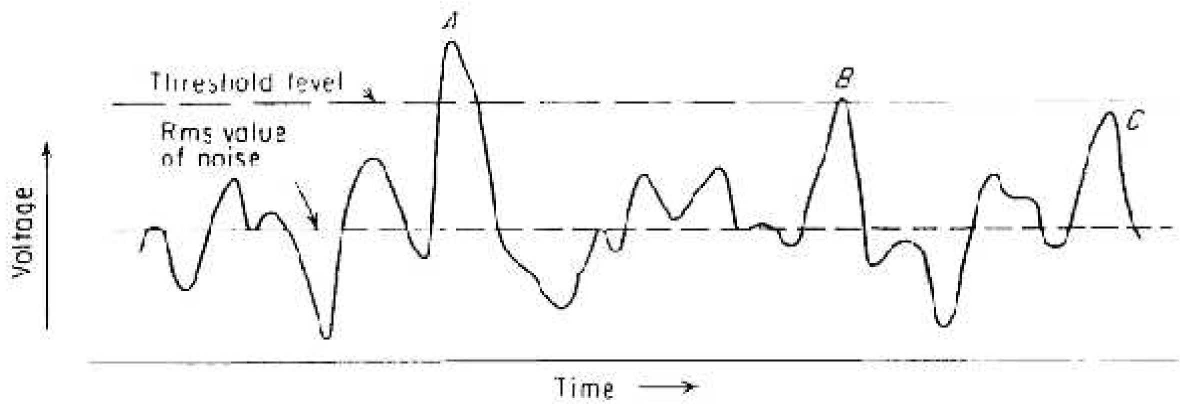


Fig. 3.4 Typical envelope of the radar receiver output as a function of time. A, and B, and C represent signal plus noise. A and B would be valid detections, but C is a missed detection.

A target is said to be detected if the envelope crosses the threshold. if the signal is large such as at A, it is not difficult to decide that a target is present. But consider the two signals at B and C, representing target echoes of equal amplitude. The noise voltage accompanying the signal at B is large enough so that the combination of signal plus noise exceeds the threshold.

Weak signals such as C would not be lost if the threshold level were lower. But too low a threshold increases the likelihood that noise alone will rise above the threshold and be taken for a real signal. Such an occurrence is called a false alarm.

CHAPTER 4: CONTINUOUS WAVE AND FREQUENCY MODULATED RADAR

4.1 THE DOPPLER EFFECT

It is well known in the fields of optics and acoustics that if either the source of oscillation or the observer of the oscillation is in motion, an apparent shift in frequency will result. This is the doppler effect and is the basis of CW radar.

If R is the distance from the radar to target, the total number of wavelengths (λ) contained in the two-way path between the radar and the target is 2R/λ. The distance R and the wavelength (λ), are assumed to be measured in the same units. Since one wavelength corresponds to an angular excursion of 2π radians, the total angular excursion ϕ made by the electromagnetic wave during its transit to and from the target is $4\pi R / \lambda$.

If target is in motion the range R and phase φ is continually changing. Thus the change in phase with respect to time can be given as frequency.

$$\frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} \qquad \ldots (1)$$

Range with respect to time can be defined as the radial velocity of the target. Thus the Doppler angular frequency can be given as:

$$\omega_d = 2\pi f_d = \frac{4\pi}{\lambda} v_r \qquad \ldots (2)$$

Where f_d is Doppler frequency and v_r is the radial velocity of the target with respect to radar. The Doppler frequency can be related with transmitter frequency f_0.

$$f_d = \frac{2v_r}{\lambda} = \frac{2v_r f_0}{c} \qquad \ldots (3)$$

When v_r is given in knots then the Doppler frequency can be given as:

$$f_d = \frac{1.03 v_r\,(knots)}{\lambda(m)} \qquad \ldots (4)$$

The relative velocity may be written $v_r = v\cos\theta$ where v is the target speed and θis the Angle made by the target trajectory and the line joining radar and target. When θ = 0, the doppler frequency is maximum. The doppler is zero when the trajectory is perpendicular to the radar line of sight (θ= 90°).

A plot of doppler frequency shifts as a function of radial velocity and the radar frequency bands is given in fig. 4.2. This figure illustrates that as the target radial velocity get increases the Doppler frequency shifts get increases with higher radar frequencies.

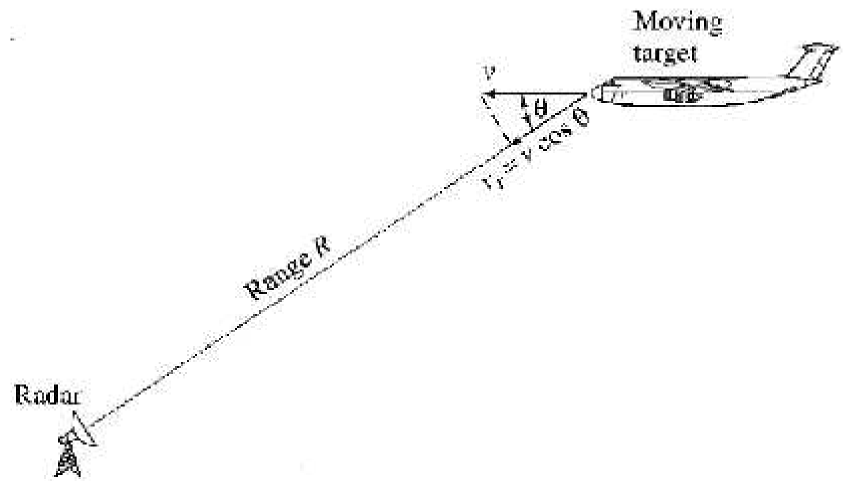


Fig. 4.1 Geometry of Radar and target in deriving the Doppler shifts

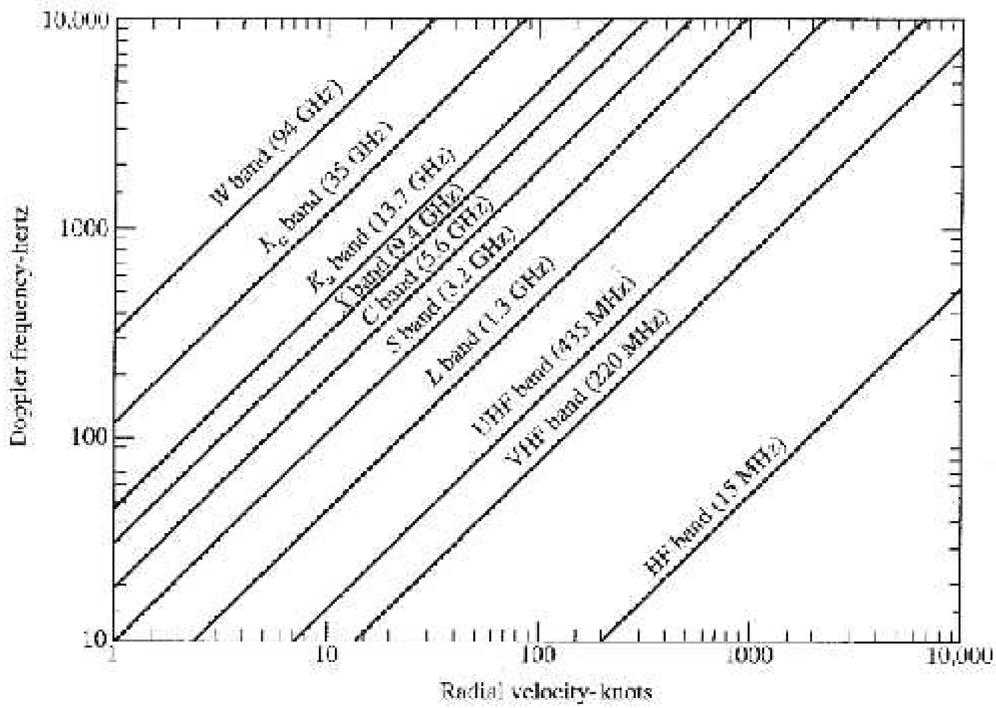


Fig. 4.2 Doppler frequency shifts for a moving target as a function of v_r and radar frequency band.

4.2 Continuous Wave Radar (CW Radar):-

A block diagram of simple CW radar is shown in Fig. 4.3. The transmitter generates a continuous (unmodulated) oscillation of frequency f_0, which is radiated by the antenna. A portion of the radiated energy is intercepted by the target and is scattered, some of it in the direction of the radar, where it is collected by the receiving antenna.

If the target is in motion with a velocity v_r relative to the radar, the received signal will be shifted in frequency from the transmitted frequency f_0 by an amount $\pm f_d$ as given by Eq. (4).

- The plus sign associated with the doppler frequency applies if the distance between target and radar is decreasing (closing target), that is, when the received signal frequency is greater than the transmitted signal frequency.
- The minus sign applies if the distance is increasing (receding target).

The received echo signal at a frequency $f_0 \pm f_d$ enters the radar via the antenna and is heterodyned in the detector (mixer) with a portion of the transmitter signal/o to produce a doppler beat note of frequency f_d The sign f_d is lost in this process.

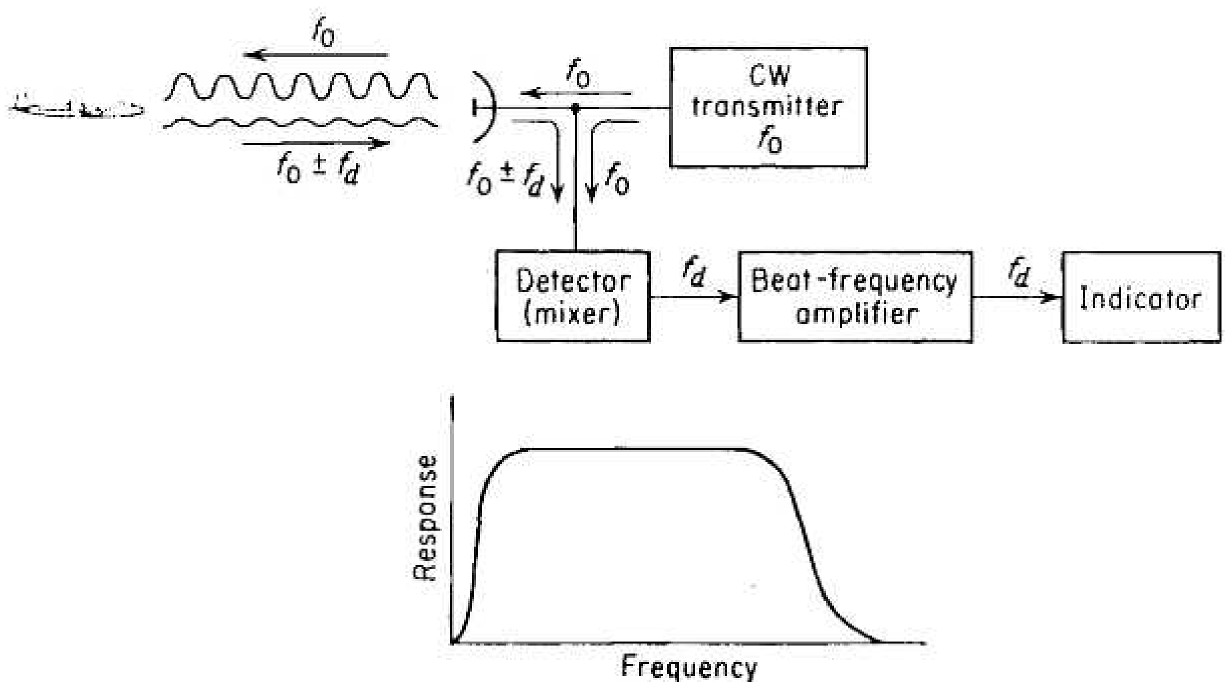


Fig. 4.3 CW Radar with frequency response

Pulse Radar: Pulse radar that extracts the Doppler frequency-shifted echo signal. A simple way to convert the CW radar to the pulse radar by turning on and off CW oscillator to generate pulses. This way of generation of pulses removes the reference signal, which is required to recognize the Doppler shifts. One way to introduce the reference signal is shown in fig. 4.4. Here the power amplifier is turned on and off to generate the high power pulses. The received echo signal is mixed with the output of CW oscillator, which acts as coherent reference to allow the recognition of any change in the frequency. Here coherent means that the transmitted pulses are synchronously used as reference signal. The change in frequency is detected through Doppler filter.

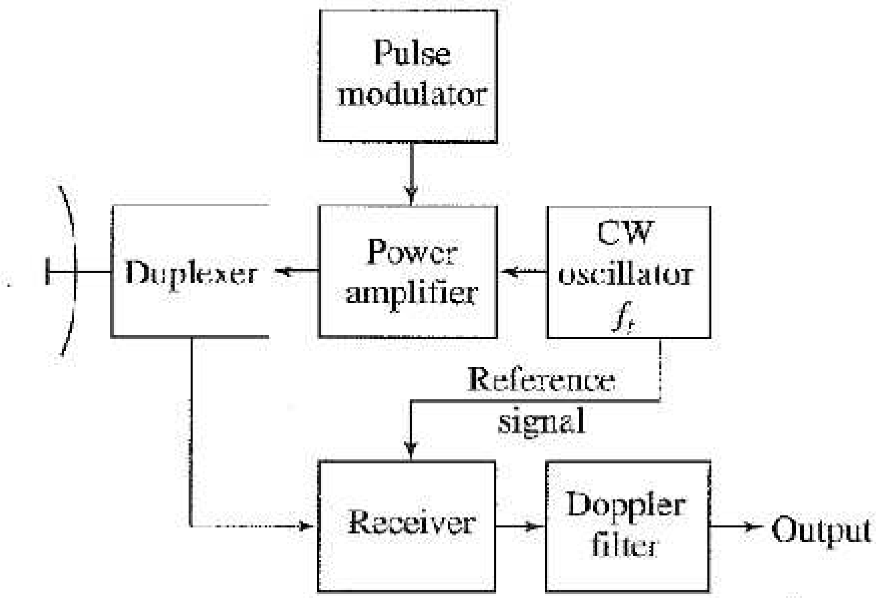


Fig. 4.4 Block diagram of simple Pulse Radar

Sweep to sweep subtraction:

The bipolar video (signal has positive and negative values) from two successive sweeps of MTI radar is shown in fig. 4.5. If one sweep is subtracted from the previous sweep, fixed clutter echoes will get cancel, and will not detected. On the other hand, moving target change its amplitude from sweep to sweep due the Doppler frequency shift. If one sweep is subtracted from other, the result will be canceled residue as shown in fig. 3.5.

Subtraction of the echoes from two successive sweeps is accomplished in delay line cancellers as shown in fig. 4.6. The delay-line canceller acts as a filter to eliminate the doc component of fixed targets and to pass the a-c components of moving targets. The video portion of the receiver is divided into two channels. One is a normal video channel. In the other, the video signal experiences a time delay equal to one pulse-repetition period (equal to the reciprocal of the pulse-repetition frequency). The outputs from the two channels are subtracted from one another. The fixed targets with unchanging amplitudes from pulse to pulse are canceled on subtraction.

However, the amplitudes of the moving-target echoes arc not constant from pulse 10 pulse, and subtraction results in an uncanceled residue.

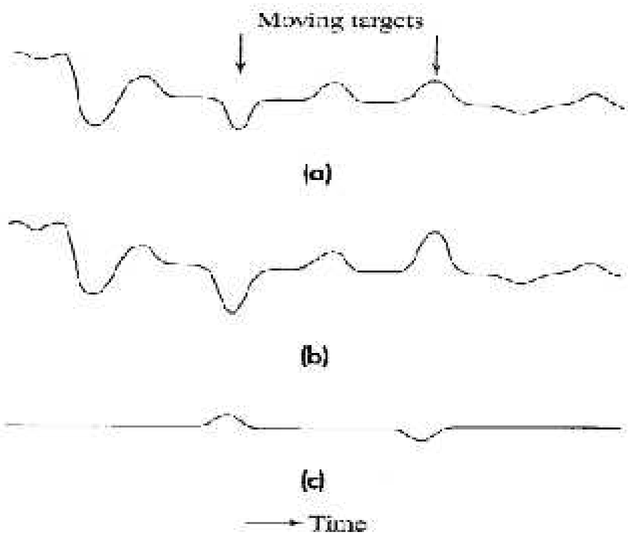


Fig. 4.5 Sweep to sweep subtraction

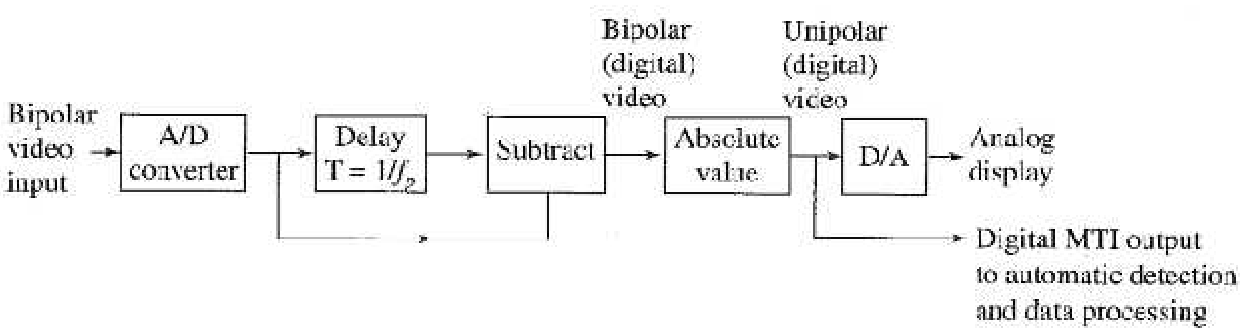


Fig. 4.6 Block diagram of single delay line canceller

MTI Radar Block Diagram:-

The doppler frequency shift [Eq. (3.2)] produced by a moving target may be used in a pulse radar. just as in the CW radar discussed in Chap. 3, to determine the relative velocity of a target or to separate desired moving targets from undesired stationary objects (clutter). Such a pulse radar that utilizes the doppler frequency shift as a means for discriminating moving from fixed targets is called an MTI (moving target indication) or a pulse doppler radar.

The block diagram of a more common MTI radar employing a power amplifier is shown in Fig. 4.5. The significant difference between this MTI configuration is the manner in which the reference signal is generated. In Fig. 4.7, the coherent reference is supplied by an oscillator called the coho, which stands for coherent oscillator.

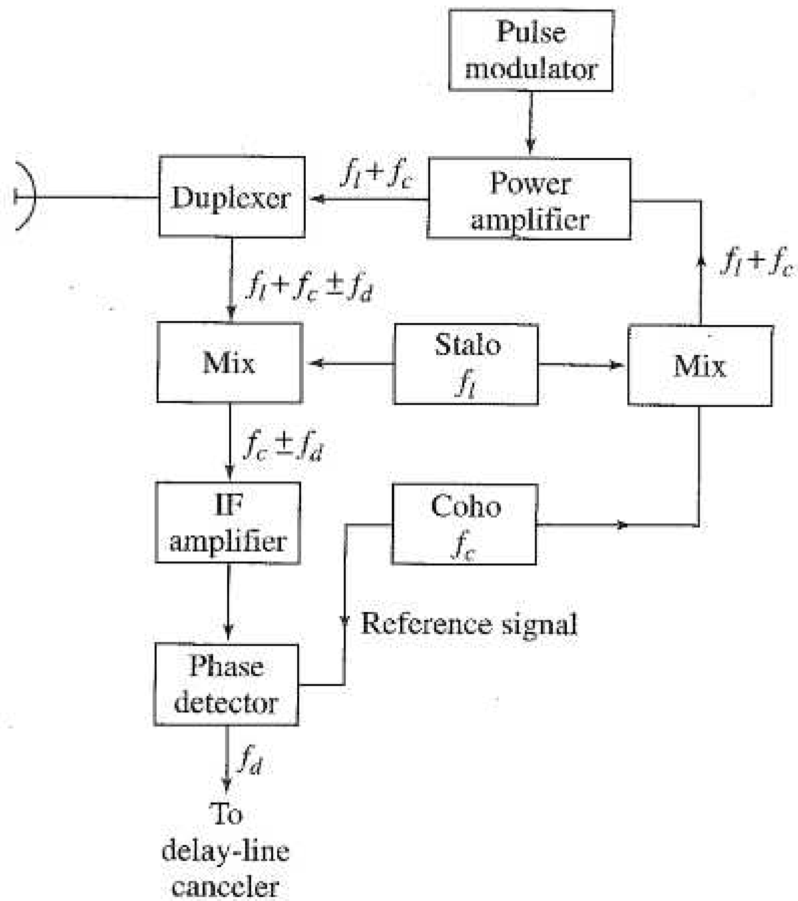


Fig. 4.7 Block diagram of MTI radar with power-amplifier transmitter

- The coho is a stable oscillator whose frequency is the same as the intermediate frequency used in the receiver. In addition to providing the refcrence signal. the output of the coho. fc is also mixed with the local-oscillator frequency fl.

- The local oscillator must also be a stable oscillator and is called stalo, for stable local oscillator.

- The stalo, coho, and the mixer in which they are combined plus any low-level amplification are called the receiver-exciter because of the dual role they serve in both the receiver and the transmitter.

- The characteristic feature of coherent MTI radar is that the transmitted signal must be coherent (in phase) with the reference signal in the receiver.

- The reference signal from the coho and the I F echo signal are both fed into a mixer called the phase detector. The phase detector differs from the normal amplitude detector since its output is proportional to the phase difference between the two input signals.

Delay Line Canceller:-

The simple MTI delay-line canceller shown in Fig. 4.6 is an example of a time-domain filter.The capability of this device depends on the quality of the medium used as the delay line. Thedelay line must introduce a time delay equal to the pulse repetition interval. For typical ground-based air-surveillance radars this might be several milliseconds. Delay times of this magnitude cannot be· achieved with practical electromagnetic transmission lines. By converting the electromagnetic signal to an acoustic signal it is possible to utilize delay lines of a delay line must introduce a time delay equal.

One of the advantages of a time-domain delay-line canceler as compared to the more conventional frequency-domain filter is that a single network operates at all ranges and does not require a separate filter for each range resolution cell. Frequency-domain doppler filter banks are of interest in some forms of MTI and pulse-doppler radar.

Frequency Response of Delay Line canceller

The delay-line canceler acts as a filter which rejects the d-c component of clutter. Because of its periodic nature, the filter also rejects energy in the vicinity of the pulse repetition frequency and its harmonics.

The signal from a target at range R_0, the output of the phase detector can be given as:

$$V_1 = k\sin(2\pi f_d t - \phi_0) \qquad \dots (5)$$

Where f_d is Doppler frequency, ϕ_0 constant phase of $4\pi R_0 / \lambda$. The signal from the previous radar transmission is similar, which is delayed by time T_P

$$V_2 = k\sin[2\pi f_d (t - T_P) - \phi_0] \qquad \dots (6)$$

Everything else is assumed to remain essentially constant over the interval T_P so that k is the same for both pulses. The output from the subtractor is

$$V = V_1 - V_2 = 2k\sin(\pi f_d T_P)\cos\left[2\pi f_d (t - \frac{T_P}{2}) - \phi_0\right] \qquad \dots. (7)$$

The magnitude of the relative frequency-response of the delay-line canceler [ratio of the amplitude of the output from the delay-line canceler, $2k\sin(\pi f_d T_P)$, to the amplitude of the normal radar video k] is shown in Fig. 4.8.

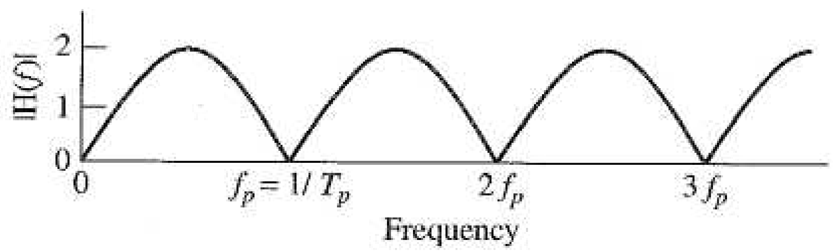


Fig. 4.8 Frequency response of the single delay-line canceller; T = delay time $= 1/f_p$

Blind Speed:-

The response of the single-delay-line canceler will be zero whenever the argument $\pi f_d T_P$ in the amplitude factor of Eq. (7) is 0, π, 2 π, ... , etc., or when

$$f_d = \frac{2V_r}{\lambda} = \frac{n}{T_P} = nf_p \qquad n = 0,1,2,3,\dots\dots \qquad \dots (8)$$

The delay-line cancela not only eliminates the d-c component caused by clutter (n = 0), but unfortunately it also rejects any moving target whose doppler frequency happens to be the same as the prf or a multiple

there of. Those relative target velocities which result in zero MTI response are called blind speed and can be given as:

$$v_n = \frac{n\lambda}{2T_p} = \frac{n\lambda f_p}{2} \qquad n = 0,1,2,3,\dots\dots \qquad \dots (9)$$

where v_n is the nth blind speed. If λ is measured in meters,fp in Hz, and the relative velocity in knots, the blind speeds are

$$v_n = \frac{n\lambda f_p}{1.02} \approx n\lambda f_p \qquad \dots (10)$$

The blind speeds are one of the limitations of pulse MTI radar which do not occur with CW radar. They are present in pulse radar because doppler is measured by discrete samples (pulses) at the prf rather than continuously.

Pulse Doppler Radar:-

A pulse radar that extracts the doppler frequency shift for the purpose of detecting moving targets in the presence of clutter is either an MTI radar or a pulse doppler radar.

The distinction between them is based on the fact that in a sampled measurement system like a pulse radar, ambiguities can arise in both the doppler frequency (relative velocity) and the range (time delay) measurements. Range ambiguities are avoided with a low sampling rate (low pulse repetition frequency), and doppler frequency ambiguities are avoided with a high sampling rate. However, in most radar applications the sampling rate, or pulse repetition frequency, cannot be selected to avoid both types of measurement ambiguities.

The pulse doppler radar is more likely to use range-gated doppler filter-banks than delay-line cancelers. Also, a power amplifier such as a klystron is more likely to be used than a delay-line cancelers. A pulse doppler radar operates at a higher duty cycle than does an MTI. Although it is difficult to generalize, the MTI radar seems to be the more widely used of the two, but pulse doppler is usually more capable of reducing clutter. .

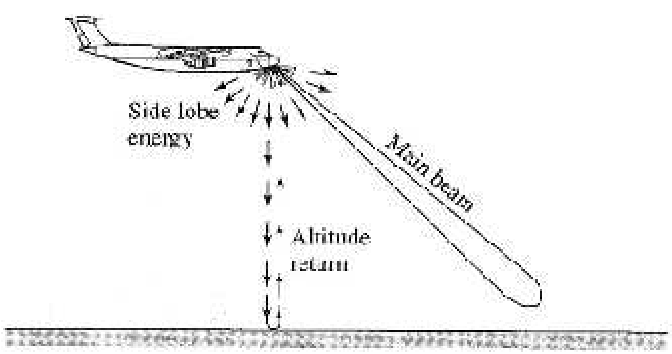


Fig. 4.9 Sketch of airborne Pulse Doppler radar

- A radar that increases its prf high enough to avoid the problems of blind speeds is called as Pulse radar.
- A high-prf pulase Doppler radar is one with no blind speeds with in the Doppler space.
- A medium-prf pulase Doppler radar is one get operated at slightly lower prf and accepts both range and Doppler ambiguities.
- A brief comparison between different Doppler pulse radar is given in table 4.1

Table. 4.1:- Comparison of different pulse Doppler radar

Radar	prf*	Duty Cycle*
X-band high-prf pulse doppler	100–300 kHz	< 0.5
X-band medium-prf pulse doppler	10–30 kHz	0.05
X-band low-prf pulse radar	1–3 kHz	0.005
UHF low-prf AMTI	300 Hz	Low

References

1- www.wikipedia.com

2- Introduction to Radar Systems by Merrill I. Skolnik, 3rd Edition, PHI Publications.